A Numerical Study of the Particle-Like Excitations in Nonlinear Dispersion Matter

E.V. Zemlyanaya, I.V. Barashenkov
Laboratory of Information Technologies, JINR
N.V. Alexeeva
Department of applied mathematics, University of Cape Town, South Africa

Since 1997, in the framework of the Agreement for co-operation, concluded between the JINR and the University of Cape Town, the particle-like excitations of nonlinear dispersion matter have been studying in the frame of models of condensed matter theory and nonlinear optics based on the nonlinear Schrödinger equation (NLS).

In [1], a class of multisoliton solutions in damped driven nonlinear Schrödinger equation has been studied. This equation arises in different models of condensed matter physics and nonlinear optics physics. It is shown that two or more damped driven solitons can form (stable and unstable) bound states. A continuation scheme for numerical analysis of multisoliton complexes is presented. Numerical results for cases of the parametrical driving and the external driving are demonstrated.

In [2],[3], the theoretical and numerical investigation of travelling solutions in nonlinear damped driven nonlinear Schrödinger equation has been performed. This equation has a number of applications in the fluid dynamics models, nonlinear optics, ferromagnets theory etc. The existence of the travelling damped solitons has been demonstrated. A continuation scheme for numerical analyzes of the traveling damped solitons is developed. It is shown that two or more solitons of parametrically driven, damped nonlinear Schrödinger equation can form a complex travelling with zero momentum at a nonzero constant speed.

In [4], nonlinear damped driven nonlinear Schrödinger equation with the defocusing type of nonlinearity has been studied. We show that unlike the bright solitons, the parametrically driven kinks of the NLS equation are immune from instabilities for all damping and forcing amplitudes; they can also form stable bound states. In the undamped case, the two types of kinks and their complexes can stably travel with nonzero velocities. The bistability of the Bloch and Néel walls within the NLS contrasts the properties of these solutions within the Ginzburg-Landau equation, where they cannot stably coexist.

In [5], the parametrically driven damped NLS equation serves as an amplitude equation for a variety of resonantly forced oscillatory systems on the plane. In the frame of the nonlinear Faraday resonance, the following equation was studied:

$$i\psi_t + \nabla^2\psi + 2\left|\psi\right|^2\psi - \psi = h\psi^* - i\gamma\psi, \tag{1}$$

where h and γ are, respectively, parameters of driving and damping.

We consider radially-symmetric nodal soliton solutions of Eq. (1). It is shown that although the nodal solitons are stable against radially-symmetric perturbations for sufficiently large damping coefficients, they are always unstable to azimuthal perturbations. The corresponding break-up scenarios are studied using direct numerical simulations. Typically, as it is demonstrated at the Fig.1, the nodal solutions break into symmetric "necklaces" of stable nodeless solitons.
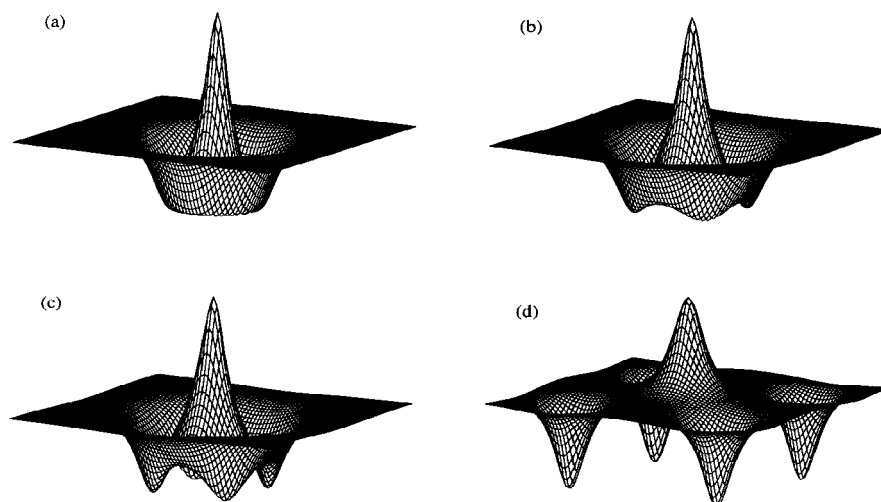
Fig. 1: Evolution of the azimuthal instability of the one-node soliton. (a): the initial condition; (b) and (c): dissociation of the ring-like "valley" into 4 nodeless solitons; (d): divergence of the fragments. Here $\gamma = 3.5$ and $h = 3.6$; shown is $\Re\,\psi$

References

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