Ward-Takahashi Identity, Soft Photon Theorem and the Magnetic Moment of the Δ Resonance

1. Formulation without quark degrees of freedom
2. Formulation with quark degrees of freedom

A.I. Machavariani
Laboratory of Information Technologies, JINR

Abstract

Starting from the modified Ward-Takahashi identity for the on-shell radiative πN scattering amplitude a generalization of the soft photon theorem approach is obtained for an arbitrary energy of an emitted photon. The external particle radiation part of the $\pi N \to \gamma' \pi' N'$ amplitude is analytically reduced to the double Δ exchange amplitude with the intermediate $\Delta \to \gamma' \Delta'$ vertex.

We have shown that the double Δ exchange amplitudes with the intermediate Δ radiation is connected by current conservation with the corresponding part of the external particle radiation terms. Moreover, according to current conservation the internal and external particle radiation terms with the $\Delta - \gamma' \Delta'$ vertex have a opposite sign i.e. they must cancel each other. Therefore we have a screening of the internal double Δ exchange diagram with the $\Delta - \gamma' \Delta'$ vertex by the external particle radiation. This enables to obtain a model independent estimation of the dipole magnetic moment of Δ^+ and Δ^{++} resonances μ_Δ through the anomalous magnetic moment of the proton μ_p as $\mu_{\Delta^+} = \frac{M_\Delta}{m_p} \mu_p$ and $\mu_{\Delta^{++}} = \frac{3}{2} \mu_{\Delta^+}$ in agreement with the values obtained from the fit of the experimental cross section of the $\pi^+ p \to \gamma' \pi^+ p$ reaction.

Considering pions and nucleons as bound systems of quarks in the conventional quantum field theory, a generalized Ward-Takahashi identity for the on shell πN radiation amplitude is derived. This identity presents a general scheme of the current conservation which allows to obtain the model independent relations between the external and internal particle radiation amplitudes. The resulting equations for the external and internal particle radiation amplitudes of the πN bremstrahlung reaction have the same form as in formulation without the quark degrees of freedom [15]. Therefore current conservation and the Δ resonance pole position of the πN scattering amplitude determines analytically the dipole magnetic moment of the Δ resonances μ_Δ through the anomalous magnetic moment of the proton μ_p.

The present investigation of the πN radiation reaction based on the Ward-Takahashi identity for the on shell amplitudes. It generates the following model-independent relations:

(i) An amplitude of an arbitrary $a + b \longrightarrow \gamma' + f_1 + ... + f_n$ $(n = 1, 2, ...)$ reaction fulfills the generalized current conservations

$$k'_\mu < out; f_1, ..., f_n | \mathcal{J}^\mu(0) | a, b; in >= \left[\mathcal{B}_{f_1 ... f_n - ab} + k'_\mu \mathcal{E}^\mu_{\gamma' f_1 ... f_n - ab} \right]_{on\ mass\ shell\ f_1,\ ...\ f_n;\ a,\ b} = 0, \qquad (I)$$

where $\mathcal{E}^\mu_{\gamma' f_1 ... f_n - ab}$ corresponds to the complete set of Feynman (or three-dimensional time-ordered) diagrams with the photo-emission from each external particles and

$$\mathcal{B}_{f_1..f_n-ab} = \sum_{m=1(I_1\neq m...I_{n-1}\neq m)}^{n} e_m < out; f_{I_1}...f_{I_{n-1}}|J_m(0)|a,b;in >$$
$$-e_a < out; f_1...f_n|J_a(0)|b;in > -e_b < out; f_1...f_n|J_b(0)|a;in > \qquad (II)$$

stands for amplitudes of the $a+b \longrightarrow f_1+...+f_n$ reaction without γ' emission.

A special case of relation (I) is the external particle radiation terms.

Equation (I) and (II) are also valid for an arbitrary number of external photons. For instance, these equations can be used as current conservation for the pion photoproduction reaction $\gamma A \rightarrow \pi' A'$, for Compton scattering $\gamma A \rightarrow \gamma' A'$ etc.

