Three-Dimensional B-spline Approximation of Magnetic Fields in the CBM Experiment

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Abstract

A three-dimensional spline approximation of a magnetic field for the CBM dipole magnet is elaborated. This allows one to use high-accuracy numerical methods for approximation of a charged particle trajectory in the inhomogeneous magnetic field.

In this note we consider numerical problems related to the track fitting problem in the CBM experiment. This problem is connected with solving motion equations of charged particles in the magnetic field. In practice, we have delt with a non-homogeneous field known from field measurement or mathematical calculations on some set of points. The problem of continuation of the field is usually solved by applying piece-wise linear node functions. Such field approximations provide neither the existence of high-order derivatives of the field nor even the existence of first derivatives. It is very important when we apply high-accuracy numerical methods for solving the motion equation. These methods give good convergence of numerical solutions to continue ones only in the case of existence of corresponding derivatives of the field. The best way of differentiable field continuation is the spline approximation method. In this note we discuss the application of B-spline method [1] for description of the magnetic field in CBM experiment.

First of all we consider a general problem of a three-dimensional B-spline approximation of a function $f(u, v, t)$ in the rectangular region $\Omega = [U_1, U_2] \times [V_1, V_2] \times [T_1, T_2]$.

Let $\{u_k, k = \overline{1, K}\}$, $\{v_l, l = \overline{1, L}\}$, $\{t_m, m = \overline{1, M}\}$ be sets of nodes of variables u, v, t. We assume that

$$u_k < u_{k+1}, \quad v_l < v_{l+1}, \quad t_m < t_{m+1};$$

and

$$u_1 = U_1, \quad u_K = U_2, \quad v_1 = V_1, \quad v_L = V_2, \quad t_1 = T_1, \quad t_M = T_2.$$

Let $\{f^{k,l,m}\}$ be

$$f^{k,l,m} = f(u_k, v_l, t_m), \qquad k = \overline{1, K}, l = \overline{1, L}, m = \overline{1, M}.$$

The numerical scheme involves three steps. At the first step we define $L \times M$ B-spline approximation functions $\hat{f}(u, v_{l_0}, t_{m_0})$ for all $\{l_0, m_0, l_0 = \overline{1, L}, m_0 = \overline{1, M}\}$ on the basic function $\{f^{k,l_0,m_0}, k = \overline{1, K}\}$. We use B-spline node functions of the third order. We add two additional nodes u_1, u_0 and two additional equations for boundary derivative values to exclude the indefinition in B-spline constructing. After this in each interval $[U_1, U_2]$ function $\hat{f}(u, v_{l_0}, t_{m_0})$ is expressed as follows:

$$\hat{f}(u, v_{l_0}, t_{m_0}) = \sum_{k=-1}^{K} a_{k,l_0,m_0} f_k(u),$$

where $f_k(u)$ is a node function associated with the node x_k. It should be noted that function $\hat{f}(u, v_{l_0}, t_{m_0})$ has a third derivative and a continuous second derivative with respect to u.

Then we repeat this procedure for variable v.

Using $\{a_{k_0,l,m_0}, l = \overline{1, L}\}$ we calculate the coefficients $\{b_{k_0,l,m_0}$ for each $\{k_0, m_0, k_0 = \overline{-1, K}, m_0 = \overline{1, M}\}$.

For all $\{m_0 = \overline{1, M}\}$ in each rectangular region $[U_1, U_2] \times [V_1, V_2]$ the function $\hat{f}(u, v, t_{m_0})$ is expressed as follows

$$\hat{f}(u, v, t_{m_0}) = \sum_{k=-1}^{K} \sum_{l=-1}^{L} b_{k,l,m_0} f_k(u) g_l(v),$$

where $g_l(u)$ is a node function associated with node v_l.

Application of a similar procedure to the variable t will give us set of coefficients $\{c_{k,l,m}, k = \overline{-1, K}, l = \overline{-1, L}, m = \overline{-1, M}\}$.

