Robust Non-stationary Singular Spectrum Analysis of Chaotic Time Series

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Abstract

We consider the problem of forecasting chaotic time-series that contains short-time surges with high amplitudes. New forecasting schemes based on robust non-stationary and singular spectrum analysis methods are presented. In order to provide the robustness of these schemes, the M-estimation with the decision functions of Tukey's and Huber's types are applied. The multidimensionality of the presented models gives a possibility to take into account the influence of non-stationary factors on the analyzed time-series by getting additional information from the related time-series set.

For the last years methods and schemes of studying and forecasting chaotic time processes [1,2] have been intensively developed. As a rule, when investigating and forecasting many time series, one has an opportunity to consider not only a separate set, but a family of time series that influence each other. Under these conditions it is worthwhile to investigate the whole family of series at once in view of their mutual influence rather than each set separately. Clearly, the correct scheme of accounting the mutual influence of chaotic time series allows one to use, when studying each series, additional information contained in other time series. Moreover, in the framework of the suggested non-stationary singular spectrum analysis (NSSA) it is possible to take into account the influence of non-stationary dynamic economic factors on the analyzed time series. The latter provides a way for essential improvement of the accuracy of forecasting analyzed time processes.

For forecasting chaotic time series, SSA is often used, including a multidimensional singular spectrum analysis (MSSA) [3-4]. However, the traditional scheme of SSA application is not stable with respect to big scatters, so the forecast received with the help of traditional nonrobust SSA or MSSA schemes, is often not acceptable.

We have developed two schemes based on the robust non-stationary singular spectrum analysis (RNSSA) for forecasting time series, also taking into account their mutual influence. The both schemes comprise two stages. At the first stage, determined components are allocated that are stable against the big scatters in the initial time series. At the second stage, one of two NSSA schemes is applied to the received set of the determined components.

Fig. 1 shows results of forecasting chaotic time series

$$y_i = a \cdot y_{i-1} + b \cdot y_{i-2} + c + d \cdot i + \epsilon_i$$

with the help of the first scheme. It is seen that even in the conditions of the available chaotic component a good forecasting accuracy is provided (on the presented figure $\sigma(\epsilon_i) = 1, M = 20, m = 5, n = 250$).

Fig. 2 presents results of forecasting a financial time series for the closing prices of the General Motors shares.

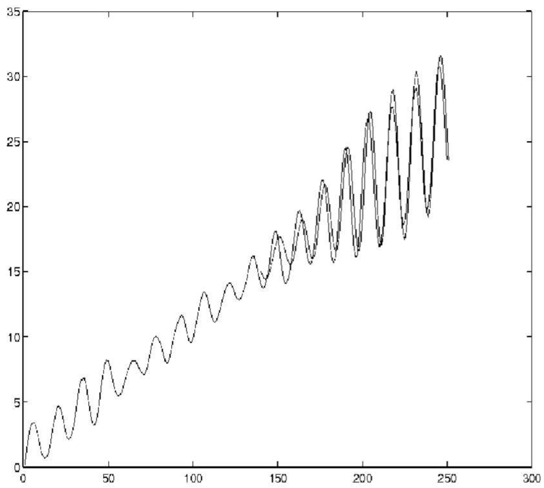

Fig. 1

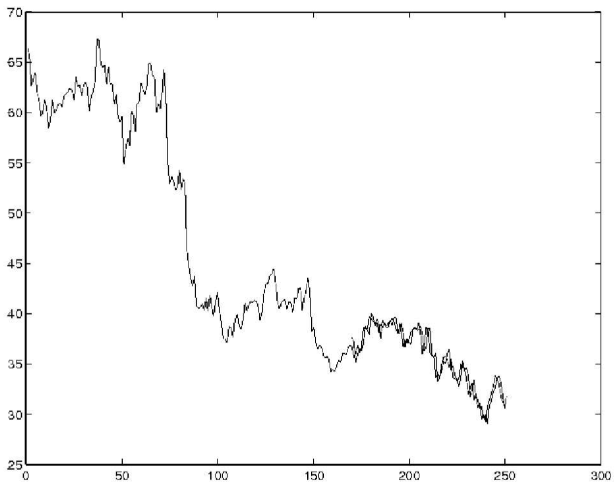

Fig. 2

The second scheme supposes that non-stationary time seria $x_{ji}, j = 1, \ldots, M, i = 1, \ldots, n$ are interconnected, i.e. influence each other. Fig. 3 gives results of forecasting the determined time series with no chaotic component

$$y_{1,i} = a_1 \cdot y_{1,i-1} + b_1 \cdot y_{1,i-2} + c_1 \cdot y_{1,i-3} + d_1 \cdot y_{2,i-1} + e_1 \cdot \sin(\varphi \cdot i) + \epsilon_{1,i};$$

$$y_{2,i} = a_2 \cdot y_{2,i-1} + b_2 \cdot y_{2,i-2} + c_2 \cdot y_{1,i-1} + \epsilon_{2,i}$$

with the help of the second scheme. One can see that in the conditions of absence of the chaotic component ($\epsilon_{1i}, \epsilon_{2i} = 0$) practically a 100% accuracy of the forecast is provided.

