The Optimization of Fast-Cycling 4T Superconducting $Cos\theta$ Style Dipole Magnet

P.G. Akishin
Laboratory of Information Technologies, JINR
A.V. Butenko, A.D. Kovalenko, V.A. Mikhaylov
Veksler and Baldin Laboratory of High Energies, JINR

A concept of the fast-cycling 4 T superconducting $Cos\theta$ style dipole based on a 12-16 turns coil made from a hollow NbTi cable was first presented at EUCAS2001 [1]. During the last years, research work on improvement the dipole have been carried out by the JINR-GSI collaboration under partial supportprovided by the Russian Foundation for Basic Research (grant 03-01-00290). The results of optimization of the magnetic field in the magnet aperture were presented at EUCAS 2003 [2], EPAC 2004 [3], EUCAS 2005 [4]. In this note we describe basic mathematical methods that were used for the magnetic field optimization of a low turn winding dipole with a circular aperture. One fourth part of the typical dipole magnet cross section is shown in Fig. 1.

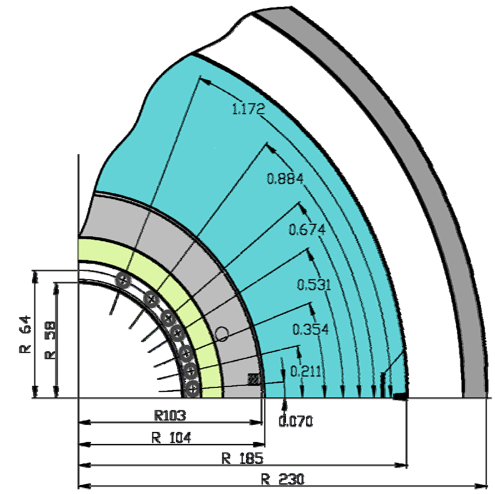

Fig.1 Cross sections of 1/4 rapid cycling dipole magnet

At first we examine the optimization problem of the two-dimensional magnetic field distribution with the aid of the variation of the current winding position , without changing of the iron screen configuration. The basic purpose consists in obtaining a field with assigned uniformity in the operating region of the magnet, which has a form the circle of radius r. We suppose that there are N current turns, whose centers are located on the circle of radius R. The position of each i-winding is characterized by angle ϕ_i. We will solve the optimization problem by variation of the angles ϕ_i. The expansion of the magnetic field components in Fourier series is one of the most convenient methods for estimation of the field distribution quality. The complexity of the task consists in the fact that it is necessary to obtain the field uniformity for a broad range of current change. The constant permeability models work quite well for low level field. It is possible to use a constant (sufficiently high) value of magnetic permeability μ in iron yoke for low current in the windings. The advantage of this model over the others consists in the possibility to obtain a magnetic field as a linear vector function of the winding current This fact allows one to reduce the computational time at the optimization of the magnetic field substantially. Let G be a region filled with iron, $\mathbf{B}(\mathbf{x})$ - induction of magnetic field, $\mathbf{H}(\mathbf{x})$ tension, $\mathbf{M}(\mathbf{x})$ - magnetic moment, $\mu = \mu(x)$ - magnetic permeability, $\mathbf{H}^S(\mathbf{x})$ - field of the current elements at the point$\mathbf{x}$. The integral equation describing the problem in the two-dimensional space is

$$\mathbf{H}(\mathbf{a}) = \mathbf{H}^s(\mathbf{a}) + \frac{1}{2\pi}\nabla_{\mathbf{a}} \int\limits_G (\mathbf{M}(\mathbf{x}), \nabla_x \ln|\mathbf{x}-\mathbf{a}|)\, dS_{\mathbf{x}}, \tag{1}$$

The vectors $\mathbf{B}(\mathbf{x})$, $\mathbf{H}(\mathbf{x})$ and $\mathbf{M}(\mathbf{x})$ are connected by the next relations

$$\mathbf{H}(\mathbf{a}) = \frac{\mathbf{B}(\mathbf{a})}{\mu\left(|\mathbf{B}(\mathbf{a})|\right)\mu_0}, \qquad \mathbf{M}(\mathbf{a}) = \frac{\mathbf{B}(\mathbf{a})}{\mu_0} - \mathbf{H}(\mathbf{a}) \tag{2}$$

