

Focus Points and the Lightest Higgs Boson Mass in the Minimal Supersymmetric Standard Model

E. Jurcisinova

Laboratory of Information Technologies, JINR

M. Jurcisin

Laboratory of Theoretical Physics, JINR

Abstract

We investigate focus points of the renormalization group equations of the Minimal Supersymmetric Standard Model. We show that within this model the up- and down-type Higgs mass soft supersymmetry breaking parameters have focus point behavior at the electroweak scale simultaneously when appropriate conditions are fulfilled. The focus point scenario is holding for large $\tan\beta$. This two focus point scenario allows one to fix the pole top-quark mass which is within the experimentally allowed interval. The main goal of this paper is the investigation of the influence of the existence of focus points on the determination of the mass of the lightest Higgs boson.

When working within the Minimal Supersymmetric Standard Model (MSSM), one encounters large parameter freedom which is mainly due to the so-called soft SUSY breaking terms [1]. At the same time, a large number of free parameters decreases the predictive power of a theory. The simplest way to reduce this freedom is to make some assumptions at a high energy scale (for example, at the Grand Unification (GUT) scale or at the Planck scale). Then, treating the MSSM parameters as running variables and using the renormalization group equations (RGEs), one can drive their values at a low-energy scale. The most common assumption is the so-called universality of the soft supersymmetry breaking terms, which means an equality of some parameters at a high energy scale. Adopting the universality, one reduces the parameter space to a five-dimensional one given by: a common scalar mass m_0 , a common gaugino mass $m_{1/2}$, a common trilinear scalar coupling A , a supersymmetric Higgs-mixing mass parameter μ , and a bilinear Higgs coupling B . The last two parameters can be eliminated in favor of the electroweak symmetry breaking scale, $v^2 = v_1^2 + v_2^2 = (174.1\text{GeV})^2$, and the Higgs fields vevs ratio $\tan\beta = v_2/v_1$, when using minimization conditions of the Higgs potential.

Further reduction of the parameter space of the MSSM can be achieved using the concept of the so-called infrared quasi fixed points (IRQFPs) [2]. Over the last ten years a great interest was paid to the phenomenological consequences of the IRQFP behavior of the corresponding system of the RGEs within the MSSM [3, 4, 5].

However, the IRQFP scenario can be directly used and works properly only in the case of small $\tan\beta \sim 1$ regime (see, e.g., discussion in Ref. [5]). On the other hand, the small $\tan\beta$ scenario is already excluded by the recent experimental data [6], therefore the moderate and large $\tan\beta$ regimes come to be investigated from the phenomenological point of view.

Recently the idea of so-called focus point was used in the investigation of the system of RGEs of the MSSM [7]. This means that the RG trajectories of some parameter of the model may meet at a “focus point”, where their values are independent of their ultraviolet boundary values. In the Refs. [7] the focus point behaviour of the up-type Higgs mass parameter was investigated. The different aspects of this idea was then discussed in several

