Evolution of the Universe with Perfect Fluid and Dark Energy

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The study of late time acceleration of the universe together with the initial singularity and isotropization process remains among the attractive problems of modern cosmology. The problem of singularity has been thoroughly addressed by us in a number of papers for the last few years both plane symmetric and Bianchi type universes. In Refs.[1, 2, 3, 4] we considered the interacting spinor and scalar fields in a plane symmetric space-time and investigated the possiblity of the formation of soliton-like configurations. In [5, 6, 7] we studied the role of nonlinear spinor field in elimination of space-time singularity for Bianchi universes, whereas a self-consistent system of scalar, spinor, electro-magnetic and gravitational field given by a Bianch type I model was studied in [8]. It has been shown that for a non-singular mode of evolution one has to use a cosmological term describing an additional gravitational energy. In a recent paper [9] we consider a universe with perfect fluid and Λ term. In this case we find following relation between the Hubble constant, energy density and volume scale:

$$\kappa\varepsilon = 3H^2 - \Lambda - C_1/(3\tau^2), \quad C_1 = \text{const.} \tag{1}$$

Since with the increase of volume scale τ, the energy density ε decreases and an infinitely large τ corresponds to a ε close to zero, at some stage of evolution ε becomes too small to be ignored and from (1) follows

$$3H^2 - \Lambda \to 0. \tag{2}$$

From (2) one concludes the followings: (i) in this case Λ is essentially non-negative; (ii) in absence of a Λ term H becomes trivial, hence beginning from some value of τ the evolution of the Universe comes stand-still, i.e., τ becomes constant; (iii) in case of a positive Λ (which corresponds to a repulsive force) the process of evolution of the Universe never comes to a halt. Moreover it is believed that the presence of the dark energy (which can be given in the form of a positive Λ) results in the accelerated expansion of the Universe. As far as negative Λ (additional gravitational force) is concerned, its presence imposes some restriction on ε, namely, ε can never be small enough to be ignored. In case of the perfect fluid given by $p = \zeta\varepsilon$ there exists some upper limit for τ as well (note that τ is essentially nonnegative, i.e. bound from below). A suitable choice of parameters in this case may give rise to an oscillatory mode of expansion, whereas in case of a Van der Waals fluid the highly nonlinear equation of state may result in an exponential expansion as well.

Unlike the models with baritropic perfect fluid and spinor or scalar fields, where a negative Λ results in a non-periodic or oscillatory mode of expansion, if the Universe is filled with a Van der Waals fluid, independent to the sign of Λ the Universe expands exponentially [cf. Fig. 1]. As one sees from Fig. 2, in case of a Van der Waals gas the pressure is initially negative that becomes positive in the course of time (evolution).

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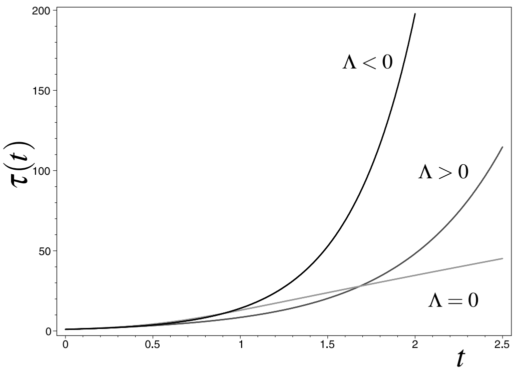

Figure 1: Evolution of $\tau(t)$ with the BI Universe filled with Van der Waals fluid

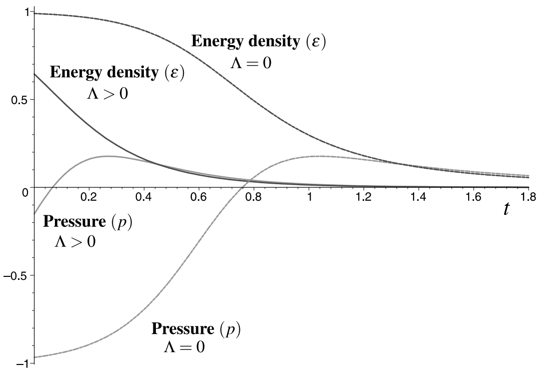

Figure 2: View of energy density ε and pressure p in case of Van der Waals fluid with $\Lambda \geq 0$

This negative pressure (repulsive force) can be viewed as a source of the initial inflation. In course of time the pressure becomes positive and the speed of expansion slows down. The discovery that the expansion of the Universe is accelerating has promoted the search for new types of matter that can behave like a cosmological constant by combining positive energy density and negative pressure. This type of matter is often called *quintessence*. An alternative model for the dark energy density was used by Kamenshchik *et al.*, where the authors suggested the use of some perfect fluid but obeying "exotic" equation of state. This type of matter is known as *Chaplygin gas*. In doing so the authors considered mainly a spatially flat, homogeneous and isotropic Universe described by a Friedmann-Robertson-Walker (FRW) metric. In a recent paper we studied the evolution of an initially anisotropic Universe given by a BI spacetime and a bimnary mixture of a perfect fluid obeying the equation of state $p = \zeta\varepsilon$ and a dark energy given by either a quintessence or a Chaplygin gas [10]. It was shown that the inclusion of dark energy does not eliminate initial singularity of the model and the space-time in those cases is ever-expanding. In order to obtain a singularity-free Universe we introduced a modified version of quintessence-like dark energy, which at the same time is able to explain the accelerated expansion [11]. In the modified model of quintessence the dark energy and the corresponding pressure obeys the following equation of state:

