Vacuum Instability and Pair Production in Strong QED

A.M. Chervyakov

e-mail: chervyak@physik.fu-berlin.de. Laboratory of Information Technologies, JINR, Dubna

Abstract.[1] Strong external electromagnetic fields make the QED vacuum unstable which decays by emitting significantly boson or fermion particle-antiparticle pairs. I report here the recent progress in studying the particle-antiparticle pair production phenomenon: 1. New exact formulas for production rates of boson and fermion pairs by a smooth potential step $\phi(\mathbf{x}) \propto \tanh kz$ in three dimensions. 2. Exact expressions for reflection and transmission coefficients, as well as for average numbers of produced pairs and for pair production intensities obtained via the studying scattering versus tunneling process by this potential. 3. On this basis, re-examining and proof the standard spin-statistics relation, a highly nontrivial task due to the vacuum instability.

I report here the recent progress in studying the vacuum instability with the consequent production of particle-antiparticle pairs in the present of an external and strong electric field of the spatial pulse shape $\phi(\mathbf{x}) \propto \tanh kz$ in three dimensions. For a constant electric field, this process known as Schwinger mechanism of electron-positron pair production [1], whose basic physics is a tunneling of a particle through an energy barrier of $2mc^2$ from the negative energy levels of the Dirac sea to the positive ones. The efficient method to solve this problem exactly is based on the use of causal Green functions to derive the pair production probabilities by means of the asymptotic solutions of wave equations in terms of scattering data [2].

In recent papers [3, 4], we study the scattering process of a single particle satisfying the relativistic Klein-Gordon and Dirac wave equations in an external Sauter potential of the form $e\phi(z) = v \tanh kz$ corresponding to nonuniform electric field along z-direction. The parameter k defines the inverse width of the electric field, whereas the parameter v governs its size $|E| = vk/e$, whose maximum is given by the critical value $|E_c| \equiv m^2c^3/e\hbar \simeq 1.3\times10^{18}$ V/m. In the transverse direction the particle propagates freely as a plane wave. The typical scattering process by this potential involves an incoming particle coming in from the left which is partly reflected back to the left and partly transmitted forth to the right through the potential barrier. For this process we find both the reflection and transmission coefficients exactly.

The physically more interesting situation comes then into about when the hight of the potential barrier v becomes larger than mc^2. In this case, the transmission coefficient of boson (fermion) particles becomes negative implying the Klein paradox. As its resolution, we must find the incoming antiparticle in the right region of space instead of outgoing particle. This happens because the usual separation of positive and negative energy states occurring in the absence of external fields is no longer ensured. Instead, there can be the region where an overlap of these states is allowed. In this level-crossing region, by means of tunneling between negative and positive states, the pair production of charged particles takes place with the rate determined by the transmission probability for a particle to cross the forbidden region.

Taking into account the conservation of the total probability, we express the production rate of boson and fermion pairs via the logarithm of reflection coefficient as an ordinary energy-momentum integral over the level-crossing (Klein) region as

$$w_\perp = (-1)^{2s+1}\,\frac{(2s+1)}{8\pi^2\hbar^3}\int_0^{(v^2-m^2)} dp_\perp^2$$

$$\int_{-v+\sqrt{p_\perp^2+m^2}}^{v-\sqrt{p_\perp^2+m^2}} dp_0\,\ln R(p_0,\,p_\perp^2)\,, \qquad (1)$$

where s being the spin of a particle and the integration is done over the Klein region in the $(p_\perp^2, p_0)$-plane as shown in Fig. 1.

Interchanging the order of integration, we perform the momentum integral in Eq. (1) to obtain for pair production rates the exact representation

$$w_\perp = \frac{(2s+1)}{3\pi}k^3\int_{-\bar\xi}^{\bar\xi} d\xi\, g(\xi)\,\frac{1}{e^{-2\pi(\xi-\kappa)}+(-1)^{2s}}\,, \quad (2)$$

where $\bar\xi \equiv \sqrt{v^2-m^2}/\hbar k$, the parameter κ being $\kappa = \sqrt{v^2/k^2\hbar^2 - 1/4}$ for bosons and $\kappa = v/k\hbar$ for fermions respectively, and the function $g(\xi)$ is the density of boson (fermion) states

$$g(\xi) = \frac{\xi\left(\bar\xi^2-\xi^2\right)^{3/2}}{\left(\bar\xi^2-\xi^2+m^2/\hbar^2k^2\right)^{1/2}} = -g(-\xi)\,. \qquad (3)$$