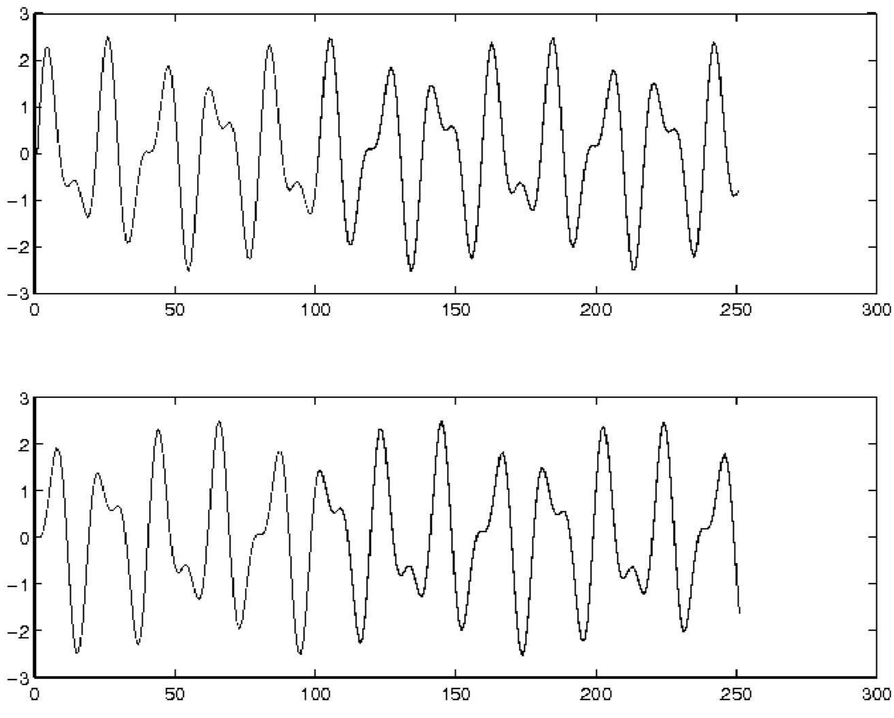

Fig. 3

Fig. 4 demonstrates (from top to bottom) results of forecasting financial time series for the opening, minimal, maximal and closing prices of the General Motors shares with the help of the second scheme.

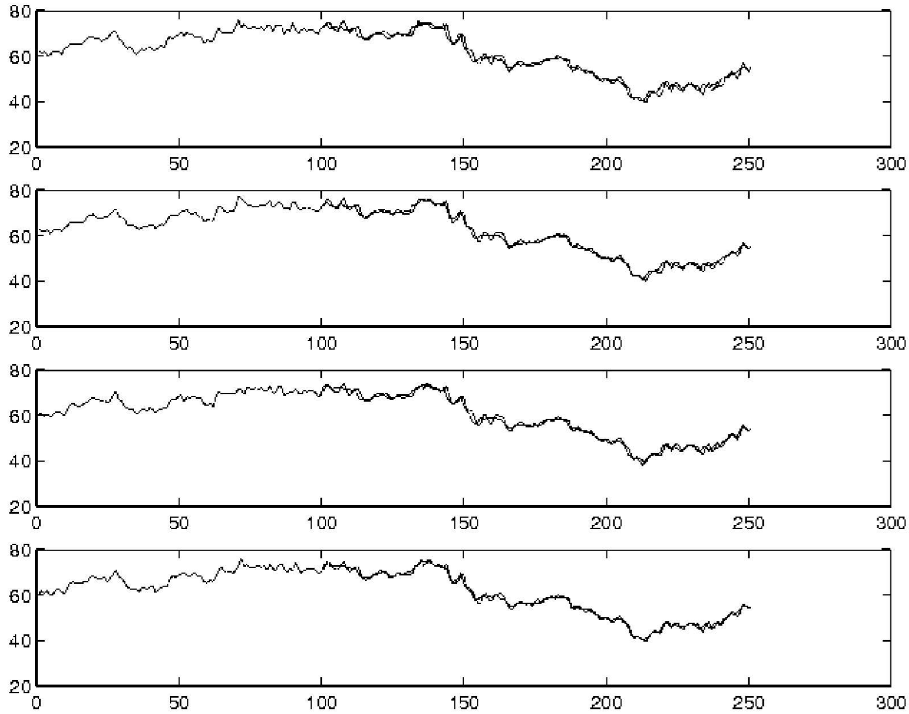

Fig. 4

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