$$p_{\rm DE} = -w(\varepsilon_{\rm DE} - \varepsilon_{\rm cr}), \tag{3}$$

where the constant $w \in [0,\, 1)$. Here $\varepsilon_{\rm cr}$ some critical energy density. Setting $\varepsilon_{\rm cr} = 0$ one obtains ordinary quintessence. It is well known that as the Universe expands the (dark) energy density decreases. As a result, being a linear negative function of energy density, the corresponding pressure begins to increase. In case of an ordinary quintessence the pressure is always negative, but for a modified quintessence as soon as $\varepsilon_{\rm q}$ becomes less than the critical one, the pressure becomes positive [cf. Fig. 4]. As oppose to the ordinary quintessence a modified quintessence imposes some restriction on the maximum value of τ. As a result the Universe initially expands, but after reaching some maximum it begins to contract. Depending on the choice of the constant, which can be viewed as an energy level, it either shrinks into a point thus giving rise to space-time singularity, or begins to expand again after reaching some non-zero minimum, i.e., the Universe in this case experiences the oscillatory mode of expansion [cf. Fig. 3]. Note that to each $\mathcal{E}$ corresponds a particular pair $(\tau_{\rm min}, \tau_{\rm max})$. In case of the modified quintessence the pressure becomes positive in the process of evolution that gives rise to the oscillatory mode of expansion.

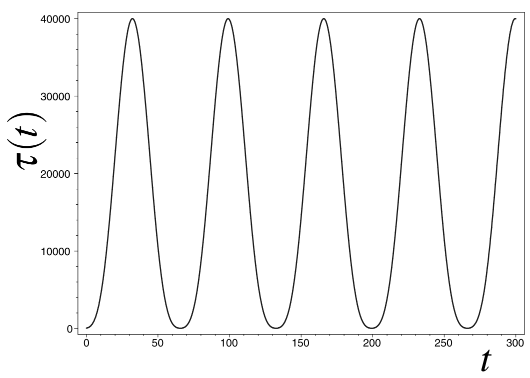

Figure 3: A modified quintessence provides an oscillatory mode of expansion of the Universe

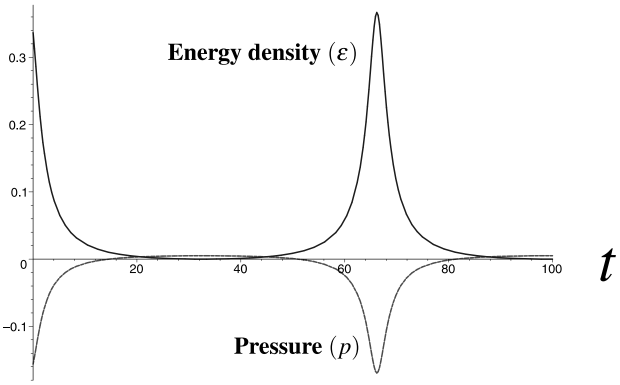

Figure 4: Evolution of energy density and pressure given by a modified quintessence

The modification of the ordinary quintessence lies in the fact that its pressure becomes positive if the (dark) energy density exceeds some critical value. The introduction of critical density though gives rise to a singularity-free universe, its physical origin is yet to be clarified. The problem of isotropization in this case remains unclear as well.

References

[1] Saha, Bijan and Shikin, G.N.: *Static plane-symmetric nonlinear spinor and scalar fields in GR* (to be published in Intern. J. Theor. Phys.), (2005).

[2] Saha, Bijan and Shikin, G.N.: *Plane-symmetric solitons of spinor and scalar fields* Chezkoslovak J. Phys., **54** (6), 597-620, (2004).

[3] Saha, Bijan and Shikin, G.N.: *Exact self-consistent plane-symmetric solutions to the spinor and scalar field equations* (to be published in Bulgarian J. Phys.).

[4] Saha, Bijan and Shikin, G.N.: *Nonlinear Spinor and Scalar Fields in General Relativity* (to be published in Vestnik PFU) [arXiv: gr-qc/0104089].

[5] Saha, Bijan and Todor Boyadjiev: *Bianchi type-I cosmology with scalar and spinor fields* Phys. Rev. D **69**, 124010, (2004) [arXiv: gr-qc/0311045].

[6] Saha, Bijan: *Nonlinear Spinor Field in cosmology* Phys. Rev. D **69**, 124006, (2004), [arXiv: gr-qc/0308088].

[7] Saha, Bijan and Todor Boyadjiev: *Interacting spinor and scalar fields in a Bianchi type-I Universe: Oscillatory solutions* in the “Gravity, Astrophysics and Strings @ the Black Sea” (Eds. P.P. Fiziev and M.D. Todorov) St. Kliment Ohridski University Press, Sofia, 226-233, (2005) [arXiv: gr-qc/0308061].

[8] Saha, Bijan: *Interacting scalar and spinor fields in Bianchi type I universe filled with magneto-fluid* J. Astrophys. Space Sci. **299** (1), 149-158, (2005) [arXiv: gr-qc/0309062].

[9] Saha, Bijan: *Anisotropic cosmological models with a perfect fluid and a Λ term* (to be published in Astrophys. Space Sci.) [arXiv: gr-qc/0411080].

[10] Saha, Bijan: *Anisotropic cosmological models with perfect fluid and dark energy* (to be published in Chinese J. Phys.) [arXiv: gr-qc/0412078].

[11] Saha, Bijan: *Anisotropic cosmological models with perfect fluid and dark energy reexamined* (submitted to International J. Theor. Phys.) [arXiv: gr-qc/0501067].