Anisotropic Cosmological Models with Viscous Fluid, Spinor and Scalar Fields

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We consider a system of nonlinear spinor and Bianchi type-I gravitational filed in presence of a viscous fluid and Λ term. The nonlinearity of the spinor field is occurred as a result of a self-action or interaction with a scalar field. In doing so we consider following Lagrangian:

$$\mathcal{L} = \frac{i}{2}\left[\bar\psi\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma^\mu\psi\right] - m\bar\psi\psi + \lambda F, \tag{1}$$

describing a spinor field with self-action, and

$$\mathcal{L}_{ss} = \frac{i}{2}\left[\bar\psi\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma^\mu\psi\right] - m\bar\psi\psi + \frac{1}{2}\varphi_{,\alpha}\varphi^{,\alpha}(1+\lambda F), \tag{2}$$

describing an interacting system or spinor and scalar fields. Here $F = F(I, J)$ with $I = I_S = S^2 = (\bar\psi\psi)^2$ and $J = I_P = P^2 = (i\bar\psi\gamma^5\psi)^2$. The gravitational filed was given by the metric

$$ds^2 = dt^2 - a^2dx^2 - b^2dy^2 - c^2dz^2, \tag{3}$$

with a, b, c being the functions of time t only.

Using variational principle corresponding equations for spinor, scalar and gravitational fields were obtained and employing the method suggested and developed by one of the authors [cf. e.g. [1]] expressions for spinor, scalar and metric functions were found in terms volume scale $\tau = abc$. Further from Einstein equation and Bianchi identity we we deduce the system of equations for determination of τ:

$$\dot\tau = 3H\tau, \tag{4a}$$

$$\dot H = \frac{1}{2}\big(3\xi H - \omega\big) - \big(3H^2 - \varepsilon - \Lambda\big) + F_1, \tag{4b}$$

$$\dot\varepsilon = 3H\big(3\xi H - \omega\big) + 4\eta\big(3H^2 - \varepsilon - \Lambda\big) - F_2. \tag{4c}$$

where

$$F_1 = 0, \quad F_2 = 0, \tag{5}$$

describes the system without spinor field,

$$F_1 = \frac{\kappa}{2}\Big(\frac{m}{\tau} + \frac{\lambda(n-2)}{\tau^n}\Big), \quad F_2 = 4\eta\kappa\Big[\frac{m}{\tau} - \frac{\lambda}{\tau^n}\Big], \tag{6}$$

describes a spinor filed with self-action, and

$$F_1 = \frac{\kappa}{2}\Big[\frac{m}{\tau} + \frac{\lambda n}{2}\frac{\tau^{n-2}}{(\lambda+\tau^n)^2}\Big], \quad F_2 = 4\eta\Big[\frac{m}{\tau} + \frac{\tau^{n-2}}{2(\lambda+\tau^n)}\Big], \tag{7}$$

describes an interacting system of spinor and scalar fields. Here H is the Hubble parameter, η and ξ are the bulk and shear viscosity, respectively, κ is the Einstein gravitational constant, m is the spinor mass, n is the power of nonlinearity, λ is the coupling constant, and Λ is the cosmological constant.

In order to understand the role of spinor field in the evolution of the Universe we first solve the Einstein gravitational field equations with the source field given by a viscous fluid. Corresponding system was exactly solved in [2]. It should be noted that exact solutions of the system can be obtained only for some special choice of viscosity. To obtain an overall picture we performed a qualitative analysis of the system in [3].

Further we introduced a nonlinear spinor field given by (1) into the system. For some special cases this system was exactly solved in [4, 5]. It was shown that in absence of shear viscosity, if bulk viscosity ξ is taken to be inverse proportional to Hubble constant H, the system allows exact solution in quadrature. In this case independent to the sign of Λ we have the expanding mode of evolution, though a positive Λ accelerates the process, while the negative one decelerates. In a more general case when bulk viscosity ξ is constant and shear viscosity η is proportional to H exact solutions can be obtained only for some special choice of nonlinear spinor term, namely for $F = F_0 S^{2(\kappa-1)/\kappa}$. In case of $\kappa < 1$ evolution is non-periodic for all Λ, while in case of $\kappa > 1$ evolution is non-periodic if $\Lambda < 0$ and ever expanding for $\Lambda \geq 0$. For a better knowledge of the system in general we have solved it qualitatively. The results were reported in several seminar and XLII All Russia Conference on the problem of Mathematics, Informatics, Physics and Chemistry, Russian Peoples' Friendship University, Moscow, Russia [6]. A comprehensive version of this study is now under active consideration in Journal Physics A: Mathematical and General and available in ArXiv [7].

