

# Heavy-Quarkonium Interaction in QCD at Finite Temperature<sup>1</sup>

Yu. L. Kalinovsky

*Laboratory of Information Technologies, JINR*

## Abstract

We explore the temperature dependence of the heavy-quarkonium interaction based on the Bhanot - Peskin leading order perturbative QCD analysis. The Wilson coefficients are computed solving the Schrödinger equation in a screened Coulomb heavy-quark potential. The inverse Mellin transform of the Wilson coefficients then allows for the computation of the 1S and 2S heavy-quarkonium gluon and pion total cross section at finite screening/temperature. As a phenomenological illustration, the temperature dependence of the 1S charmonium thermal width is determined and compared to recent lattice QCD results.

The Debye screening between two opposite color charges is clearly seen in the QCD static potential computed at finite temperature  $T$  on the lattice [1]. Consequently, heavy quark bound states (which we call  $\Phi$ ) may no longer exist well above the deconfinement critical temperature  $T_c$ , of order 200–300 MeV [2]. This has made the heavy-quarkonium suppression in high energy heavy-ion collisions (as compared to proton-proton scattering) one of the most popular signatures for quark-gluon plasma formation [3, 4]. On the experimental side, a lot of excitement came out a few years ago after the NA50 collaboration reported a so-called “anomalous” suppression in the  $J/\psi$  channel in the most central lead-lead collisions ( $\sqrt{s} \simeq 17$  GeV) at the CERN SPS [5]. At RHIC energy ( $\sqrt{s} = 200$  GeV),  $J/\psi$  production has been measured recently by the PHENIX collaboration although the presently too large statistical and systematic error bars prevent one from concluding anything yet quantitative from these data [6].

The NA50 measurements triggered an intense theoretical activity and subsequently a longstanding debate on the origin of the observed  $J/\psi$  suppression. However, it became unfortunately rapidly clear that no definite conclusion could be drawn as long as theoretical uncertainties exceed by far that of the high statistics data. Indeed, both the realistic description of the space-time evolution of the hot and dense medium as well as the interaction of heavy-quarkonia with the relevant degrees of freedom (let them be pions or gluons) are required to be known. While the former can be constrained by global observables, the latter needs to be computed theoretically. Several approaches have been suggested to determine heavy-quarkonium total cross sections, from meson exchange [7] or constituent quark models [8] to the perturbative framework developed by Bhanot and Peskin [9, 10] upon which the present paper relies. Let us remark in particular that many recent phenomenological applications have used the latter perturbative  $\Phi$  – gluon cross section to estimate the heavy-quarkonium dissociation or formation in heavy-ion collisions [11].

However, although derived from first principles in QCD perturbation theory, the Bhanot - Peskin result describes the interaction of Coulombic bound states, that is for which the heavy-quark potential is well approximated by the perturbative one-gluon exchange potential. As indicated from spectroscopic studies this may be too crude an assumption to describe bound states in the charm or (even) the bottom sector. Furthermore,

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it does not take into account the possible effects of the medium on the heavy-quarkonium interaction. It is the aim of this Letter to explore how the  $\Phi$  interaction with gluons and pions gets modified at finite temperature.

The meson ( $\Phi$ ) interaction with gluons and pions has been computed so far using a heavy-quark screened Coulomb potential characterized by one parameter  $\mu$ . Interpreting  $\mu$  as the screening mass in a gluon plasma, the model for the finite temperature  $Q\bar{Q}$  potential now looks like

$$V_s = -\frac{g^2(r, T) N_c}{8\pi r} \exp(-m_D(T) r) \quad (1)$$

At short distance and/or low temperature, we shall consider a frozen coupling constant

$$g^2(r, T) = g^2 \text{ for } rT \ll \Lambda \quad (2)$$

and recover the Coulomb potential behavior, while the QCD coupling starts to run with  $T$  at large distance and/or high temperature. At two loops, we have

$$g^2(r, T) \equiv \tilde{g}^2(T) = \left( \frac{11}{8\pi^2} \ln \left( \frac{2\pi T}{\Lambda_{\overline{\text{MS}}}} \right) + \frac{51}{88\pi^2} \ln \left[ 2 \ln \left( \frac{2\pi T}{\Lambda_{\overline{\text{MS}}}} \right) \right] \right)^{-1} \text{ for } rT \gg \Lambda \quad (3)$$

with  $T_c/\Lambda_{\overline{\text{MS}}} = 1.14$ . The Debye mass  $m_D$  is related to the temperature through the leading-order perturbative result,

$$m_D(T) = \tilde{g}(T) T.$$

The  $\Lambda$  dimensionless parameter introduced in Ref. [12] separates somewhat arbitrarily the short from the long distance physics at finite temperature. Fitting pure gauge SU(3) heavy quark potential, they obtained the empirical value  $\Lambda = 0.48 \text{ fm} \times T_c$ . Following [12], we shall take the 2-loop running coupling (3) rescaled by 2.095 and interpolate smoothly between the short and long distance regime<sup>2</sup>.