(**ii**) Current conservation (I) requires the existence of the internal particle radiation amplitude $\mathcal{I}^{\mu}_{\gamma' f_1..f_n-ab}$ which ensures the validity of this condition

$$k'_{\mu}\mathcal{I}^{\mu}_{\gamma' f_1...f_n-ab} = \mathcal{B}_{f_1...f_n-ab}, \ or \ k'_{\mu}\mathcal{E}^{\mu}_{\gamma' f_1..f_n-ab} + k'_{\mu}\mathcal{I}^{\mu}_{\gamma' f_1..f_n-ab} = 0. \qquad (III)$$

This means that $\mathcal{E}^{\mu}_{\gamma' f_1..f_n-ab}$ and $\mathcal{I}^{\mu}_{\gamma' f_1...f_n-ab}$ have a different sign and they must be subtracted from each other. Thus we have a screening of the internal particle radiation amplitudes by the external one-particle radiation terms.

(**iii**) For the soft emitted photons $k' \rightarrow 0$ our approach immediately reproduces the low energy theorems for the bremsstrahlung reactions.

(**iv**) The external particle radiation part of the bremsstrahlung amplitude $\mathcal{E}^{\mu}$ contains the electromagnetic form factors of the external particles only in the tree approximation. This follows from the equal-time commutators which are a result of charge conservation. Thus we must modify the equal-time commutators between the Heisenberg operators of the external particles in order to apply the full electromagnetic form factors of pions and nucleons in the current conservation condition (I) or (III).

The above screening mechanism has been applied to the πN bremsstrahlung reaction with the leading double Δ exchange term. We have shown, that in the low energy region, where the electric quadrupole and the magnetic octupole momenta of Δ can be neglected, the intermediate Δ radiation radiation term is completely canceled against the corresponding part of the external particle radiation amplitude. From this cancellation follows the normalization condition for the Coulomb monopole part of the $\Delta - \gamma'\Delta'$ vertex which allows to extract the Δ^+ and Δ^{++} dipole magnetic momenta $\mu_{\Delta^+} = G_{M1}(0) = \frac{M_\Delta}{m_N}\mu_p$ and $\mu_{\Delta^{++}} = \frac{3}{2}\mu_{\Delta^+} = 5.46e/2m_p$ or $\mu_{\Delta^{++}}/\mu_p \sim 1.95$. Our result for $\mu_{\Delta^{++}}$, based on the model independent current conservation condition, is in agreement with the prediction of the naive $SU(6)$ quark model for $\mu_{\Delta^{++}} = 2\mu_p = 5.58e/2m_p$ [1, 2], with the nonrelativistic potential model [9] $\mu_{\Delta^{++}} = 4.6 \pm 0.3$. and with extraction of $\mu_{\Delta^{++}}$ from the $\pi^+ p \rightarrow \gamma\pi^+ p$ experimental cross section in the framework of the low energy photon approach $\mu_{\Delta^{++}} = 3.6 \pm 2.0$ [4], $\mu_{\Delta^{++}} = 5.6 \pm 2.1$ [5] and $\mu_{\Delta^{++}} = 4.7 - 6.9$ [7]. Our result is larger as the predictions in the modified $SU(6)$ models [10, 3] and in the soft-photon approximation $\mu_{\Delta^{++}} = 3.7 \sim 4.9e/2m_p$ [6]. On the other hand our result is smaller as the values obtained in the framework of the effective meson-nucleon Lagrangian $\mu_{\Delta^{++}} = 6.1 \pm 0.5e/2m_p$ [13], in the effective quark model $\mu_{\Delta^{++}} = 6.17e/2m_p$ [14] and in the modified bag model $\mu_{\Delta^{++}} = 6.54$ [11].

The summary of the numerical estimations of the magnetic moments of Δ^+ and Δ^{++} resonances is given in table 1. In a number of approaches the magnetic moment of Δ is treated as an adjustable parameter in the radiative πN scattering which is determined using the most sensitive configurations to the $\Delta - \gamma\Delta$ vertex in the slow photon regime.

