Néel-Néel and Bloch-Bloch Interactions of Parametrically Driven Dark Solitons

I.V. Barashenkov, E.V. Zemlyanaya
Laboratory of Information Technologies, JINR
S.R. Woodford
University of Cape Town, Department of Applied Mathematics, South Africa

Аннотация

Проведено аналитическое и численное исследование взаимодействий темных солитонов нелинейного уравнения Шредингера с нелинейностью дефокусирующего типа, с параметрической накачкой и диссипацией. Рассмотрены два типа темных солитонов - стенка Нееля и стенка Блоха. С помощью прямого численного решения исходного уравнения в частных производных показано, что, в зависимости от значений параметров диссипации и накачки, два темных солитона могут притягиваться, отталкиваться, либо формировать устойчивые связанные состояния. Путем численного продолжения по параметрам накачки и диссипации на плоскости этих параметров определены области существования и устойчивости связанных состояний двух стенок Нееля и двух стенок Блоха. Получены приближенные аналитические выражения для определения границ этих областей.

We considered interactions between the dark solitons of the parametrically driven nonlinear Schrödinger (NLS) equation:

$$i\partial_t\Psi + \tfrac{1}{2}\partial_X^2\Psi + \Psi - |\Psi|^2\Psi = h\Psi^* - i\gamma\Psi. \tag{1}$$

Here h is the strength of the parametric driving and γ is the damping coefficient.

Equation (1) was indeed derived in a broad variety of physical situations. In fluid dynamics, the repulsive ("defocusing") parametrically driven NLS describes the amplitude of the water surface in a vibrated channel with large width-to-depth ratio [1]. The same equation arises as an amplitude equation for the upper cutoff mode in a chain of parametrically driven, damped nonlinear oscillators. In the optical context, it was derived for the doubly resonant $\chi^{(2)}$ optical parametric oscillator in the limit of large second-harmonic detuning [2]. Next, stationary solutions of Eq.(1) with $\gamma = 0$ minimise the Ginzburg-Landau free energy of the anisotropic XY-model [3]. Finally we note that Eq.(1) also arises in a completely different magnetic context — that of a weakly anisotropic easy-plane ferromagnet in a constant external magnetic field [4].

In the paper [5], interactions of the walls of the same type, i.e. Néel-Néel and Bloch-Bloch interactions were studied. The analysis of the nonsymmetric situations, i.e. Néel-Bloch interactions, required a different mathematical formalism and was presented separately in [6].

For the analysis of the Bloch-Bloch and Néel-Néel interaction we used the variational method, under the assumption of well-separated walls. The variational analysis have been verified in direct numerical simulations of the full partial differential equation. The numerical simulations allow to advance beyond the limit of well-separated walls; in particular, we use this approach to examine the outcome of soliton collisions.

For numerical investigation of stationary bound states ("bubble", see Figs. 1 and 2) in the case of dissipative dynamics, the continuation computer code [7] was applied. The continuation algorithm is presented in [8], Sect. 1.4.

The dynamics of the *repelling* solitons is, in a sense, trivial: if the walls are initially at rest, they will simply diverge to the infinities. Less obvious is what the collision of two *attracting* walls will result in. Less obvious is what the collision of two *attracting* walls will result in. We studied the asymptotic (as $t \to \infty$) attractors arising in the parametrically driven NLS. It is shown that if the dynamics are dissipative, then, depending on the strength of the driving, colliding walls either annihilate or form a stable stationary bound state. In contrast to this, undamped collisions will be found to always produce a breather, a spatially localised, temporally oscillating structure. Depending on the initial conditions, the breather propagates or remains motionless, and in either case is found to persist indefinitely.