The partonic and hadronic  $J/\psi$  and  $\Upsilon$  cross sections are computed in Figure 1 for several temperatures in units of the critical temperature for deconfinement,  $T_c = 270 \text{ MeV}$  in SU(3) pure gauge theory [2]. The temperatures selected for the bottomonium system are chosen to be slightly higher than those for the charmonium system since the larger bottom quark mass (hence, smaller size) probes more efficiently hotter QCD media.

The effects of the running coupling in Eq. (1) being quite small, rather similar features at finite temperature and at finite screening are observed. In particular, the charmonium binding energy (hence the inelastic threshold) drops by a factor of two already at  $T/T_c = 0.5$  and thus affects dramatically the  $J/\psi$  interaction in the vicinity of the threshold. At higher temperature, the  $J/\psi$  – gluon cross section is significantly enhanced at small gluon energy due to the larger charmonium size. The  $J/\psi$  –  $\pi$  cross section is also somewhat modified with a magnitude increasing noticeably with the temperature. Moving to the bottom sector (Figure 1, *right*), the  $\Upsilon$  cross sections exhibit the same general characteristics yet the medium effects at a given temperature prove much less pronounced from the smaller bottomonium size.

At high temperature, heavy-quarkonium interaction can not be described by short-distance techniques and our predictions are not valid any longer. On top of that, the

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<sup>2</sup>Similar results are obtained using the one loop running coupling with an appropriate rescaling.

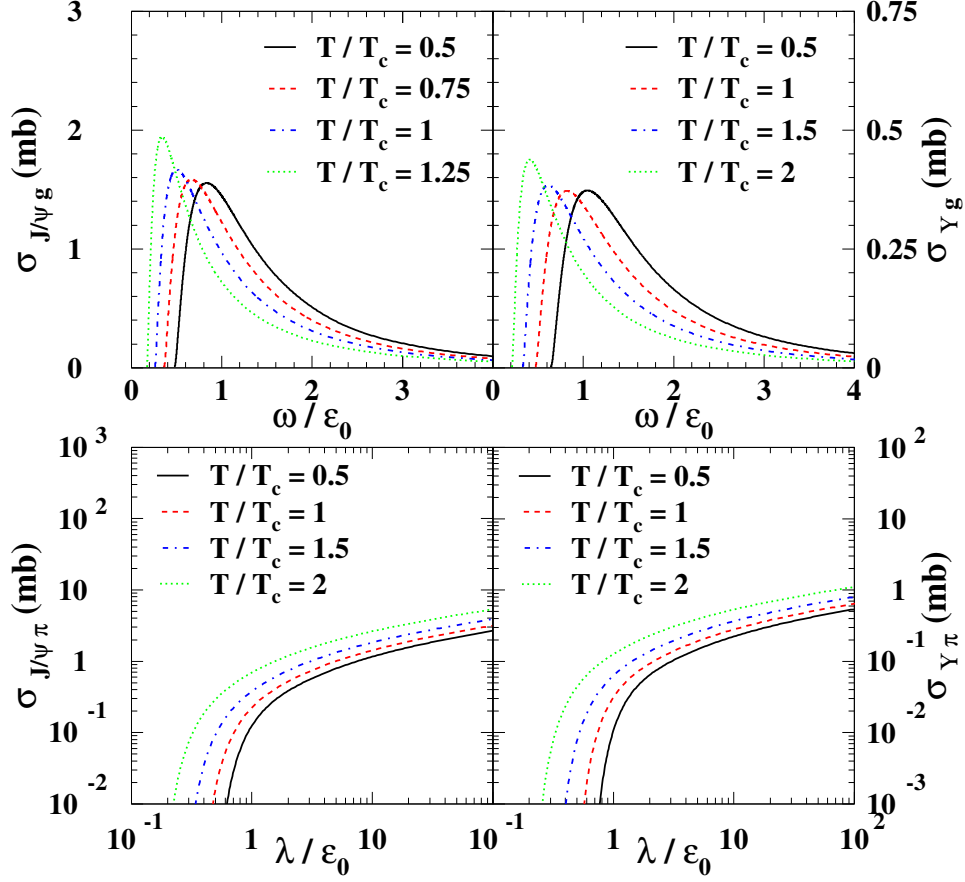


Figure 1:  $J/\psi$  (left) and  $\Upsilon$  (right) total cross sections with gluons (top) and pions (bottom) at various temperatures

process described here is the heavy-quarkonium dissociation by *hard* gluons as opposed to the soft gluons which only affect its properties. Therefore, our calculations should be valid as long as the Debye mass is kept smaller than the heavy-quarkonium Rydberg energy. This condition is fulfilled provided the bath temperature is smaller than 350 MeV. Above that scale, the screened exchanges are able to dissociate the bound states, the factorization between the heavy-quarkonium physics and the external gluon field is broken and the above QCD picture loses its significance.

