Condensate and Superfluid Fractions for Varying Interactions and Temperature in Dilute Bose Systems

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A system with Bose-Einstein condensate is considered in the frame of the self-consistent mean-field approximation, which is conserving, gapless, and applicable for arbitrary interaction strengths and temperatures. The main attention is paid to the thorough analysis of the condensate and superfluid fractions in a wide region of interaction strengths and for all temperatures between zero and the critical point t_c, where condensate disappears.

The normal and anomalous averages are shown to be of the same order for almost all interactions and temperatures, except the close vicinity of t_c. Increasing temperature influences the condensate and superfluid fractions in a similar way, by diminishing them. But their behavior with respect to the interaction strength is very different. For all temperatures, the superfluid fraction is larger than the condensate fraction. For asymptotically strong interactions, the condensate is almost completely depleted, even at low temperatures, while the superfluid fraction can be close to one.

The dimensionless sound velocity s satisfies the equation

$$s^2 = 4\pi\gamma(n_0 + \sigma) \; , \tag{1}$$

where n_0 is the normal condensate fraction and σ is the anomalous average. The fraction of uncondensed atoms, n_1, is given by the expression

$$n_1 = \frac{s^3}{3\pi^2}\left\{ 1 + \frac{3}{2\sqrt{2}} \int_0^\infty \left(\sqrt{1+x^2} - 1\right)^{1/2} \left[\coth\left(\frac{s^2 x}{2t}\right) - 1 \right] dx \right\} . \tag{2}$$

The anomalous average σ reads as

$$\sigma = \frac{2s^2}{\pi^{3/2}}\sqrt{\gamma n_0} \; - \; \frac{s^3}{2\sqrt{2}\,\pi^2} \int_0^\infty \frac{(\sqrt{1+x^2}-1)^{1/2}}{\sqrt{1+x^2}} \left[\coth\left(\frac{s^2 x}{2t}\right) - 1 \right] dx \; . \tag{3}$$

And the superfluid fraction n_s takes the form

$$n_s = 1 \; - \; \frac{s^5}{6\sqrt{2}\,\pi^2 t} \int_0^\infty \frac{(\sqrt{1+x^2}-1)^{3/2} x \, dx}{\sqrt{1+x^2}\,\sinh^2(s^2 x/2t)} \; . \tag{4}$$

Equations (1) to (4), together with the relation $n_0 = 1 - n_1$, define all characteristics we wish to investigate. Our aim is to study Eqs. (1) to (4) for the varying interaction strength $\gamma \geq 0$ and temperature $t \geq 0$.

First, we find analytic expressions for low temperatures. At sufficiently low temperature, such that $\frac{t}{s^2} \ll 1$, we find from Eqs. (1) to (3) the asymptotic expansions, in terms of s, for the fraction of uncondensed atoms

$$n_1 \simeq \frac{s^3}{3\pi^2} + \frac{t^2}{12s} \; ,$$

the anomalous average

$$\sigma \simeq \frac{2s^2}{\pi^{3/2}} \sqrt{\gamma n_0} \; - \; \frac{t^2}{12s} \; ,$$

and for the superfluid fraction

$$n_s \simeq 1 \; - \; \frac{2\pi^2 t^4}{45 s^5} \; .$$

When the interaction is weak, such that $\gamma \to 0$, then the sound velocity s is

$$s \simeq 2\sqrt{\pi}\,\gamma^{1/2} \; - \; \frac{t^2}{12} \; .$$

The considered expansions are valid for the low temperatures $t \ll \gamma \ll 1$. Then for the anomalous average σ and the condensate fraction n_0, we get

$$\sigma \simeq \frac{8\gamma^{3/2}}{\sqrt{\pi}} \left(1 \; - \; \frac{t^2}{192\gamma^2} \right) \; , \qquad n_0 \simeq 1 \; - \; \frac{8\gamma^{3/2}}{3\sqrt{\pi}} \left(1 + \frac{t^2}{64\gamma^2} \right) \; ,$$

which is in agreement with the known temperature expansion for the weakly interacting Bose gas. For the superfluid fraction n_s, we have

$$n_s \simeq 1 \; - \; \frac{\gamma^{3/2}}{720\sqrt{\pi}} \left(\frac{t}{\gamma} \right)^4 \; .$$

In the case of very strong interactions, when $\gamma \to \infty$,

$$s \simeq \left(3\pi^2\right)^{1/3} - \frac{t^2}{36} \; , \qquad \sigma \simeq \frac{(9\pi)^{1/3}}{4} \left(\frac{1}{\gamma} \; - \; \frac{t^2}{9\pi} \right) \; , \qquad n_0 \simeq \frac{\pi}{64}\,\gamma^{-3} + O(t^4) \; ,$$

and the superfluid fraction behaves as

$$n_s \simeq 1 \; - \; \frac{2}{135\pi(9\pi)^{1/3}} \left(1 + \frac{5\pi}{192\gamma^3} \right) t^4 \; .$$

