Bianchi Type-I Viscous Cosmology

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The investigation of relativistic cosmological models usually has the energy momentum tensor of matter generated by a perfect fluid. To consider more realistic models one must take into account the viscosity mechanisms, which have already attracted the attention of many researchers. In a recent paper [1] we consider a system of nonlinear spinor and a Bianchi type I gravitational fields in presence of viscous fluid. The nonlinear term in the spinor field Lagrangian is chosen to be λF, with λ being the self-coupling constant and F being a function of the invariants constructed from bilinear spinor forms S and P. We consider the cases when F is the power law of its arguments. Self-consistent solutions to the spinor and BI gravitational field equations are obtained in terms of τ, where τ is the volume scale of BI universe. The system of equations for τ and ε in presence of a λ term in this case can be written as

$$\dot\tau = 3H\tau, \tag{1a}$$

$$\dot H = \frac{1}{2}\bigl(3\xi H - \omega\bigr) - \bigl(3H^2 - \varepsilon + \Lambda\bigr) + \frac{\lambda_{\rm sp}}{2}\Bigl(\frac{m}{\tau} + \frac{\lambda(n-2)}{\tau^{n-1}}\Bigr), \tag{1b}$$

$$\dot\varepsilon = 3H\bigl(3\xi H - \omega\bigr) + 4\eta\bigl(3H^2 - \varepsilon + \Lambda\bigr) - 4\eta\lambda_{\rm sp}\Bigl[\frac{m}{\tau} - \frac{\lambda}{\tau^n}\Bigr]. \tag{1c}$$

Here η and ξ are the bulk and shear viscosity, respectively and they are both positively definite, i.e., $\eta > 0$ and $\xi > 0$. They may be either constant or function of time or energy, e.g.,

$$\eta = |A|\varepsilon^\alpha, \quad \xi = |B|\varepsilon^\beta. \tag{2}$$

with A, B, α and β being some constants. In (1), H is the Hubble constant. Note that the constant $\lambda_{\rm sp}$ we introduce for convenience. Setting $\lambda_{\rm sp} = 0$ we come to the case where the BI universe is filled with viscous fluid only. The system of equations (1) is solved exactly for some special choices of η and ξ. In particular, in [2] it has been shown that though the viscosity cannot remove the cosmological singularity, it plays a crucial part in the formation of a qualitatively new behavior of the solutions near singularity. It is shown that the introduction of the Λ term can be handy in the elimination of the cosmological singularity. In particular, in case of a bulk viscosity, it provides an everlasting process of evolution ($\Lambda < 0$), whereas, for some positive values of Λ and the bulk viscosity being inverse proportional to the expansion, the BI Universe admits a singularity-free oscillatory mode of expansion. In case of a constant bulk viscosity and share viscosity being proportional to expansion, the model allows both non-periodic and inflationary expansion independent to the sign of Λ term [cf. Figs. 1 and 2]. A detailed qualitative and numerical study of the system (1) was further performed by us in [3]. Here, in Figs. 3, 4 and 5 we illustrate some interesting cases as $H - \varepsilon$ phase diagrams.

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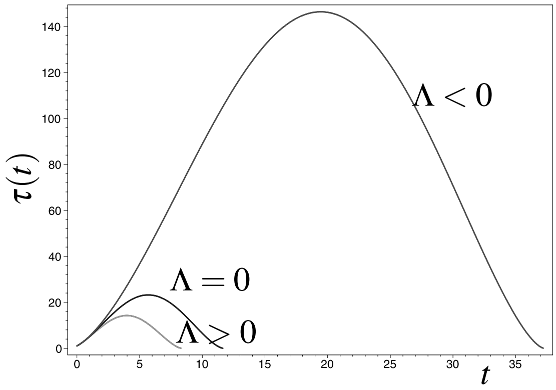

Figure 1: View of $\tau(t)$ for $C_4 = 0.1,\ \zeta = 0.33, \kappa = 0.1$, and $\xi = 0.1$ with $\Lambda = -0.03$ and $\Lambda = 0.03$, respectively

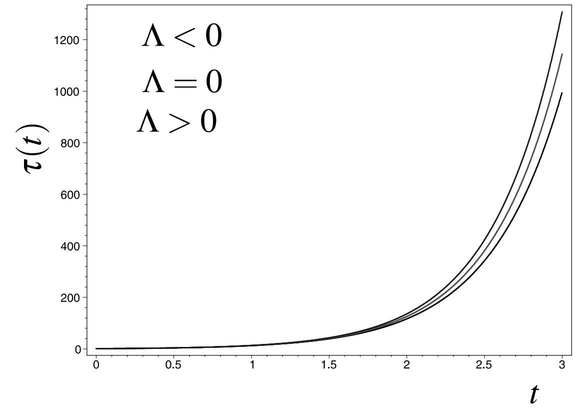

Figure 2: Evolution of the BI space-time with the parameters $C_4 = 0.1,\ \zeta = 0.33, \kappa = 1$, and $\xi = 1$ with $\Lambda = -0.03$ and $\Lambda = 0.03$, as in previous case

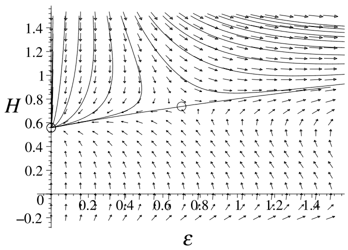

Figure 3: Phase diagram on $H-\varepsilon$ plane for $\beta = 1.5$, $\Lambda = -.933,\ B = .720$

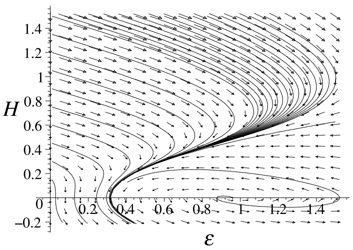

Figure 4: Phase diagram on $H-\varepsilon$ plane for $\beta = .05$, $\Lambda = .317,\ B = .667$

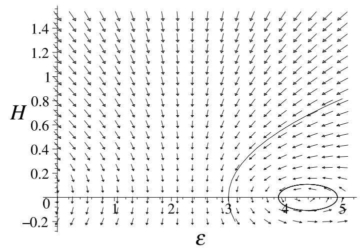

Figure 5: Phase diagram on $H - \varepsilon$ plane for $\Lambda = 3$, $\zeta = 0.333,\ C_2 = 1,\ C_3 = 1$

References

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[2] Saha, Bijan: *Bianchi type Universe with viscous fluid* Modern Physics Letters A **20** (28) 2127-2143, (2005) [arXiv: gr-qc/0409104].

[3] Saha, Bijan and V. Rikhvitsky: *Bianchi type I universe with viscous fluid: A qualitative analysis* (submitted to Physica D) [arXiv: gr-qc/0410056].