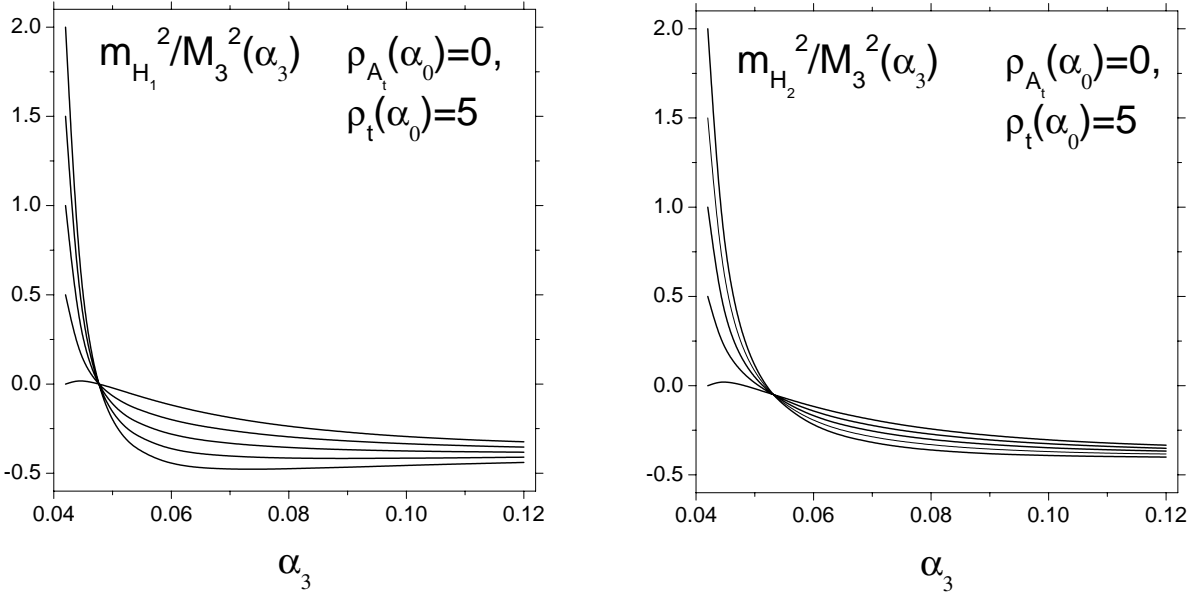


Fig. 1: The focus points behavior for soft mass parameters $m_{H_1}^2$ and $m_{H_2}^2$ for the concrete universal values of Yukawa coupling constants and trilinear scalar couplings (that means $\rho_t(\alpha_0) = \rho_b(\alpha_0) = \rho_\tau(\alpha_0)$ and $\rho_{A_i}(\alpha_0) = \rho_{A_b}(\alpha_0) = \rho_{A_\tau}(\alpha_0)$). The value $\alpha_3 \approx 0.042$ corresponds to the GUT scale, and the value $\alpha_3 \approx 0.12$ corresponds to the electroweak scale

papers [8]. In what follows we adopt the strategy based on the focus point behavior of the RGEs in the analysis of the mass of the lightest Higgs boson. It is found that within the MSSM the up- and also down-type Higgs mass soft parameters have focus point behavior at the electroweak scale simultaneously when appropriate conditions are fulfilled [9]. This leads to the determination of the top-quark mass. Thus, if the focus point scenario works, the lightest Higgs boson mass is determined more precisely. As we shall see the mass of the lightest Higgs boson determined by this method is allowed by recent experiment [6].

In what follows we shall use the notations as in Ref. [4]. Besides, for Yukawa coupling constants $h_i, i = t, b, \tau$ we also use expressions $\rho_i = Y_i/\tilde{\alpha}$ where $Y_i = h_i^2/(4\pi)^2$ and $\tilde{\alpha}_3 = \alpha_3/(4\pi) = g_3^2/(4\pi)^2$ is strong coupling constant (t, b, τ correspond to top quark, bottom quark, and τ lepton, respectively). For trilinear scalar coupling A_i we also use definition $\rho_{A_i} = A_i/M_3$, where M_3 represents the gluino mass. $\alpha_0 = \alpha_{30}$ is the unified coupling constant at the GUT scale.

By complete analysis of the system of the one-loop RGEs in the MSSM one can find that focus point behavior is connected with the Higgs mass soft parameters $m_{H_1}^2$ and $m_{H_2}^2$ (the explicit form of all one-loop RGEs in the MSSM can be found, e.g., in Ref. [1]). In Fig. 1 is present their running for different initial values at the GUT scale ($M_{GUT} = 2 \cdot 10^{16}$) and where the focus points are shown explicitly.

Our aim is to analyze if it is possible to have both focus points of the RGEs for up- and down-type Higgs mass parameter at the electroweak scale simultaneously. Using numerical calculations it is shown that such situation is possible if the Yukawa coupling constants Y_i have appropriate initial values at the GUT scale. In our investigation we suppose the universal behaviour of the soft SUSY breaking parameters at the GUT scale. Thus, the parameter space of the model is almost completely reduced. In Fig. 2 are present the simultaneous values of the Yukawa coupling constants at the GUT scale which leads to the focus points for soft Higgs mass parameters at the electroweak scale. It can be