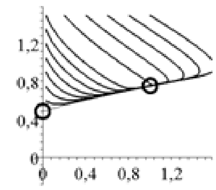
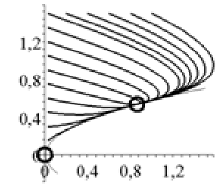
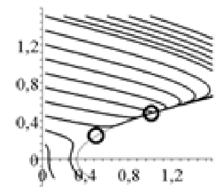
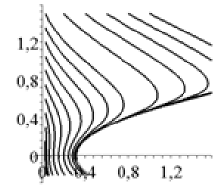
Finally in [8] we have considered an interacting system of spinor and scalar fields in a Bianchi type-I cosmological model in presence of viscous fluid and a Λ term. Proceeding as in previous case we find the analogical results obtained for the spinor field only if the shear viscosity is overlooked and the bulk viscosity is taken to be inverse proportional to Hubble parameter. In the more general case, i.e., when bulk viscosity is constant and shear viscosity is proportional to H we find

$$t = \begin{cases} \frac{1}{\sqrt{B^2-4AC}} \ln\left|\frac{2AH+B+\sqrt{B^2-4AC}}{2AH+B-\sqrt{B^2-4AC}}\right|, & B^2 > 4AC, \\ \frac{2}{\sqrt{4AC-B^2}} \arctan\frac{2AH+B}{\sqrt{4AC-B^2}}, & B^2 < 4AC, \\ -\frac{2}{2AH+B}, & B^2 = 4AC. \end{cases} \tag{8}$$

It shown that in the case considered the behavior of evolution does not depends solely on the sign of Λ but the sign of $\delta = B^2 - 4AC$. For $\delta = 0$ evolution is non-periodic for $\Lambda < 0$ and ever-expanding for $\Lambda \geq 0$. For $\delta > 0$ we have a expanding mode of evolution, while for $\delta < 0$ evolution is non-periodic, independent to the sign of Λ.

Given its richness of the corresponding system was qualitatively studied and reported in XLIII All Russia Conference on the problem of Mathematics, Informatics, Physics and Chemistry, Russian Peoples' Friendship University, Moscow, Russia [9].

In the tables we give the classification of the results on different plane for different choice of problem parameters.

<table>
<tr><th></th><th></th><th>$\Lambda > 0$</th><th>$\Lambda = 0$</th><th>$\Lambda < 0$</th></tr>
<tr><td>$\beta < 1/2$</td><td></td><td rowspan="2">
a)</td><td>
b)</td><td rowspan="3">

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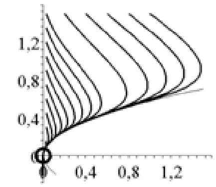
<tr><td rowspan="2">$\beta = 1/2$</td><td>$\frac{1+\zeta}{B} > \sqrt{3\kappa}$</td><td>
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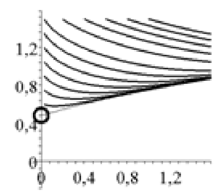
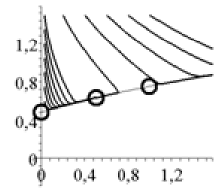
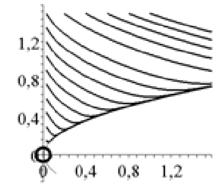
<tr><td>$\frac{1+\zeta}{B} < \sqrt{3\kappa}$</td><td rowspan="3">

e)</td><td>
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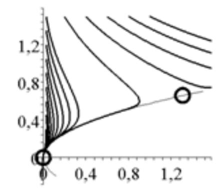
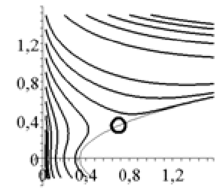
<tr><td>$1/2 < \beta < 1$</td><td></td><td rowspan="4">
g)</td><td rowspan="4">
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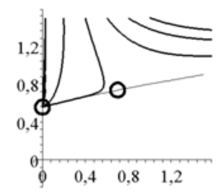
<tr><td>$\frac{1+\zeta}{B} < 3\Lambda$</td><td rowspan="2">
i)</td></tr>
<tr><td>$\beta > 1$</td><td></td></tr>
</table>

Table 1. Classification of qualitatively different types of evolution (phase portrait) depending on the parameters β, Λ and $(1+\zeta)/B$ in plane $\nu = 0$