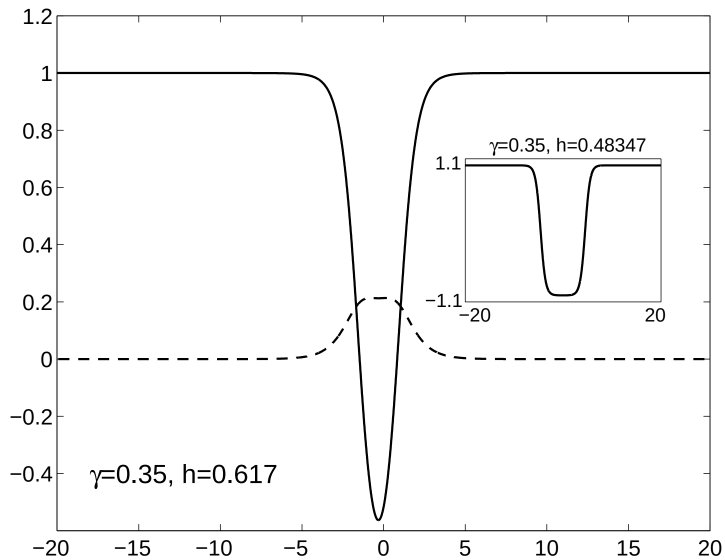

Fig. 1: The bubble with a small distance between the walls (a tightly bound complex). The solid and the dashed lines stand for the real and imaginary part of ψ, respectively. For comparison, we also show a bubble with a wide interwall separation arising for h^2 close to $\gamma^2 + 1/9$ (inset). Here only the real part is shown as $\operatorname{Im}\psi(x) = 0$

Main results of our study are following [5].

In the dissipative situation ($\gamma \neq 0$), the only available solitons are the Néel walls. When two Néel walls are very far apart, their interaction is simple: the walls repel if $h^2 < \gamma^2 + \frac{1}{9}$ and attract if $h^2 > \gamma^2 + \frac{1}{9}$. The repelling walls diverge to infinities; as for the case of attraction, there are two possible scenarios. One is led to consider the situation where the walls are closer to each other. At these shorter distances, the dynamics is influenced by two bound states, a stable and an unstable one. The stable bound state exists for h between $\sqrt{\gamma^2 + 1/9}$ and a threshold driving strength h_{sn}, and the unstable one exists for all $h < h_{sn}$. For small γ, the curve $h_{sn}(\gamma)$ can be described explicitly. In their region of coexistence, the stable complex has a larger separation: $2z_s > 2z_u$.

When h is smaller than $\sqrt{\gamma^2 + 1/9}$, the walls repel if their separation distance $2z(0)$ is greater than $2z_u$, the interwall separation in the unstable complex. If $2z(0) < 2z_u$, the walls converge and annihilate. (This verdict does not extend to the region where *both* h and γ are small. In this region the variational analysis of the small-separation dynamics is inconclusive.) When $\sqrt{\gamma^2 + 1/9} < h < h_{sn}$, pairs of Néel walls with separations $2z(0)$

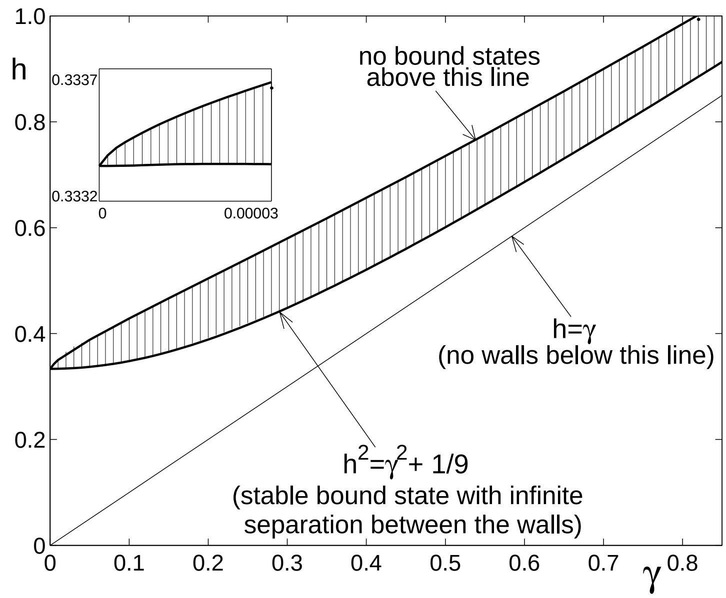

Fig. 2: The existence and stability diagram of the bound states on the (γ, h)-plane (obtained numerically.) The shaded corridor marks the region of coexistence of the stable and unstable bubble. Its upper boundary corresponds to the turning point where the two bubbles merge; there are no bound states above this line. The dotted curve gives the variational approximation to the line of turning points. The lower boundary of the shaded area corresponds to the stable bound state of two infinitely separated walls (an infinitely long stable bubble). This numerical curve is visually indistinguishable from $h = \sqrt{\gamma^2 + 1/9}$. In the strip between $h = \sqrt{\gamma^2 + 1/9}$ and $h = \gamma$, only the unstable bubble is found. Inset: a blow-up of the small-γ region. Here, the dotted curve is plotted using the asymptotic formula

larger than $2z_u$ evolve towards the stable bound state while those with $2z(0) < 2z_u$ converge and annihilate.