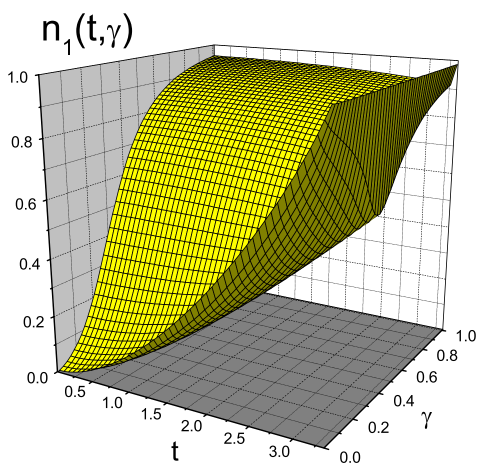

Fig. 1: Fraction of uncondensed atoms $n_1 = n_1(t, \gamma)$ as a function of the dimensionless temperature t and of the interaction strength γ

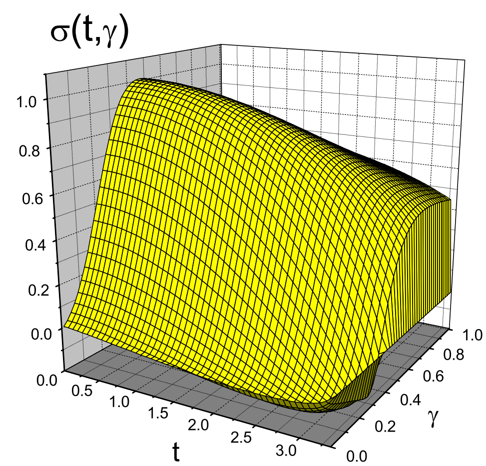

Fig. 2: Anomalous average $\sigma = \sigma(t, \gamma)$ as a function of the variables t and γ

The analysis of Eqs. (1) to (4) demonstrates that there is the critical temperature t_c

$$t_c = \frac{2\pi}{[\zeta(3/2)]^{2/3}} = 3.312498 \; ,$$

where n_0, n_s, σ, and s all tend to zero.

In order to investigate the behavior of the characteristic quantities in the whole range of temperatures $t \in [0, t_c]$ and for arbitrary interactions $\gamma \geq 0$, we solve numerically the system of equations (1), (2), (3), and (4), together with the relation $n_0 = 1 - n_1$. Figure 1 presents the normal fraction n_1 as a function of the dimensionless temperature t and the interaction strength γ. In Fig. 2, we show the anomalous average σ as a function of the same variables t and γ. The condensate fraction n_0 is depicted in Fig. 3, and the superfluid fraction n_s is shown in Fig. 4.

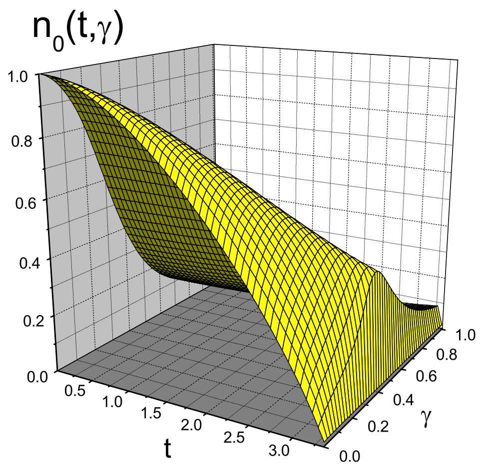

Fig. 3: Condensate fraction $n_0 = n_0(t, \gamma)$ as a function of the variables t and γ

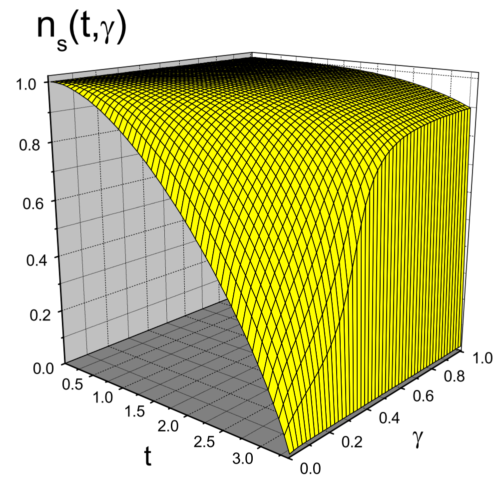

Fig. 4: Superfluid fraction $n_s = n_s(t, \gamma)$ as a function of the variables t and γ

References

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[2] V.I. Yukalov, E.P. Yukalova. *Condensate and superfluid fractions for varying interactions and temperature.* Phys. Rev. A **76**, 013602-9 (2007).