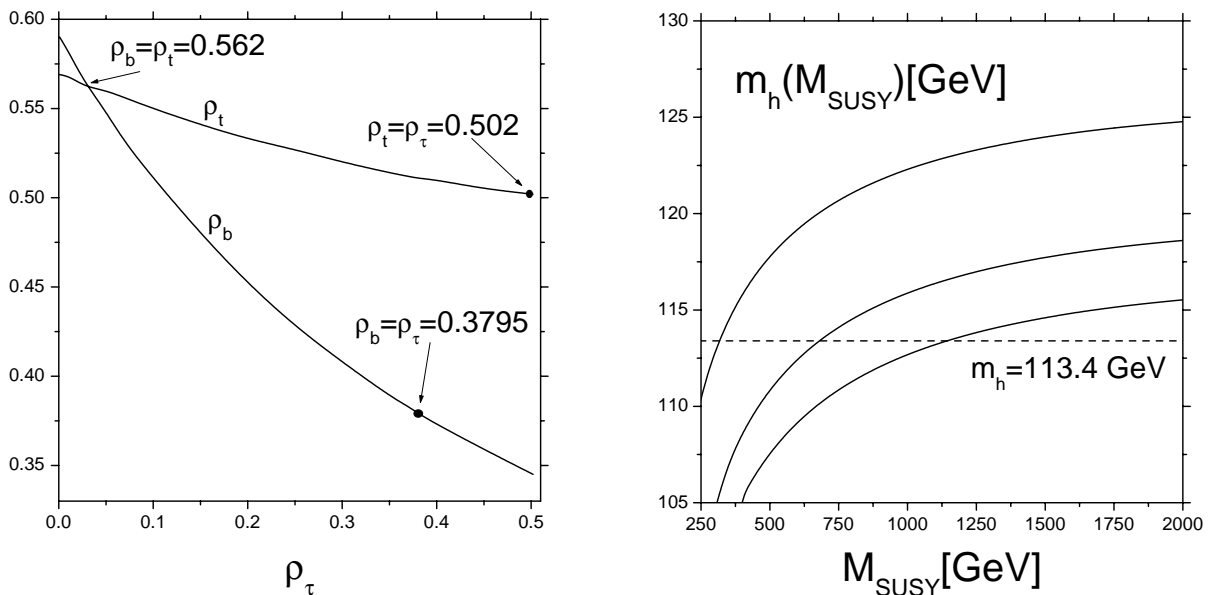


Fig. 2: (Left) The values of the Yukawa couplings for which the up- and down-quark Higgs mass soft parameters have focus points at the electroweak scale simultaneously. The marked points and the crossing of the lines correspond to the situation when two initial values of the Yukawa couplings at the GUT scale are the same. (Right) The mass of the lightest Higgs boson in the focus point scenario with universal assumption about soft SUSY breaking parameters. The central line is related to the central values of the parameters. The upper and lower lines correspond to the maximal allowed deviations of the soft parameters. The dash line represents the experimental restriction of the Higgs boson mass [6]

shown that the influence of the initial values of the gaugino mass soft parameters and the trilinear scalar couplings on the position of the focus points is negligible. On the other hand, as we shall see in the next section, their values will have some non-negligible impact on the determination of the lightest Higgs boson mass.

Now we have all necessary tools at hand to discuss the lightest Higgs boson mass based on the focus point scenario. Our strategy is as follows: As input parameters we take the known values of the top-quark, bottom-quark and τ -lepton pole masses ($m_t^{\text{pole}} = (172.7 \pm 2.9)$ GeV [10], $m_b^{\text{pole}} = (4.94 \pm 0.15)$ GeV [11], $m_\tau^{\text{pole}} = (1.7771 \pm 0.0005)$ GeV [12]), the experimental values of gauge couplings [13] $\alpha_3 = 0.120 \pm 0.005$, $\alpha_2 = 0.034$, $\alpha_1 = 0.017$ and the sum of the Higgs vev's squared $v^2 = v_1^2 + v_2^2 \approx 174.1$ GeV². Using the focus point scenario which determine the values of the Yukawa coupling constants at the GUT scale, we proceed to the determination of the $\tan \beta$ and top and bottom quark masses which are related by well-known relations

$$m_t = h_t v \sin \beta \quad (1)$$

$$m_b = h_b v \cos \beta \quad (2)$$

$$m_\tau = h_\tau v \cos \beta \quad (3)$$

where m_i , $i = t, b, \tau$ are running quark and lepton masses. The aim is to find such values of $\tan \beta$ and top-quark, bottom-quark and τ -lepton masses to be inside the intervals allowed by experiment and, at the same time, to fulfill Eqs. (1-3) in the framework of our focus point scenario. It can be shown that this problem has solutions with $\tan \beta \simeq 60$. The most

problematic in the process of calculation is to obtain the proper mass of the τ -lepton. In our calculations we determine the top-quark and bottom-quark running masses from the corresponding pole masses taking into account QCD and SUSY corrections (for details see Ref. [4])

$$m_i(m_i) = \frac{m_i^{pole}}{1 + \left(\frac{\Delta m_i}{m_i}\right)_{QCD} + \left(\frac{\Delta m_i}{m_i}\right)_{SUSY}}, \quad i = t, b. \quad (4)$$

