Topological Coherent Modes in Bose-Condensed Trapped Atoms

E.P. Yukalova
Laboratory of Information Technologies, JINR
V.I. Yukalov
Bogoliubov Laboratory of Theoretical Physics, JINR

Trapped Bose atoms cooled down to temperatures below the Bose-Einstein condensation temperature were considered [1–5].

Stationary solutions to the Gross-Pitaevskii equation define the topological coherent modes, representing nonground-state Bose-Einstein condensates. These modes can be generated by means of alternating fields whose frequencies are in resonance with the transition frequencies between two collective energy levels corresponding to two different topological modes. The theory of resonant generation of these modes was generalized in several aspects: Multiple-mode formation was described; a shape-conservation criterion was derived, imposing restrictions on the admissible spatial dependence of resonant fields; evolution equations for the case of three coherent modes were investigated; the complete stability analysis was accomplished; the effects of harmonic generation and parametric conversion for the topological coherent modes were predicted [1,2,4]. All considerations were realized both by employing approximate analytical methods as well as by numerically solving the Gross-Pitaevskii equation. Numerical solutions confirmed all conclusions following from analytical methods.

Dilute Bose-condensed gases at low temperature are characterized by a coherent field $\varphi(\mathbf{r}, t)$, which is a wave function satisfying the Gross-Pitaevskii equation

$$i\hbar \, \frac{\partial}{\partial t} \, \varphi(\mathbf{r}, t) = \left(\hat{H}[\varphi] + \hat{V} \right) \, \varphi(\mathbf{r}, t) \; , \tag{1}$$

where the nonlinear Hamiltonian

$$\hat{H}[\varphi] = -\hbar^2 \, \frac{\nabla^2}{2m_0} + U(\mathbf{r}) + N \, A_s \, |\varphi(\mathbf{r}, t)|^2 \tag{2}$$

contains a trapping potential $U(\mathbf{r})$ and the interaction intensity

$$A_s \equiv 4\pi\hbar^2 \, \frac{a_s}{m_0} \; , \tag{3}$$

with m_0 being atomic mass; a_s, scattering length; and N, the total number of atoms. The wave function is normalized to unity, so that $||\varphi|| = 1$. The confining potential can be modulated by applying an additional field $\hat{V} = V(\mathbf{r}, t)$.

The topological coherent modes are the solutions to the stationary Gross-Pitaevskii equation

$$\hat{H}[\varphi_n] \, \varphi_n(\mathbf{r}) = E_n \, \varphi_n(\mathbf{r}) \; , \tag{4}$$

where n is a labelling multi-index. Being the solutions to the nonlinear eigenproblem (4), these modes can be represented in the form

$$\varphi(\mathbf{r}, t) = \sum_n c_n(t) \, \varphi_n(\mathbf{r}) \, \exp\left(- \, \frac{i}{\hbar} \, E_n \, t \right) \; , \tag{5}$$

where $c_n(t)$ are unknown complex functions of time.

We considered the case of three coupled topological coherent modes. Substituting into (1) the presentation (5), driving field

$$V(\mathbf{r},t)=\sum_j\left[V_j(r)\cos(\omega_j t)+V_j'(\mathbf{r})\sin(\omega_j t)\right]$$

with resonant frequencies ω_j, $j=1,2,3$, and employing the averaging techniques, we received the system of equations for the coefficient functions $c_1(t)$, $c_2(t)$, $c_3(t)$.

Numerical simulations for the Gross-Pitaevskii equation and the system of equations for $c_j(t)$ with realistic physical parameters show that simultaneous generation of several topological modes is feasible.

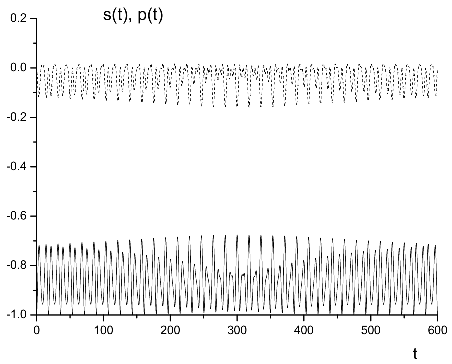
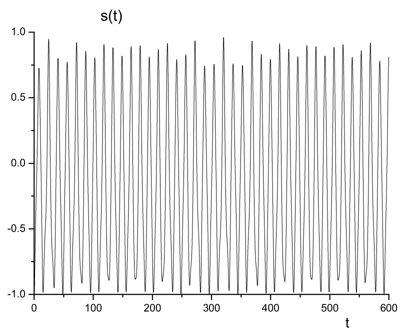


Figure 1: Mode locked regime for the case of three coupled nonlinear modes. Shown are the population differences $s(t)\equiv|c_2|^2-|c_1|^2$ (solid line) and $p(t)\equiv|c_3|^2-|c_2|^2$ (dashed line)



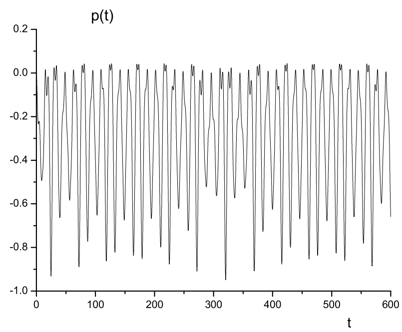


Figure 2: Mode unlocked regime for the case of three coupled nonlinear modes. Shown are the population differences $s(t)\equiv|c_2|^2-|c_1|^2$ and $p(t)\equiv|c_3|^2-|c_2|^2$

The condensate with topological coherent modes exhibits a variety of nontrivial effects. It was demonstrated [5] that the dynamical transition between the mode-locked and mode-unlocked regimes was accompanied by noticeable changes in the evolutional entanglement production.

A mixture of the multicomponent Bose-Einstein condensate was also considered, where each component moved with its own velocity. As a result of the relative motion, the mixture stratified when the relative velocity reached a critical value. Stability conditions for a binary moving mixture were derived and the critical velocity was found [3].

References

[1] V.I. Yukalov, K.P. Marzlin, and E.P. Yukalova, Laser Phys. **14**, 565–570 (2004).
Multiple coupling of topological coherent modes of trapped atoms.

[2] V.I. Yukalov, K.P. Marzlin, and E.P. Yukalova, Phys. Rev. A **69**, 023620–16 (2004).
Resonant generation of topological modes in trapped Bose-Einstein gases.

[3] V.I. Yukalov and E.P. Yukalova, Laser Phys. Lett. **1**, 50–53 (2004).
Stratification of moving multicomponent Bose-Einstein condensates.

[4] V.I. Yukalov, K.P. Marzlin, E.P. Yukalova, and V.S. Bagnato, Am. Inst. Phys. Conf. Proc. **770**, 218–227 (2005).
Topological coherent modes in trapped Bose gas.

[5] V.I. Yukalov and E.P. Yukalova, J. Low Temp. Phys. **138**, 657–662 (2005).
Dynamics of nonground-state Bose-Einstein condensates.