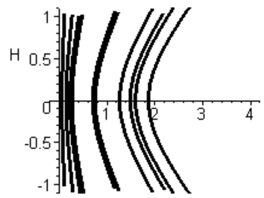
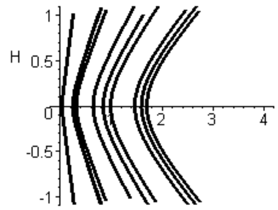
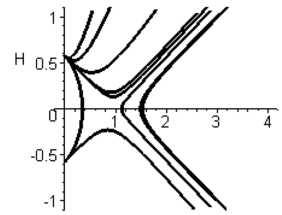
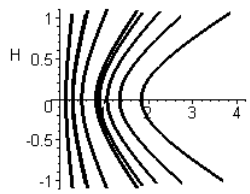
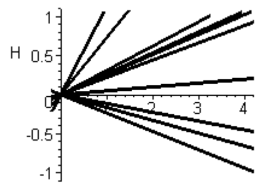
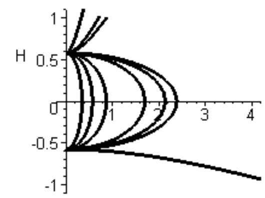
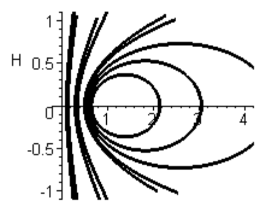
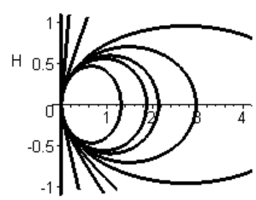
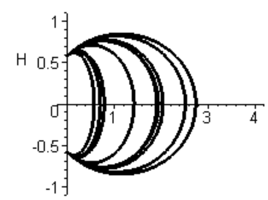
	$\Lambda < 0$	$\Lambda = 0$	$\Lambda > 0$
$m < 0$	 a)	 b)	 c)
$m = 0$	 d)	 e)	 f)
$m > 0$	 g)	 h)	 i)

Table 2. Classification of qualitatively different types of evolution (phase portrait) on $\varepsilon = 0$ plane for $n = 2$

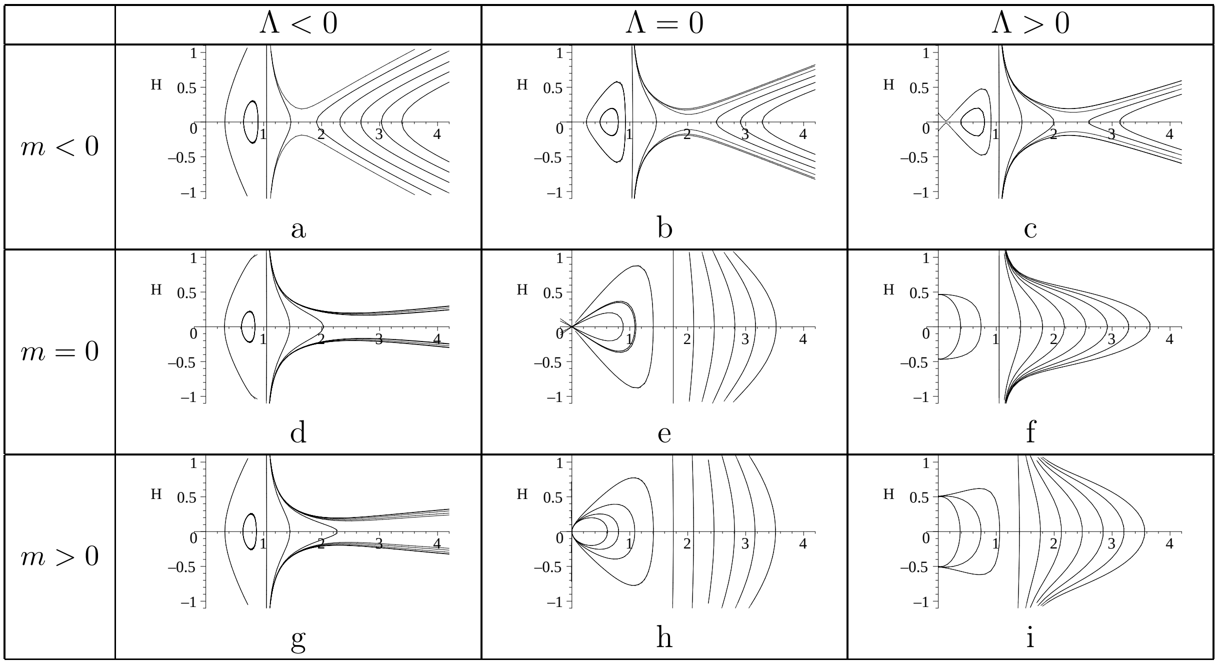


Table 3. Phase portrait on $\varepsilon = 0$ plane. Case with $n = 2$ and $\lambda < 0$

