3D B-spline Approximation of Magnetic Fields in Inclined Dipole Magnets

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Abstract

A problem of the magnetic fields description for high energy physics experiments is considered. As an example we present a three-dimensional B-spline approximation of the magnetic field in the working region of the CBM dipole magnet. Such an approximation allows one both to calculate accurately the values of the magnetic field in each point of the working region and to determine the derivatives of the magnetic field components up to the third order. This provides a possibility to use high-accuracy numerical methods for approximation of a charged particle trajectory in the inhomogeneous magnetic field.

In this paper we consider a numerical problem of the three-dimensional (3D) magnetic field approximation. This is a typical problem not only for the CBM experiment [2], but for many experiments in high energy physics where one deals with a track fitting task. It arises in solving the motion equations of charged particles in the magnetic field. In practice, we deal with a non-homogeneous field known from field measurements or from mathematical calculations on some set of points. The problem of continuation of the field is usually solved by applying the piece-wise linear node functions. However, such field approximations provide neither only the existence of high-order derivatives of the field nor even the existence of first derivatives. The presence of high-order derivatives is very important when we apply high-accuracy numerical methods for solving the motion equation. These methods provide a good convergence of numerical solutions to continuous ones only in the case of existence of corresponding derivatives of the field. The best approach for continuation of the differentiable field is the spline approximation method. In this paper we discuss the application of B-spline methods [1] for description of magnetic fields in the CBM experiment.

Let us consider a general problem of three-dimensional B-spline approximation of a function $f(u, v, t)$ in rectangular region $\Omega = [U_1, U_2] \times [V_1, V_2] \times [T_1, T_2]$. Let $\{u_k, k = \overline{1, K}\}$, $\{v_l, l = \overline{1, L}\}$, $\{t_m, m = \overline{1, M}\}$ be sets of nodes for variables u, v, t. We suppose that

$$u_k < u_{k+1}, \quad v_l < v_{l+1}, \quad t_m < t_{m+1};$$

and

$$u_1 = U_1, \quad u_K = U_2, \quad v_1 = V_1, \quad v_L = V_2, \quad t_1 = T_1, \quad t_M = T_2.$$

Let $\{f^{k,l,m}\}$ be

$$f^{k,l,m} = f(u_k, v_l, t_m), \qquad k = \overline{1, K}, l = \overline{1, L}, m = \overline{1, M}.$$

The B-spline approximation procedure includes three steps. At first, we define $L \times M$ B-spline approximation functions $\hat{f}(u, v_{l_0}, t_{m_0})$ for all $\{l_0, m_0, l_0 = \overline{1, L}, m_0 = \overline{1, M}\}$ on the base $\{f^{k,l_0,m_0}, k = \overline{1, K}\}$. We use B-spline node functions of the third order. We also add two additional nodes u_0, u_{K+1} and two additional equations for the boundary derivative values to exclude the indefinition in the B-spline constructing. Then on the interval $[U_1, U_2]$ the function $\hat{f}(u, v_{l_0}, t_{m_0})$ may be expressed

$$\hat{f}(u, v_{l_0}, t_{m_0}) = \sum_{k=0}^{K+1} a_{k,l_0,m_0} f_k(u), \tag{1}$$

where $f_k(u)$ is the node function associated with the node x_k. It should be noted that the function $\hat{f}(u, v_{l_0}, t_{m_0})$ has a third derivative and a continuous second derivative as a function of

u. After this we repeat the procedure for variable v. For all $\{k_0, m_0, k_0 = \overline{0, K+1}, m_0 = \overline{1, M}\}$ on the base $\{a_{k_0,l,m_0}, l = \overline{1, L}\}$ we get a set of coefficients $\{b_{k_0,l,m_0}\}$. For all $\{m_0 = \overline{1, M}\}$ in rectangular region $[U_1, U_2] \times [V_1, V_2]$ the function $\hat{f}(u, v, t_{m_0})$ may be expressed

$$\hat{f}(u, v, t_{m_0}) = \sum_{k=0}^{K+1} \sum_{l=0}^{L+1} b_{k,l,m_0} f_k(u) g_l(v), \tag{2}$$

where $g_l(u)$ is the node function associated with the node v_l.

