Torsional Shear Oscillations in the Neutron Star Crust Driven by Restoring Force of Elastic Stresses

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Abstract

The issues considered in this paper are motivated by recent discovery of quasiperiodic oscillations in light curves of X-ray flaring magnetars – neutron stars endowed with ultrapowerful magnetic fields $B \sim 10^{15}$ Gauss. The detailed analysis is given of the old standing problem of pulsar asteroseismology regarding exact form of spectral formulae for the frequency of torsional vibrations of a neutron star, both global and trapped in the crust, whose exact solution has been obtained for the first time. From quite general viewpoint of nuclear matter theory, this mode of nonradial differentially rotational vibrations of a neutron star – self-gravitating mass of degenerate nuclear matter dominated by neutron component – is of particular interest in that it exhibits elastic properties of nuclear material which can be identified, thereby, with elastically deformable Fermi-solid whose macroscopic motions are properly modeled by the continuum equations of solid-mechanics or elastodymanics, rather than Fermi-liquid macroscopic behavior is described by equations of fluid mechanics or hydrodynamics.

1. Introduction. Ever since the identification of pulsars with neutron stars, the nonradial torsional shear oscillations restored by bulk forces of different in physical nature internal stresses have been and still are among the most important issues in the study of the interconnection between the electromagnetic activity and asteroseismology of pulsars. Recently, increasing interest in this domain of research has been prompted by the discovery of quasiperiodic oscillations (QPOs) on the lightcurve tail of SGR 1806-20 and SGR 1900+14. An extensive survey of observational data and theoretical works devoted to this issue can be found in recent papers [1,2]. Motivated by the above interest, we focus here on the mathematical physics of the eigenfrequency problem for the torsional shear oscillations governed by equation of Newtonian solid mechanics. The restoring force is the bulk force of Hookean elastic shear stresses. Emphasis is laid on the boundary conditions which must be imposed on the toroidal field of material displacement at the edges of the seismogenic layer, that is, on the core-crust boundary and the star surface. Working from the homogeneous crust model we show that these boundary conditions substantially affect the asymptotic spectral formulae for the frequency of torsional shear oscillations.

2. Spectral formulae of torsional nodeless oscillations.The point of departure is the generally accepted attitude that the quake induced vibrations of crustal matter can be properly modeled by equation of solid mechanics for solenoidal field of material displacement u_i

$$\rho \ddot{u}_i = \nabla_k \, \sigma_{ik} \qquad \sigma_{ik} = 2\mu u_{ik} \tag{1}$$

$$u_{ik} = \frac{1}{2}[\nabla_i u_k + \nabla_k u_i] \quad u_{kk} = \nabla_k u_k = 0 \tag{2}$$

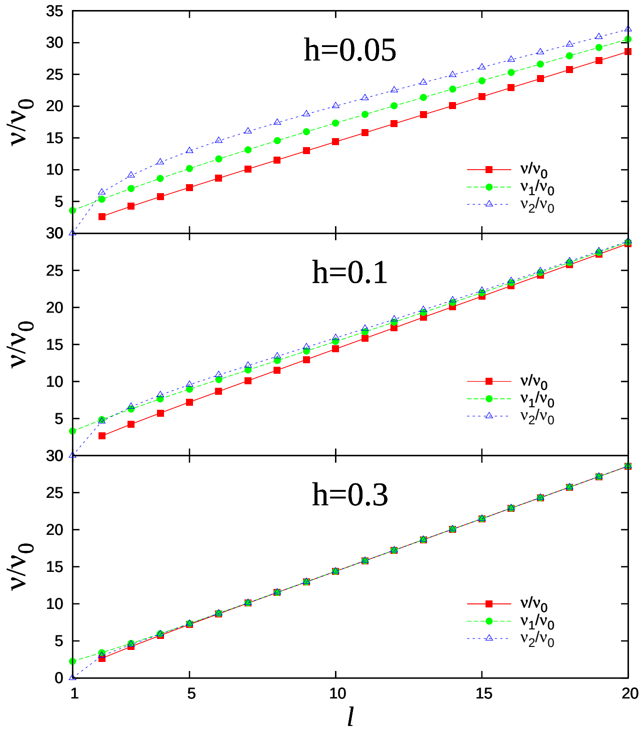

Fig. 1: Fractional frequencies of nodeless torsional shear oscillations as functions of multipole degrees $1 < \ell < 20$ plotted at $h = \Delta R/R = 0.05, 0.1$ and 0.3. ν/ν_0 for global nodeless torsional mode in entire volume of homogeneous neutron star model; ν_1/ν_0 and ν_2/ν_0 for the frequency of torsional modes trapped in the peripheral layer of homogenous model of crust computed with boundary conditions of first and second examples