Table 1: Magnetic moments of Δ^+ and Δ^{++} in units of the nuclear magneton $\mu_N = e/2m_N$. The ref. in front of the index f indicates the theoretical model which is used to fit of the experimental data and to extract the magnetic moment μ_Δ

MODELS	*This work*	$SU(6)$	*Potential and K-matrix appr.*	*Modified Bag*	*Soft photon theorem*	*Eff. πN Lagran.*	*Eff. quark*
μ_{Δ^+}	3.64	2.79 [1, 2]					2.79[14]
$\mu_{\Delta^{++}}$	5.46	5.58 [1, 2] 4.25[3] 4.41-4.89[10]	6.9-9.7[8]f 4.6±0.3[9]f 5.6-7.5[12]f	6.54[11]	3.6±2.0[4]f 5.6±2.1[5]f 4.7-6.9[7]f 3.7-4.9[6]f	6.1±0.5[13]f	6.17[14]

Corresponding results obtained from the experimental cross sections of the $\pi^+ p \to \gamma\pi^+ p$ reaction are indicated in the table 1 with the index f. It must be emphasized, that only our approach and naive $SU(6)$ quark model gives an analytical form for μ_{Δ^+} and $\mu_{\Delta^{++}}$. But our result for μ_{Δ^+} is $M_\Delta/m_p \sim 1.31$-times larger as $\mu_{\Delta^+} = \mu_p = 2.79e/2m_p$ in refs. [1, 14].

This screening mechanism can be observed in the cross sections of the πN bremsstrahlung reaction or in the $\gamma p \to \gamma\pi^o p$ reaction by comparison of the cross sections in and outside the Δ resonance region. Due to the importance of the double Δ exchange diagram (Fig. 2B) one must have a different $1/k'$ behavior of the bremsstrahlung amplitude in and outside the Δ resonance region.

Next we have extended our work [15] of the analytic extraction of the dipole magnetic moments of the Δ resonances on the base of the modified Ward-Takahashi identities for the on shell πN bremsstrahlung amplitude. This extension is done in the framework of the general field-theoretical approach [17, 18, 19, 16], where particles are constructed as the bound (composite) states of quarks and gluons. The creation and annihilation operators of the composite pions and nucleons enables to construct the pion-nucleon radiation amplitude $< out; \mathbf{p'}_N \mathbf{p'}_\pi | J^\mu(0) | \mathbf{p}_\pi \mathbf{p}_N; in >$ with on mass shell pions and nucleons in the asymptotic $''in''$ and $''out''$ states and $J^\mu(0)$ current operator of photon. Afterwards the Ward-Takahashi identity follows from the current conservation $k'_\mu \; < out; \mathbf{p'}_N \mathbf{p'}_\pi | J^\mu(0) | \mathbf{p}_\pi \mathbf{p}_N; in >= 0$. Besides we have used the charge conservation which determines the equal-time commutation rules for the photon current operator and quark field operators. A model-independent connection between the external $\mathcal{E}^\mu$ and internal $\mathcal{I}^\mu$ particle radiation terms follows from the corresponding Ward-Takahashi identity which has the same form as in the formulation without quark-gluon degrees of freedom. In particular, $\mathcal{E}^\mu$ and $\mathcal{I}^\mu$ have the opposite sign because they satisfy the condition $k'_\mu < out; \mathbf{p'}_N \mathbf{p'}_\pi | J^\mu(0) | \mathbf{p}_\pi \mathbf{p}_N; in >= k'_\mu \mathcal{E}^\mu + k'_\mu \mathsf{I}^\mu = 0$. Therefore after the same transformations as in [15] one obtains $\mathsf{D}^\mu(\Delta\Delta) = -\mathsf{I}^\mu(\Delta\Delta)$ Thus the internal particle radiation part $\mathsf{I}^\mu(\Delta\Delta)$ and the corresponding double Δ exchange part of the external particle radiation amplitude $\mathcal{E}_2^\mu$ cancel. In other words the same screening of the internal particle radiation terms by the external particle radiation diagrams must be observed in the bremstrahlung reactions in the formulations with and without quark degrees of freedom. This screening mechanism allows us determine the dipole magnetic moments of the Δ resonances via the magnetic moments of the external nucleons in the same way as it was done in our previous paper without quark degrees of freedom [15].

The general current conservation for the bremstrahlung reactions with and without quarks were studied in the framework of the 3D time-ordered field theoretical approach which was developed in refs. [20, 21, 22, 23].

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