The last step for variable t will give us a set of coefficients $\{c_{k,l,m}, k = \overline{0, K+1}, l = \overline{0, L+1}, m = \overline{0, M+1}\}$. As a result, we have the following approximation $\hat{f}(u, v, t)$ of the function $f(u, v, t)$

$$\hat{f}(u, v, t) = \sum_{k=0}^{K+1} \sum_{l=0}^{L+1} \sum_{m=0}^{M+1} c_{k,l,m} f_k(u) g_l(v) h_m(t), \tag{3}$$

where $h_m(t)$ is the node function associated with the node t_m. By differentiating this equality, one can get the approximation of the derivatives of the function $f(u, v, t)$ up to the third order as a function of u, v, t.

It is very difficult to construct for some magnets the differentiable field approximations using only Cartesian coordinates. One can see 3D models of inclined magnets on Figs. 1-3. These magnets are considered as possible variants of the dipole magnet for the CBM experiment. The inclined poles version is also very popular for separator magnets. That is why we adopt the approach considered above for this case.

It is impossible to apply directly the above scheme for such type magnets. In this connection, we use special variables for the field description. Let **O** be the place of a target, see Fig. 4. Let the axes **Oz** , **Oy** be the direction of the beam and the ray perpendicular to the median plain of the dipole magnet. We define the axis **Ox** as a perpendicular line to the axes **Oz** and **Oy**. Let Ψ be the angle between the plain passing through the point $\mathbf{a} = (x_a, y_a, z_a)$, the axis **Ox** from one side and the median plane **Oxz** on the other side. We have

$$y_a = tan(\Psi) z_a. \tag{4}$$

Let Ψ_{max} be the acceptance of the magnet in the plain **Oyz**, $\pm X_{max}$ and $\pm Z_{max}$ be the maximum deviations for coordinates x_a, y_a, correspondingly. Then, we can use new variables $\{\Psi, X, Z\}$ which belong to the rectangular region

$$\Omega = [-\Psi_{max}, \Psi_{max}] \times [-X_{max}, X_{max}] \times [-Z_{max}, Z_{max}]$$

for field components $\{\mathbf{B_i}\}$ description:

$$\mathbf{B_i}(\Psi, x, z) = \sum_{k=0}^{K+1} \sum_{l=0}^{L+1} \sum_{m=0}^{M+1} c^i_{k,l,m} f_k(\Psi) g_l(x) h_m(z). \tag{5}$$

We have the inclination of the magnet working region in the plane **Oyz** only here, but one can see the magnet with the inclination in two planes **Oyz** and **Oxz** in Fig. 5. Let Φ be the angle between the plain passing through the point $\mathbf{a} = (x_a, y_a, z_a)$, the axis **Oy** and the plane **Oyz**, see Fig. 6. We have