expressing the second law of Newtonian elastodynamics. The general solution of (1) describing axisymmetric differentially rotational harmonic in time material oscillations in the star is given by the toroidal vector field

$$\mathbf{u}(\mathbf{r},t) = \nabla \times \mathbf{r}U(\mathbf{r},t) = \nabla U(\mathbf{r},t) \times \mathbf{r} \quad U(\mathbf{r},t) = f_\ell(kr)\, P_\ell(\cos\theta)\, \exp(i\omega t) \tag{3}$$

$$f_\ell(kr) = [A_\ell\, j_\ell(kr) + B_\ell\, n_\ell(kr)] \tag{4}$$

$$u_r = 0,\ u_\theta = 0,\ u_\phi = f_\ell(kr)\, P_\ell^1(\zeta)\exp(i\omega t) \tag{5}$$

$$P_\ell^1(\zeta) = (1-\zeta^2)^{1/2}\frac{dP_\ell(\zeta)}{d\zeta} \quad \zeta = \cos\theta. \tag{6}$$

By $j_\ell(kr)$ and $n_\ell(kr)$ are denoted the spherical Bessel and Neumann functions, respectively, and by $P_\ell(\cos\theta)$ the Legendre polynomial of multipole degree ℓ.

The prime purpose here is to obtain the frequency spectra of long wavelength differentially rotational oscillations, when $kR << 1$. In so doing we adopt boundary conditions which are currently utilized in the works studying quake-induce oscillations in the neutron star crust. Namely the condition of stress-free-surface for both core-crust boundary and surface of the star and non-slip condition on the the core-crust interface. It is shown that for no-slip boundary condition on the core-crust interface and no-stress on the star surface the spectral formula for the fractional frequencies can be represented in the form

$$\frac{\nu_1^2}{\nu_0^2} = (2\ell+3)(\ell-1)\left[1 + \frac{\ell+2}{\ell-1}\lambda^{2\ell+1}\right] \tag{7}$$

$$\nu_1 = \frac{\omega_1}{2\pi} \quad \nu_0 = \frac{\omega_0}{2\pi} \quad \lambda = \frac{R_c}{R} = 1-h \quad h = \frac{\Delta R}{R}. \tag{8}$$

For the stress free boundary conditions on both core-crust interface and the neutron star surface the spectral formulae is given by

$$\frac{\nu_2^2}{\nu_0^2} = (2\ell+3)(\ell-1)\left[\frac{1+\lambda^{2\ell+1}}{1-\lambda^{2\ell+3}}\right] \qquad 0 \le \lambda < 1. \tag{9}$$

In the limit $\lambda = (R_c/R) \to 0$, the above equations are i reduced to spectral formula for the frequency of global torsional oscillations which can be represented in the following equivalent forms which are of particular interest for identification of torsion elastic modes in the detected quasiperiodic oscillations of X-ray luminosity of magnetars. First is given in the form

$$\frac{\nu^2({}_0t_\ell)}{\nu_0^2} = 2(\ell+2)(\ell-1)\left[1-\frac{1}{2(\ell+2)}\right] \;\to\; [2(\ell+2)(\ell-1)] \qquad \ell >> 1. \tag{10}$$

Second, the form

$$\frac{\nu^2({}_0t_\ell)}{\nu_0^2} = 2\ell(\ell+1)\left[1-\frac{1}{\ell(\ell+1)}\right]\left[1-\frac{1}{2(\ell+2)}\right] \;\to\; [2\ell(\ell+1)] \qquad \ell >> 1. \tag{11}$$

The comparison of obtained spectral formulae is shown in Fig.1.

3. Summary. In conclusion of this paper, which continues our long term investigations of transport properties of nuclear matter [3-7], it is argued that obtained asymptotic spectral formulae for the frequency of nodeless torsion oscillations have been presented in the form that can be conveniently applied to a wide class of celestial objects. The practical usefulness of presented exact solutions for the toroidal field of material displacements is that they can be utilized in the study of torsional vibrations restored by forces of Newtonian gravitation field stresses and Maxwellian magnetic field stresses, not only Hookean elastic stresses considered in this work.

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