Two Approaches for e/π Identification Applying the CBM TRD

E.P. Akishina, T.P. Akishina, O.Yu. Denisova, V.V. Ivanov
Laboratory of Information Technologies, JINR

Abstract

Comparative analysis of two approaches for e/π identification based on the energy losses in the n-layered CBM TRD is presented.

We consider two approaches for e/π identification using the Transition Radiation Detector (TRD) in the CBM experiment. They are based on the measurements of ionization losses (dE/dx) for π (top plot) and e (bottom plot), including energy losses on the transition radiation, in a one-layer TRD prototype: beam-test in GSI, $p = 1.5$ GeV/c, February 2006 (Fig. 1). These measurements have been used for simulation of energy

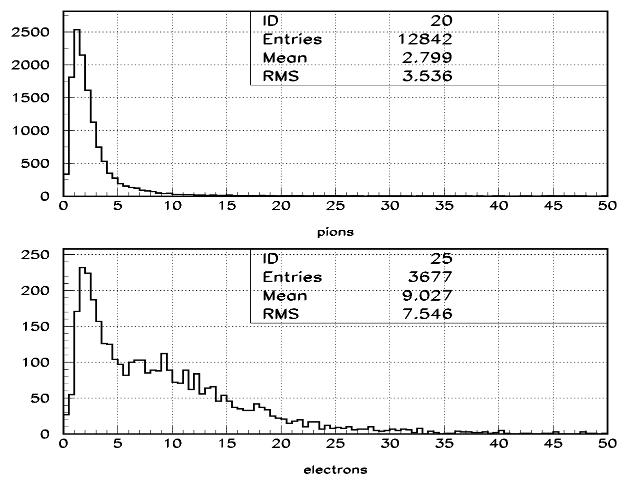

Figure 1: Distributions of energy losses of π (bottom plot) and e (top plot), including the transition radiation, in the TRD prototype: $p = 1.5$ GeV/c

losses by e and π during their passing through the n-layered TRD.

In the first approach, a method of ratio of likelihood functions is used for particles identification: see, for example, [1, 2]. While applying the likelihood test to the problem considered, the value

$$L = \frac{P_e}{P_e + P_\pi}, \qquad P_e = \prod_{i=1}^{n} p_e(\Delta E_i), \qquad P_\pi = \prod_{i=1}^{n} p_\pi(\Delta E_i), \tag{1}$$

is calculated for each set of energy losses, where $p_\pi(\Delta E_i)$ is the value of the density function p_π in the case when π loses energy ΔE_i in the i-th absorber, and $p_e(\Delta E_i)$ is a similar value for e.

In order to calculate correctly the value of variable L, it is necessary to construct the density functions which with a good accuracy must reproduce the distributions of energy losses of π and e (Fig. 1).

We have found that the distribution of ionizing energy losses of π is quite well approximated by a log-normal function

$$f_1(x) = \frac{A}{\sqrt{2\pi}\sigma x} \exp^{-\frac{1}{2\sigma^2}(\ln x - \mu)^2}, \tag{2}$$

where σ is the dispersion, μ is the mean value, and A is a normalizing factor (Fig. 2), and

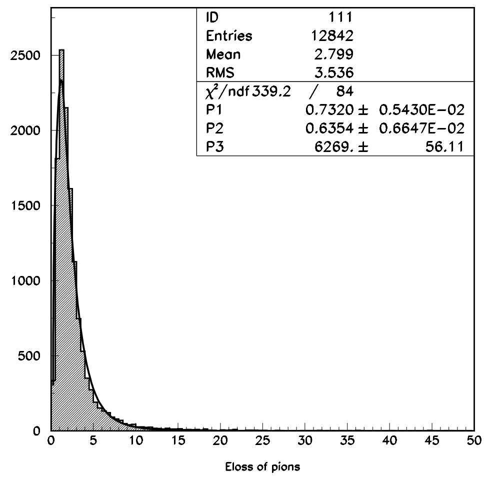

Figure 2: Approximation of the distribution of pion energy losses in the TRD prototype by a log-normal function (2)

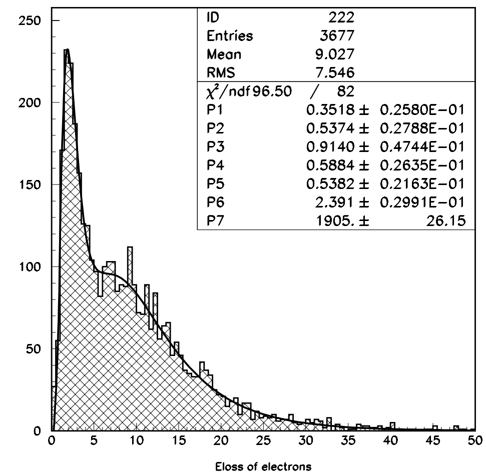

Figure 3: Approximation of the distribution of electron energy losses in the TRD prototype by a weighted sum of two log-normal functions (3)

electron energy losses are approximated with a high accuracy by a weighted sum of two log-normal distributions (Fig. 3)

$$f_2(x) = B\left(\frac{a}{\sqrt{2\pi}\sigma_1 x} \exp^{-\frac{1}{2\sigma_1^2}(\ln x - \mu_1)^2} + \frac{b}{\sqrt{2\pi}\sigma_2 x} \exp^{-\frac{1}{2\sigma_2^2}(\ln x - \mu_2)^2}\right), \tag{3}$$

here σ_1 and σ_2 are dispersions, μ_1 and μ_2 are mean values, a and $b = 1 - a$ are the contributions of the first and second log-normal distributions, correspondingly, and B is a normalizing factor.