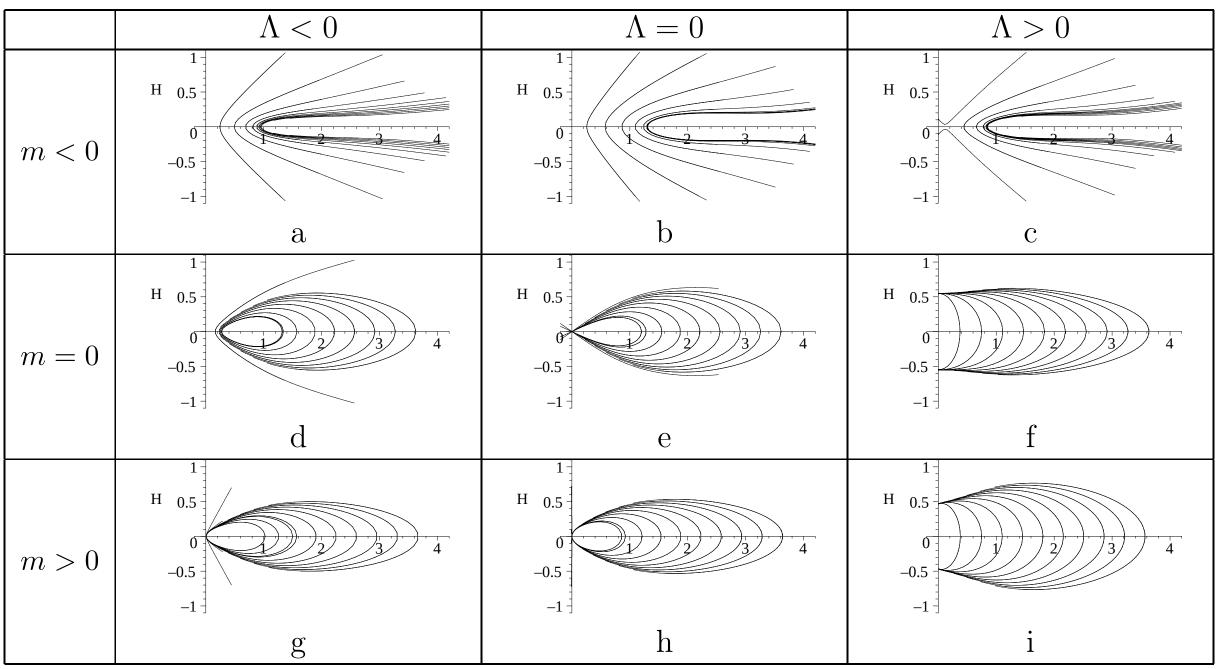


Table 4. Phase portrait on $\varepsilon = 0$ plane. Case with $n = 2$ and $\lambda > 0$

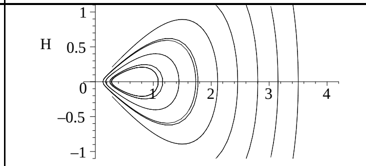
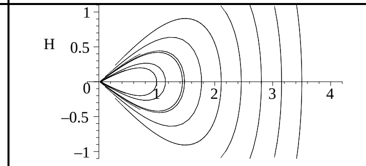
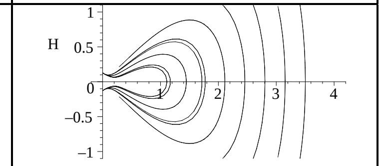
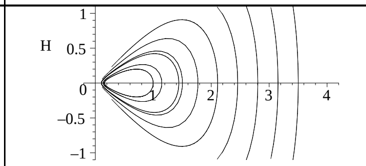
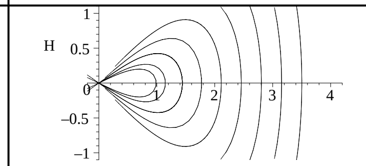
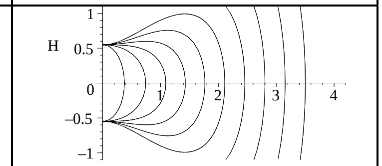
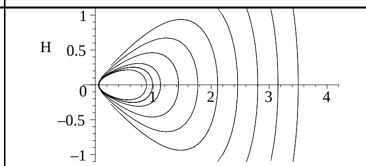
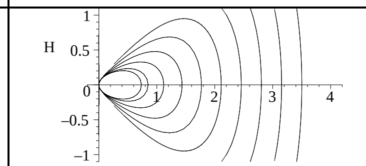
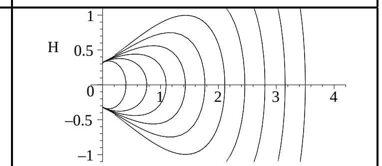
	$\Lambda < 0$	$\Lambda = 0$	$\Lambda > 0$
$m < 0$	 a	 b	 c
$m = 0$	 d	 e	 f
$m > 0$	 g	 h	 i

Table 5. Phase portrait on $\varepsilon = 0$ plane. Case with $n = 2$ and $\lambda = 0$

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