As a result, we obtain the approximation $\hat{f}(u, v, t)$ of the original function $f(u, v, t)$

$$\hat{f}(u, v, t) = \sum_{k=-1}^{K} \sum_{l=-1}^{L} \sum_{m=-1}^{M} c_{k,l,m} f_k(u) g_l(v) h_m(t),$$

where $h_m(t)$ is a node function associated with node t_m.

Applying the formal differentiation to the last equation, one can get the approximation of derivatives of the function $f(u, v, t)$ up to the third order against variables u, v, t.

This procedure was used to construct the differentiable three-dimensional B-spline approximations for each components of the magnetic field in the working region of CBM experiment. On figure 1 one can see a distribution of the main magnetic field component in the median plane of the dipole.

The described method could be applied for approximation of a wide class of experimental data. The three-dimensional B-spline approximation has an advantage in comparison with three-dimensional cubic spline approximation, because it allows one to decrease significantly the requirements for computer memory with the same approximation accuracy.

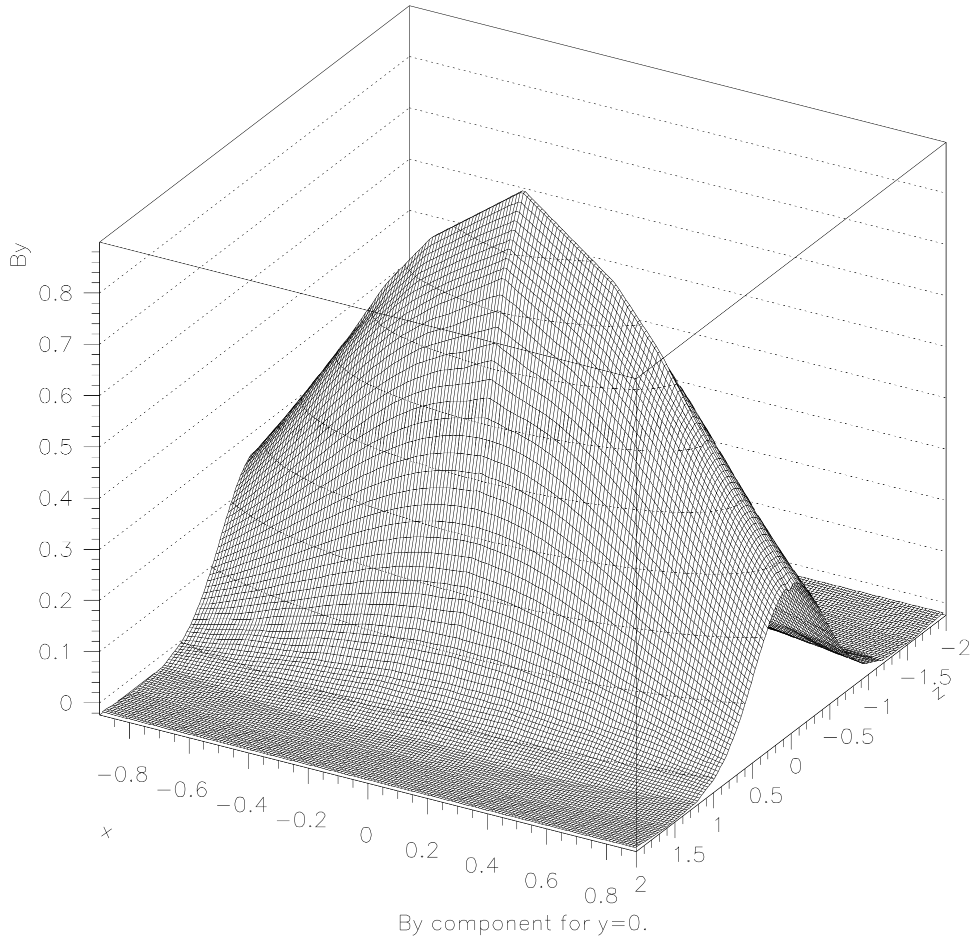

Figure 1:

References

[1] J.C.Zavialov, B.I.Kvasov, B.L.Miroshnichenko. Spline-function methods. Moscow, Nauka, 1980.