The distributions of L in cases when only π (top left plot) or only e (top right plot) pass through the n-layered TRD with are presented in Fig. 4; the bottom plot shows a summary distribution for both particles.

The efficiency of electrons registration is determined by the ratio of electrons selected in the admissible region for the preassigned significance level α (first order error) to part β of pions having hit in the admissible region (second order error).

In our case α value was set approximately equal to 10 %. In particular, the critical value $L_{cr} = 0.00035$ corresponds to the significance level $\alpha = 10.24\,\%$, thus, in the admissable region there will remain 89.76 % of electrons. In this case, the second order error $\beta = 0.0274\,\%$. Thus, the suppression factor of pions, which equals to $100/\beta$, will make up 3646.

The second approach is based on a successive application of two statistical criteria: 1) the mean value method, and 2) the ω_n^k-test.

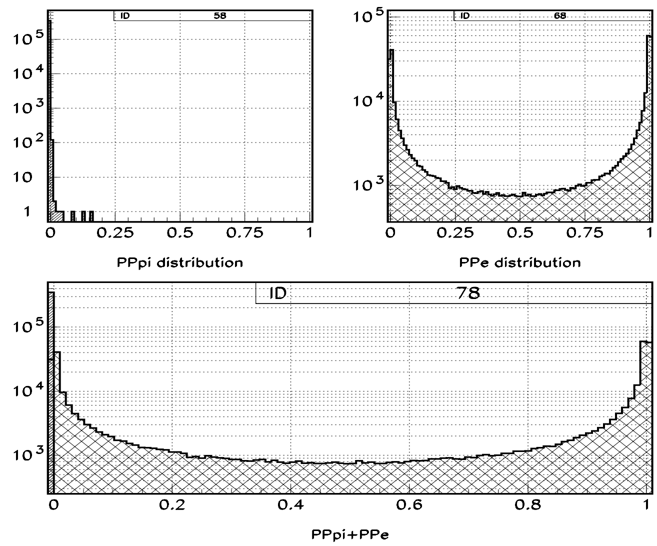

Figure 4: Distributions of L in cases when only π (top left plot) or only e (top right plot) pass through the TRD with $n = 12$ layers; the bottom plot is a summary distribution for both particles

In the mean value method a variable

$$\overline{\Delta E} = \frac{1}{n}\sum_{i=1}^{n} \Delta E_i$$

is calculated, where n is the number of layers in the TRD.

Figure 5 shows distributions of $\overline{\Delta E}$ for e (left top plot), π (right top plot), and a summary distribution for both particles (bottom plot). It is clearly seen that the pion distribution is quite well separated from the electron one. If we set the critical value $\overline{\Delta E}_{cr}$ = 6.3, then there will remain 90.62 % of e in the admissible region, and the admixture of π identified as electrons β will form 0.055 %. Thus, the factor of π suppression will constitute 1833.

This result could be significantly improved, if we apply the ω_n^k-test to the events selected in the admissible region [3, 4].

This test is based on the comparison of the distribution function $F(x)$ corresponding to a preassigned null-hypothesis (H_0) with empirical distribution function $S_n(x)$:

$$S_n(x) = \left\{ \begin{array}{rl} 0, & \text{if} \quad x < x_1; \\ i/n, & \text{if} \quad x_i \le x \le x_{i+1}, \quad i = 1, \dots, n-1. \\ 1, & \text{if} \quad x_n \le x, \end{array} \right. \tag{4}$$

Here $x_1 \le x_2 \le \ldots \le x_n$ is the ordered sample (*variational series*) of size n constructed on the basis of observations of variable x.

Energy losses for π have a form of Landau distribution. We use it as H_0 to transform the initial measurements to a set of variable λ:

$$\lambda_i = \frac{\Delta E_i - \Delta E^i_{mp}}{\xi_i} - 0.225, \qquad \text{i = 1,2,...,n,} \tag{5}$$

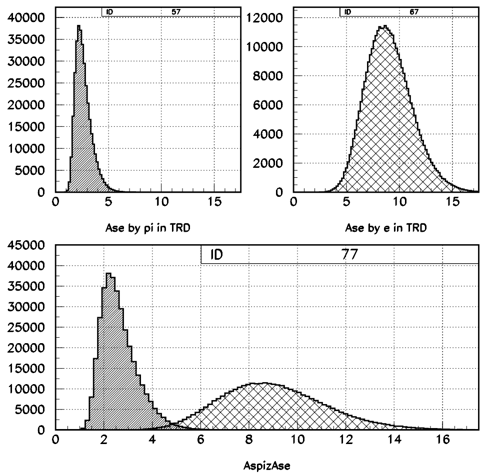

Figure 5: Distributions of variable $\overline{\Delta E}$ for e (left top plot), π (right top plot); summary distribution (bottom plot)