The results depend on the sign of the μ parameter which enters the mixing terms in the stop sector. In what follows, we shall analyze only the case $\mu > 0$. The case $\mu < 0$ is similar with the almost the same results and conclusions for the mass of the lightest Higgs boson mass. The value of the Higgs mixing parameters μ can be found from the requirement of radiative electroweak symmetry breaking and can be determined from the Higgs potential minimization condition. The one-loop minimization condition reads

$$\frac{M_Z^2}{2} + \mu^2 = \frac{m_{H_1}^2 + \Sigma_1 - (m_{H_2}^2 + \Sigma_2) \tan^2 \beta}{\tan^2 \beta - 1}, \quad (5)$$

where Σ_1 and Σ_2 are the one-loop corrections, M_Z is the Z -boson mass (for details see Ref. [4]).

In the MSSM, the Higgs sector consists of five physical states: two neutral CP-even scalars h and H , one neutral CP-odd scalar A , and two charged Higgs scalars H^\pm . In what follows we shall concentrate on the mass of the lightest Higgs boson h . At the tree level, the mass of h is smaller than the mass of Z -boson, M_Z , but the loop corrections increase it. In general, the mass matrix for the CP-even neutral Higgs bosons looks like

$$\mathcal{M} = \left[\begin{pmatrix} \tan \beta & -1 \\ -1 & \cot \beta \end{pmatrix} m_A^2 + \begin{pmatrix} \cot \beta & -1 \\ -1 & \tan \beta \end{pmatrix} M_Z^2 \right] \cos \beta \sin \beta + \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{12} & \Delta_{22} \end{pmatrix} \quad (6)$$

where m_A is the mass of the CP-odd Higgs boson and Δ 's are the radiative corrections (for details see Ref. [4] and references cited therein).

To find the Higgs boson mass, one has to diagonalize the mass matrix (6). In the present paper we use the concept of focus points and universality conditions for soft SUSY breaking parameters. The Yukawa couplings are determined by focus points to be at the electroweak scale and soft parameters are inside the following intervals at the GUT scale: $A_0 = A_{0i}/M_{03} \in \langle -3, 3 \rangle$, $i = t, b, \tau$, $m_0^2 = m_{0i}^2/M_{03}^2 \in \langle 0.25, 4 \rangle$, $i = Q, U, D, H_1, H_2$, where Q refers to the third generation squark doublet, U to the stop singlet and D to the sbottom singlet.

In Fig. 2, the dependence of the mass of the lightest Higgs boson m_h on the geometric mean of the stop masses $\sqrt{\tilde{m}_{t_1} \tilde{m}_{t_2}}$ (which is often identified with the supersymmetry breaking scale M_{SUSY}) is shown for the case when bottom-quark and τ -lepton Yukawa couplings at the GUT scale are equal. As the central values of the parameters we take: $A_{0t}/M_{03} = 0$, $m_0^2/M_{03}^2 = 1$. Taking into account corresponding deviations from the central values, the mass of the lightest Higgs boson at a typical scale $M_{SUSY} = 1$ TeV ($\mu > 0$) is

$$m_h = 115.9 \begin{matrix} +6.4 \\ -3.2 \end{matrix} \pm 0.4 \text{ GeV}, \quad \text{for } M_{SUSY} = 1 \text{ TeV}. \quad (7)$$

The first uncertainty is given by the deviations from central values of the soft breaking parameters, and the second one by uncertainty in the strong coupling constant. Our

main result is that in this approach the dependence on the top-quark mass disappeared completely. For the central value of the presented result (7) we find $m_t^{pole} = 174.2$ GeV and $\tan\beta = 58$. It is important that the top-quark mass obtained by the focus point scenario is inside of experimentally allowed interval [10]. One can see that in the focus point scenario the mass of the lightest Higgs boson typically belongs to the interval $< 113, 122 >$ GeV. If we compare our results to the experimental restriction to the mass of the lightest Higgs boson [6] $m_h > 113.4$, one can conclude that there still exists a little space to find Higgs boson related to the MSSM.

Conclusions: We have analyzed the behavior of the system of RGEs in the MSSM from the point of view of focus point behavior. We have found that such type of behavior is typical for up- and down-quark Higgs mass soft parameters and that it is possible to have both focus points at the electroweak scale simultaneously which leads to the large $\tan\beta$ regime. This non-trivial fact results in further reduction of the parameter space of the model. The most important conclusion is that the uncertainty in the top-quark mass disappeared completely. The mass of the lightest Higgs boson is obtained and it is not excluded by the experimental restrictions.

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