The former results indicate that Debye screening effects may play an important role in the heavy-quarkonium dissociation by incoming gluons or pions. In order to illustrate how medium modifications could affect the  $\Phi$  suppression in heavy-ion collisions, we compute in this section the 1S charmonium thermal width  $\Gamma_{J/\psi}$  (or equivalently its lifetime,  $\tau_{J/\psi} = \Gamma_{J/\psi}^{-1}$ ) in a hot gluon bath. Assuming the  $J/\psi$  suppression is only due to the gluon dissociation process, the width can be written

$$\Gamma_{J/\psi}(T) = \frac{1}{2\pi^2} \int_0^\infty \omega^2 d\omega \sigma_{J/\psi g}(\omega, T) n_g(\omega, T)$$

where  $n_g(\omega, T) = 2(N_c^2 - 1)/(\exp(\omega/T) - 1)$  is the gluon density in a gluon gas in thermal equilibrium.

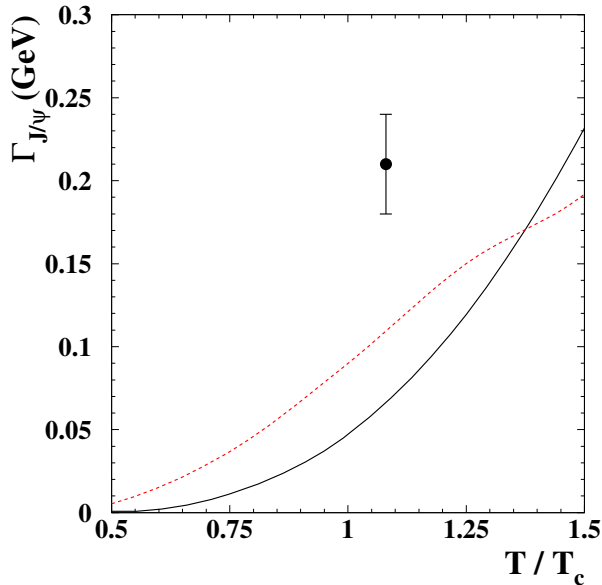


Figure 2:  $J/\psi$  thermal width as a function of the temperature with (*dotted*) and without (*solid*) modifications of the heavy-quark potential. The lattice data point obtained in Ref. [13] is also shown for comparison

The thermal width is computed in Figure 2 as a function of the temperature  $T$  assuming the vacuum (*solid*) and the in-medium (*dashed*)  $J/\psi$  – gluon cross section. At small temperature,  $T \ll \epsilon$ , most gluons are not sufficiently energetic to dissociate  $J/\psi$  states and the width remain small as the phase space selected by the  $J/\psi$  gluon threshold is restricted. When the medium gets warmer, more and more gluons are able to interact inelastically with the  $J/\psi$ , hence the thermal width increases. Interestingly enough, the in-medium  $J/\psi$  thermal width proves larger by a factor of two or more up to  $T = T_c$  due to the lower threshold in the medium modified cross sections. At even higher temperature, the medium modified result becomes smaller to that in the vacuum since *dissociating* gluons (with  $\omega$  of order  $\epsilon$ ) grow scarce. Also plotted in Figure 2 is the  $J/\psi$  width computed recently on the lattice at finite temperature in the quenched approximation [13]. Although a significant discrepancy remains between our calculations and the lattice data point, it is interesting to note that adding medium effects tends to reduce the disagreement, whose origin is not clarified.

The starting point of the calculation is the forward scattering amplitude  $\mathcal{M}_{\Phi h}$  originally derived for Coulomb bound states. To go beyond this one-gluon exchange picture would require to include light quark loops in the diagrammatics, to which the soft gluon source may couple, that we have not attempted. However, as conjectured in [10], it is appealing to guess that the generic dipole coupling appearing to leading order in  $g^2$  in the heavy-quarkonium Wilson coefficients survives perturbative and non-perturbative modifications of the  $Q\bar{Q}$  binding potential. Therefore we believe that taking the literal expression for the Coulomb states Wilson coefficients and compute them in a screened Coulomb potential appears sensible, at least as long as the screening remains reasonable,  $m_D a_0 \ll 1$ . This is certainly the case when the temperature is kept small as compared to the heavy quark mass. In that sense, the smallness of the charm and bottom quark

mass as compared to the non-perturbative scale of QCD indeed remains a problematic issue. As we have seen, typical space time scale becomes increasingly larger with the temperature, thus strongly limiting our confidence in the high temperature regime. Finally, one should keep a clear factorization between the gluon source and the heavy-quarkonium swimming in the gluon bath. We have seen that such a separation should be achieved as long as the Debye mass is small as compared to the bound state Rydberg energy, that is for temperatures  $T \leq 350$  MeV.

We presented a numerical calculation of the heavy-quarkonium cross section with gluons and pions, taking into account the possible medium-modifications of the heavy-quark potential at finite temperature. Such a work can therefore be useful to estimate heavy-quarkonium production in high energy heavy-ion collisions. In particular, we feel it would be interesting to explore the phenomenological consequences of such corrections comparing them to present calculations based on the vacuum heavy-quarkonium interaction. Finally, this very framework could be applied to study the  $\Phi$  interaction using a variety of realistic heavy quark (confining) potentials currently used in charmonium and bottomonium spectroscopy to describe more accurately, although further away from the perturbative requirement, heavy-quarkonium interaction with gluons and hadrons.

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