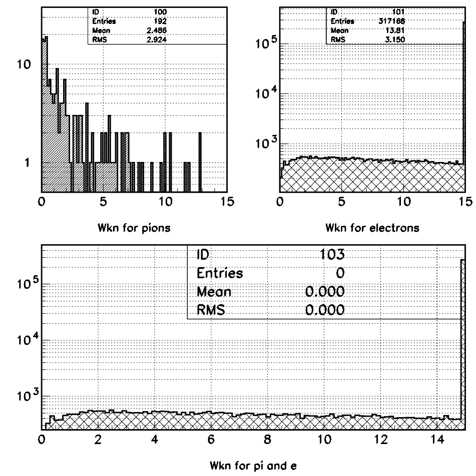

Figure 6: Distributions of ω_{12}^8 values for π (top left plot) and e (top right plot) events; summary distribution (bottom plot)

ΔE_i – energy loss in the i-th absorber, ΔE_{mp}^i – the value of most probable energy loss, $\xi_i = \frac{1}{4.02}$ FWHM of distribution of energy losses for π (see details in [2]).

The obtained $\lambda_i,\ i = 1, ..., n$ are ordered due to their values $(\lambda_j,\ j = 1, ..., n)$ and used for determination of ω_n^k

$$\omega_n^k = -\frac{n^{\frac{k}{2}}}{k+1} \sum_{i=1}^{n} \left\{ \left[\frac{i-1}{n} - \phi(\lambda_i) \right]^{k+1} - \left[\frac{i}{n} - \phi(\lambda_i) \right]^{k+1} \right\}, \tag{6}$$

where the values of Landau distribution function $\phi(\lambda)$ are calculated using the DSTLAN function (CERNLIB library).

Figure 6 shows the distributions of ω_{12}^8 values for π (top left plot) and for e (top right plot: here all values of $\omega_{12}^8 > 15$ were set equal to 15); a summary distribution is presented in the bottom plot.

Table 1 presentss the results of comparison of the given methods: α is part of lost electrons, β is the fraction of pi identified as e, pion suppression factor equals $100/\beta$.

Table 1: Comparison of the given methods

method	$\alpha, \%$	$\beta, \%$	suppression of pions
likelihood	10.24	0.0274	3646
mean value	9.38	0.055	1833
mean value + ω_n^k	10.54	0.02857	3500

These results demonstrate that, under the condition of loss of approximately 1 % of electrons, the application of the ω_n^k-test to the events selected in the admissible region permits us to increase the factor of pions suppression in almost two times. Thus, we have achieved the result which is very close to the limit value obtained by the likelihood test.

The likelihood functions ratio test could be related to Neiman-Pirson criterion, which is the most powerful criterion for testing the hypothesis H_0 against the alternative hypothesis H_1 [1]. Therefore, for the given significance level $\alpha = 10.24\,\%$ the value of $\beta = 0.0274\,\%$ could be considered as minimally possible (which corresponds to the maximum factor of pions suppression).

The bottleneck of this method is that the distribution of energy losses for electrons is strongly dependent on their momenta. At the same time, the distribution of pions energy losses weakly changes.

The second approach does not run into this issue, because for application of the ω_n^k-test, it is necessary to know only the distribution of pion energy losses. This combined approach is simpler from a practical application viewpoint, and, as it has been demonstrated, it may provide the power close to the limit value – for $\alpha = 10.54\,\%$ the value of $\beta = 0.02857\,\%$, which corresponds to the factor of pions suppression equal to 3500.

References

[1] W.T. Eadie, D. Dryard, F.E. James, M. Roos and B. Sadoulet: *Statistical Methods in Experimental Physics*, North-Holland Pub.Comp., Amsterdam-London, 1971.

[2] P.V. Zrelov and V.V. Ivanov: *The Relativistic Charged Particles Identification Method Based on the Goodness-of-Fit ω_n^3-Criterion*. Nucl. Instr. and Meth. in Phys. Res., **A310** (1991) 623-630.

[3] E.P. Akishina, T.P. Akishina, V.V. Ivanov, A.I. Maevskaya and O.Yu. Denisova: Electron/pion identification in the CBM TRD applying a ω_n^k goodness-of-fit criterion, "Particles & Nuclei, Letters" (accepted for publication).

[4] P.V. Zrelov and V.V. Ivanov: *The Small Probability Events Separation Method Based on the Smirnov-Gramer-Mises Goodness-of-Fit Criterion*. In: "Algorithms and Programs for Solution of Some Problems in Physics". Sixth Volume. Preprint KFKI-1989-62/M, Budapest, Hungary. 1989, p.127-142.