The magnetic field $\mathbf{H}^S(\mathbf{a})$ from current sources in (1) is determined by Biot-Savarr law

$$\mathbf{H}^s(\mathbf{a}) = \frac{1}{2\pi}\sum_{i=1}^{N}\int\limits_{\Omega_i} [\nabla_{\mathbf{x}} \ln|\mathbf{x}-\mathbf{a}| \times \mathbf{e_0} j_i^s(\mathbf{x})]\, dS_{\mathbf{x}}. \tag{3}$$

where $\{\Omega_i, i = \overline{1,N}\}$ are current windings;$\mathbf{e_0}$ is a vector, orthogonal to plane; $j_i^s(\mathbf{a})$ is a current density in i-th winding. In the case of constant magnetic permeability, equation (1) is transformed to the boundary integral equation

$$\mathbf{H}(\mathbf{a}) = \mathbf{H}^S(\mathbf{a}) + \frac{1}{2\pi}\nabla_{\mathbf{a}} \oint\limits_{DG} (\mathbf{M}(\mathbf{x}), \mathbf{n}_{\mathbf{x}}) \log|\mathbf{x}-\mathbf{a}| dl_{\mathbf{x}} \quad . \tag{4}$$

Let for $\mathbf{x} \in DG$ the function $\sigma(\mathbf{x})$ be $\sigma(\mathbf{x}) = (\mathbf{B}(\mathbf{x}), \mathbf{n}_{\mathbf{x}})$. Then from (2) and (4) we obtain

$$\frac{\sigma(\mathbf{a})}{\mu} = \mu_0(\sum_{i=1}^{N} \mathbf{H}_i^S(\mathbf{a}), \mathbf{n}_{\mathbf{a}}) + \frac{1}{2\pi}(1-\frac{1}{\mu})(\mathbf{n}_{\mathbf{a}}, \nabla_{\mathbf{a}} \oint\limits_{DG} \sigma(\mathbf{x})(\mathbf{x}) \log|\mathbf{x}-\mathbf{a}| dl_{\mathbf{x}}) \quad . \tag{5}$$

For a constant magnetic permeability, μ the field from several windings can be obtained as a sum of the fields from each winding individually. An obvious consequence of this fact is the possibility to calculate the Fourier coefficients for the magnetic system as a sum of Fourier coefficients for each winding individually. Let us examine the procedure of the optimization of the field distribution. One can designate $\{f_i, i = \overline{0,L}\}$ and $\{g_i, i = \overline{0,L}\}$ as the first $(L+1)$- harmonic of the component of magnetic field B_x and B_y, respectively. The functional F, describing the magnetic field homogenieity, we define as

$$F = F(\phi_1, \phi_2, \ldots, \phi_N) = \sum_{i=0}^{L}\left(\frac{f_i}{g_0}\right)^2 + \sum_{j=1}^{L}\left(\frac{g_j}{g_0}\right)^2 \quad . \tag{6}$$

The gradient descent method was used for minimization of this functional. It should be noted that from nonlinearity of the solved problem the uniformity of the field with the low values does not guarantee this one for a big field level. Therefore for the selected configuration of windings, the 2D integral equation (1) was used to analyze the effects of saturation of iron for big currents. The first results in optimization of the magnetic field in the magnet aperture were presented in [2], [3]. There was an attempt to suppress higher harmonics of the magnetic field in the aperture by means of a proper angular distribution of the coil turns only. A mathematical model and a special computer code were developed and used for that purpose [5]. The best obtained result was the following: the relative nonlinearity of the field of B_l/B_1 was about $1 * 10^{-3}$ within 75% of the magnet aperture at B = 4 T.

The further improvement of the magnetic field quality was limited by an insufficient flexibility in choosing the coil turns angular position and the large dynamic range of the operating field values as well. The next improvement step consisted in including a variation of the inner shape of the ferromagnetic boundary in the optimization process. Additional fictitious current windings were introduced for imitation of the yoke. The details of the approach are shown in [6]. The total field $\mathbf{H}(\mathbf{x})$ may be expressed through the gradient of the vector-potential $u(\mathbf{a})$:

$$u(\mathbf{a}) = \sum_{i=1}^{M} \frac{j_i^s}{2} \ln \left[\frac{((a_1 - x_1^i)^2 + (a_2 - x_2^i)^2) \times ((a_1 - x_1^i)^2 + (a_2 + x_2^i)^2)}{((a_1 + x_1^i)^2 + (a_2 - x_2^i)^2) \times ((a_1 + x_1^i)^2 + (a_2 + x_2^i)^2)} \right], \tag{7}$$

where (x_i, y_i)- the coordinates of winding. The optimization inner shape problem will be solved if the normal derivative of the total vector potential $u(\mathbf{a})$ on a boundary contour equals zero. The result of the optimization are shown in Fig. 2.