Finally, the walls with $h > h_{sn}$ converge and annihilate irrespectively of their initial separation. These results pertain to walls at *shorter distances* (which are however sufficiently far apart for the variational approximation to remain valid). In particular, they answer the question as to what finally happens to the two walls attracted from very large distances.

In the nondissipative case, the walls can move at constant speeds and the interaction pattern becomes complicated by the presence of inertia. When h is greater than $\frac{1}{3}$ (by small or large value), the Néel walls attract and converge — unless the initial condition corresponds to walls having large and opposite velocities. In the latter case the attraction is unable to stop the diverging walls and they escape to infinities. On the other hand, when h is smaller than $\frac{1}{3}$, the walls repel. The exception here is the case where h is close to $\frac{1}{3}$; in this case walls with very large separations repel whereas walls which are not so far from each other, attract.

In the dissipation-free case, the available dark solitons also include Bloch walls. The interaction between two Bloch walls depends on their relative chiralities: two initially quiescent, oppositely-handed Bloch walls attract while two quiescent walls with like chiralities placed at a large distance away from each other, repel. The exception is the case of

h close to $\frac{1}{3}$; in this limit, two walls of like chirality repel at large distances but exhibit anomalous interaction or transmute into an opposite-chirality pair and attract – when placed closer to each other.

These conclusions can be extended to the case of the *moving* Bloch walls, where one just needs to take their inertia into account. For example, two initially diverging oppositely-handed walls at large separation will continue to diverge despite the attraction whereas two likely-handed walls which were initially moving against each other, will continue to converge (until the repulsion stops them and sends away to infinities).

In addition to the interactions between well-separated walls, we investigated products of their collision. When the system is not damped, the collision of two walls results in a stationary or travelling breather. We reconstructed the numerically found breather as an asymptotic series.

When γ is nonzero, oscillations are damped and the only nontrivial product of collision of two Néel walls is a stationary bubble — the bound state of two Néel walls (see Fig. 1). Using the numerical continuation of solutions the corresponding ODE, we demarcated the bubble's domain of existence in the parameter space. [This domain is, naturally, a subset of the part of the (γ, h)-plane in which remote walls attract]. The numerical demarcation is in excellent agreement with the domain of existence obtained variationally (see Fig. 2).

References

[1] C. Elphick and E. Meron, Phys. Rev. A **40**, 3226 (1989); B. Denardo, W. Wright, S. Putterman, and A. Larraza, Phys. Rev. Lett. **64**, 1518 (1990); W. Chen, L. Lu, and Y. Zhu, Phys. Rev. E **71**, 036622 (2005)

[2] S. Trillo, M. Haelterman and A. Sheppard, Opt. Lett. **22**, 970 (1997)

[3] L.N. Bulaevskii and V.L. Ginzburg, Sov. Phys. JETP, **18**, 530 (1964)

[4] I.V. Barashenkov, S.R. Woodford, E.V. Zemlyanaya, Phys. Rev. Lett. **90**, 054103 (2003)

[5] I.V.Barashenkov, S.R. Woodford, E.V. Zemlyanaya. Interactions of Parametrically Driven Dark Solitons. I: Neel-Neel and Bloch-Bloch interactions. Arxiv: nlin/0612059; Phys. Rev. E, **75** 026603 (2007)

[6] I.V. Barashenkov and S.R. Woodford, Interactions of Parametrically Driven Dark Solitons II: Néel-Bloch interactions, Arxiv: nlin/0701005; Phys. Rev. E, **75** 026605 (2007)

[7] http://www.jinr.ru/programs/jinrlib/contin-nlin

[8] I.V. Puzynin, T.L. Boyadjiev T.L., S.I. Vinitsky, E.V. Zemlyanaya, T.P. Puzynina, O. Chuluunbaatar. Methods of computational physics for investigation of models of complex physical systems. Physics of Particles and Nuclei, **38** No.1 70-116 (2007)