Exactly Solvable Models with Time-Dependent and Time-Independent Potentials

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Many interesting quantum-mechanical effects such as Berry geometric phase, molecular Aharonov-Bohm effect, dynamic localization of particles, fractional statistic and the quantum Hall effect were observed in atomic and molecular physics, quantum chemistry, quantum optics, plasma physics, and quantum field theory. A great deal of investigations in quantum computers refresh interest on the Berry phase effect in quantum mechanics. The idea of using unitary evolution operators produced by the non-Abelian Berry phase as quantum calculations was proposed by Zanardi, Pachos, Rasetti and realized by Pachos and Chountasis in a concrete model of holonomic quantum computer. In our view, the time-dependent and time-independent exactly solvable models in quantum theory make it possible to simulate these processes in a number of cases. Besides, they serve as a promising technique for their more fruitful investigations and deeper understanding, and also in finding new properties.

The technique of Bargmann-Darboux transformations and the supersymmetry method in quantum mechanics yield a variety of soluble stationary models. Each of these models with a time-independent exactly soluble Hamiltonian can be generalized to obtain the corresponding family of time-dependent exactly soluble Hamiltonians.

The technique of the intertwining operators has been applied to the matrix Schrödinger equations. The first- and second-order matrix Darboux transformations, factorization, supersymmetry, chains of transformations have been studied. An interrelation has been found between the differential and integral transformation operators of the 1st and 2nd order. The 2nd order integral Darboux transformations turn into expressions for matrix solutions and potentials obtained by the inverse scattering problem with degenerate integral kernels in a particular case $\mathcal{V} = \nu\mathcal{I}$. The relationships between the different potential matrices and their pertinent solutions have been obtained in a more general case when the 2nd order transformations are simultaneously realized on M states $\mathcal{V}_\mu = \nu_\mu\mathcal{I}$, $\mu = 1, 2, \ldots, M$. The method is illustrated by some specific examples with transformation vector- and matrix-functions. Using unitary time-dependent transformations, we construct exactly soluble time-dependent generalizations of exactly soluble time-independent equations. A large family of time-dependent potential matrices are generated, for which a system of linear Scrödinger equations have exact solutions. Explicit expressions of the expectation values of Hamiltonians, the spin-expectation values and total, dynamical and nonadiabatic geometric phases are given in terms of the obtained cyclic solutions. In particular, we demonstrate how to construct a set of periodic time-dependent Hamiltonians whose expectation values do not depend on time. The approach opens new opportunities for modelling the quantum dynamic systems with desired properties, for instance, quantum wells with the properties of dynamic localization.

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