$$x_a = tan(\Phi) z_a. \tag{6}$$

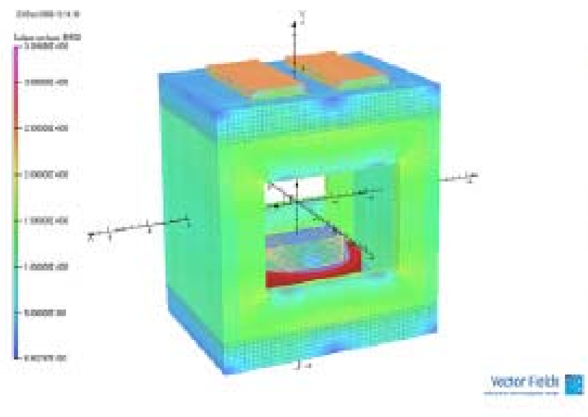

Fig. 1: 3D model of the HERA dipole magnet

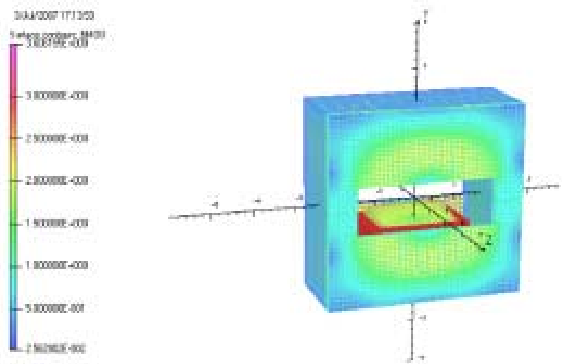

Fig. 2: 3D model of the HERMES dipole magnet

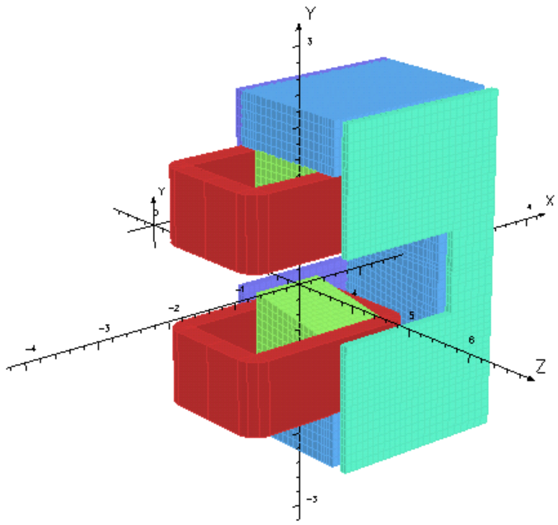

Fig. 3: Half of the HERMES dipole magnet

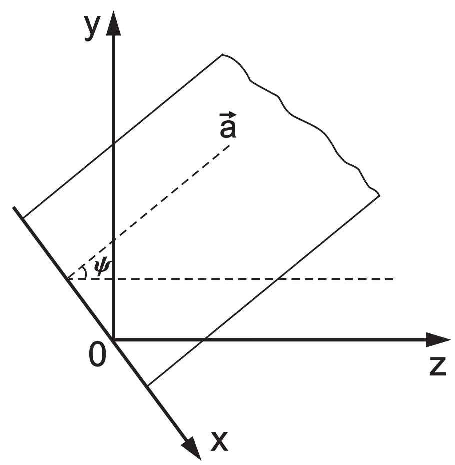

Fig. 4:

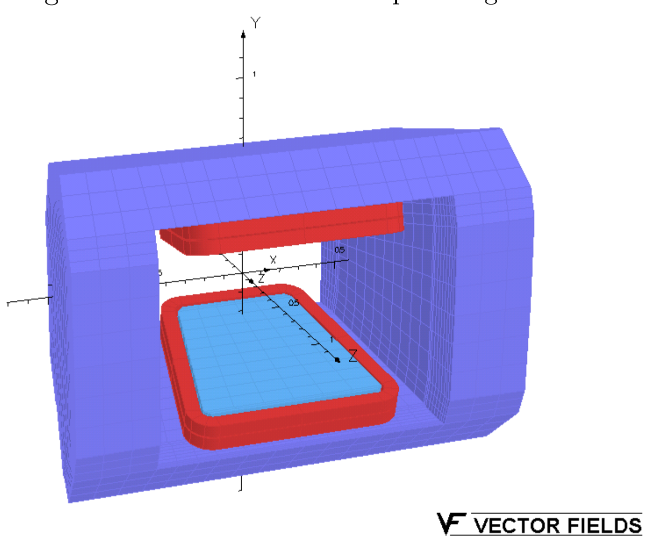

Fig. 5: 3D model of the inclined version of the CBM dipole magnet

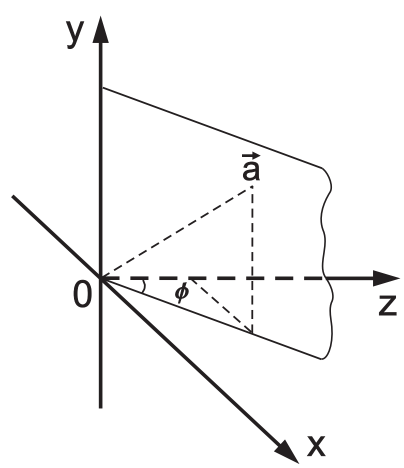

Fig. 6:

Let Ψ_{max} be the acceptance of the magnet in the plane **Oyz**, Φ_{max} be the acceptance of the magnet in the plane **Oxz**, and $\pm Z_{max}$ be the maximum deviations for coordinate z_a. Then we can use new variables $\{\Psi, \Phi, z\}$ belonging to the rectangular region

$$\Omega = [-\Psi_{max}, \Psi_{max}] \times [-\Phi_{max}, \Phi_{max}] \times [-Z_{max}, Z_{max}]$$

for field description in this case

$$\mathbf{B_i}(\Psi, \Phi, z) = \sum_{k=0}^{K+1} \sum_{l=0}^{L+1} \sum_{m=0}^{M+1} c^i_{k,l,m} f_k(\Psi) g_l(\Phi) h_m(z). \tag{7}$$

By differentiating this equality, we get the approximation for derivatives of the field components $\mathbf{B_i}$ up to the third order depending on $\{\Psi, \Phi, Z\}$.

In practice, we need the derivative not as a function of $\{\Psi, \Phi, z\}$, but depending on x, y, z. To calculate the derivatives of the function $\mathbf{B_i}$ depending on x, y, z, we can use the following equalities

$$\frac{d\mathbf{B_i}(\Psi, \Phi, z)}{dx} = \frac{d\mathbf{B_i}(\Psi, \Phi, z)}{d\Psi}\frac{d\Psi}{dx} + \frac{d\mathbf{B_i}(\Psi, \Phi, z)}{d\Phi}\frac{d\Phi}{dx}, \tag{8}$$

$$\frac{d\mathbf{B_i}(\Psi, \Phi, z)}{dy} = \frac{d\mathbf{B_i}(\Psi, \Phi, z)}{d\Psi}\frac{d\Psi}{dy} + \frac{d\mathbf{B_i}(\Psi, \Phi, z)}{d\Phi}\frac{d\Phi}{dy}, \tag{9}$$

$$\frac{d\mathbf{B_i}(\Psi, \Phi, z)}{dz} = \frac{d\mathbf{B_i}(\Psi, \Phi, z)}{d\Psi}\frac{d\Psi}{dz} + \frac{d\mathbf{B_i}(\Psi, \Phi, z)}{d\Phi}\frac{d\Phi}{dz} + \frac{d\mathbf{B_i}(\Psi, \Phi, z)}{dz}. \tag{10}$$

We have from (4) and (6)

$$\Psi = \arctan(\frac{y}{z}), \quad \Phi = \arctan(\frac{x}{z}). \tag{11}$$

It follows from here

$$\frac{d\Psi}{dx} = 0, \quad \frac{d\Psi}{dy} = \frac{z}{y^2 + z^2}, \quad \frac{d\Psi}{dz} = -\frac{y}{y^2 + z^2}, \tag{12}$$

$$\frac{d\Phi}{dx} = \frac{z}{x^2 + z^2}, \quad \frac{d\Phi}{dy} = 0, \quad \frac{d\Phi}{dz} = -\frac{x}{x^2 + z^2}. \tag{13}$$

The mixed partial derivatives of a higher order could be calculated similarly.

These procedures were used to construct a differentiable 3D B-spline approximation of the magnetic field for different types of dipole magnets for the CBM experiment.

The proposed method of the magnetic field approximation may be used for the description of wide class experiments. The 3D B-spline approximation has an advantage in comparison with the 3D cubic-spline approximation, because it permits one to decrease significantly the requirements to the computer memory for the same accuracy level.

References

[1] J.C.Zavialov, B.I.Kvasov, B.L.Miroshnichenko. Spline-function methods. Moscow, Nauka, 1980.

[2] http://www.gsi.de/fair/experiments/CBM/index