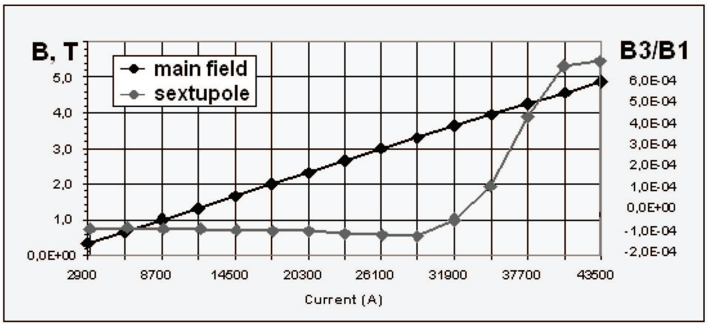

Fig. 2. The dependencies of the gap magnetic field and the field sextupole component on supply current.

The sextupole nonlinearity was suppressed to about $1 * 10^{-4}$ of the main field within 75% of the magnet aperture over the dynamic range from 0.5 T to 4 T. On the Figs. 3, 4 one can see the topography of vector potential $u(\mathbf{a})$, the placement of windings and internal profile of the dipole magnet.

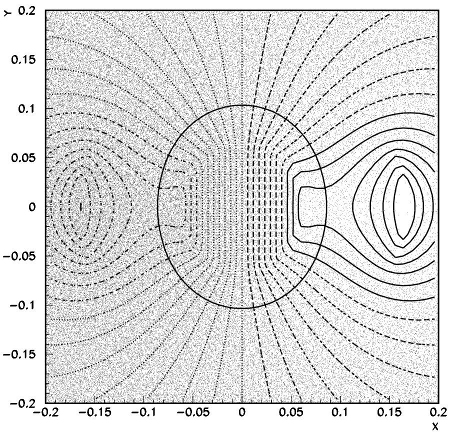

Fig. 3.

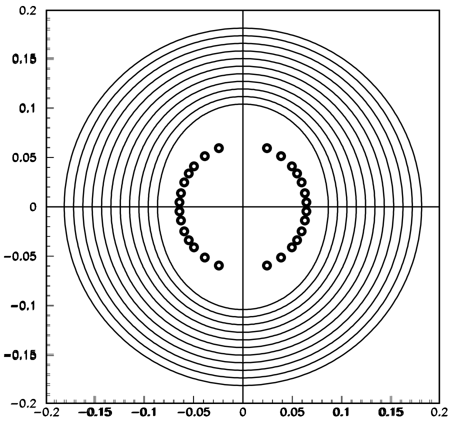

Fig. 4.

The proposed mathematical methods can be used for a wide class of the magnetic field optimization problems.

References

[1] Kovalenko A., Agapov N., Khodzhibagiyan H., Moritz G. Fast cycling superconducting magnets: new design for ion synchrotrons 2002 Physica **C** 2002,**V**.372-376, pp.1394–1397.

[2] A.Kovalenko, N.Agapov, P.Akishin, A.Butenko, H.Khodzhibagiyan, V.Mikhaylov, G.Moritz, E.Fisher. Fast cycling 4T dipole magnet based on a hollow high current superconducting NbTi cable . Proceedings of 6th European Conference on Applied Superconductivity, EUCAS 2003, Sorrento, Italy September 2003

[3] V.A.Mikhaylov, P.G.Akishin, A.V.Butenko, A.D.Kovalenko: Field Study of the 4T Superconducting Magnet for Rapid Cycling Heavy Ion Synchrotrons. EPAC 2004 — Proceedings Lucerne, Switzerland.4–9 July, 2004.

[4] H.Khodzhibagiyan, P.Akishin, A.Butenko, E.Fischer, A.Kovalenko, G.Kuznetsov, V.Mikhaylov. Progress in the Design of a Fast-Cycling Cosine θ Style Dipole Based on High Current Hollow Superconducting Cable. EUCAS 2005, 11–15 September 2005, Vienna, Austria.

[5] P.G.Akishin, A.V.Butenko, A.D.Kovalenko, V.A.Mikhaylov. The Magnetic Field Design of a 4T Fast Cycling Superconducting Dipole Magnet. JINR Communication, P9-2003-244, Dubna, 2003.

[6] P.G.Akishin, A.D.Kovalenko. Application on a Fictitious Current Windings Method for Field Optimization of Synchrotron Superconducting Magnets. JINR Communication, P5-2004-211, Dubna, 2004.