This function is the unique for bosons and fermions, and counts the number of modes per unit ξ-interval. Contrary, the second function under the integral in

[1] Collaboration with FU Berlin (Prof. H. Kleinert)

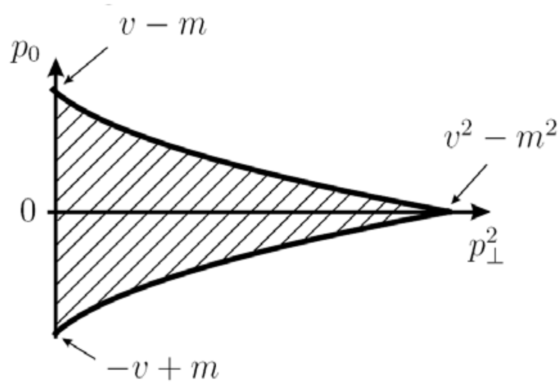

Figure 1: In the $(p_\perp^2, p_0)$-plane, the level-crossing covers the positive region restricted by two intersecting parabolas $p_0 = v - \sqrt{p_\perp^2 + m^2}$ and $p_0 = -v + \sqrt{p_\perp^2 + m^2}$ with horizontal axes of symmetry above and below the $p_\perp^2$-axis for $v > m$

Eq. (2) is different by sign for bosons and fermions. It represents the local probability for production of a single particle-antiparticle pair in a certain energy mode on this interval

$$t_s(\xi) = \frac{1}{e^{-2\pi(\xi-\kappa)} + (-1)^{2s}} = $$

$$= 1 - \frac{1}{e^{2\pi(\xi-\kappa)} + (-1)^{2s}} = (-1)^{2s}\left(1 - r_s(\xi)\right), \quad (4)$$

where $r_s(\xi)$ is the local probability that no boson (fermion) pairs are created in this energy mode. This equation implies the conservation of the total probability for transition of an arbitrary initial state into all possible final states during the scattering process.

Then we find the average number of produced boson (fermion) pairs created in a certain energy mode on ξ-interval $-\bar{\xi} \le \xi \le \bar{\xi}$ as [5, 6]:

$$\bar{n}(\xi) = \frac{t_s(\xi)}{1 + (-1)^{2s+1}\, t_s(\xi)} = \frac{t_s(\xi)}{r_s(\xi)} = \exp\left(2\pi(\xi - \kappa)\right). \quad (5)$$

It is the Bose-Einstein distribution for spin $s = 0$ particles, and the Fermi-Dirac distribution for spin $s = 1/2$ particles. This preserves therefore the normal relation between spin and statistics even though the vacuum instability invalidates the standard proof of the spin-statistics theorem of quantum field theory.

Finally, in the papers [3, 4], we calculate the integral (2) as a series expansion in powers of small dimensionless parameter $0 < \tilde{k} < 1$. Introducing the probability rate $w \equiv w_\perp / L$, we find

$$w = -(2s+1)\frac{(e|E|)^2}{8\pi^3\hbar^2}\left\{ \mathrm{Li}_2(-e^{\tilde{\rho}}) + \tilde{k}^2 \left[\frac{5\pi}{2\epsilon}\,\mathrm{Li}_1(-e^{\tilde{\rho}}) - \right.\right.$$

$$\left.\left. -\frac{3}{4}\,\mathrm{Li}_2(-e^{\tilde{\rho}}) - \frac{3\epsilon}{2\pi}\,\mathrm{Li}_3(-e^{\tilde{\rho}})\right] + \cdots \right\}, \quad (6)$$

where $\mathrm{Li}_\nu(z)$, $\nu = 1, 2, 3, \ldots$, are the polylogarithm functions, $\tilde{\rho} \simeq -\pi/\epsilon + \pi(\epsilon\tilde{k}^2)/4 + \cdots$, and $\epsilon = |E|/|E_c|$. For $\tilde{k}$ small, the leading term in Eq. (6) yields already an excellent approximation. In a constant-field limit $\tilde{k} \to 0$, we obtain from Eq. (6):

$$w \to -(2s+1)\frac{(e|E|)^2}{8\pi^3\hbar^2}\,\mathrm{Li}_2(-e^{-\pi/\epsilon}). \quad (7)$$

in the complete agreement with the result of Schwinger [1].

In the papers [5, 6], we calculate also the pair production intensities for the Sauter potential. Applying the sane small-$\tilde{k}$ expansion, we find

$$n = (2s+1)\frac{(e|E|)^2}{8\pi^3\hbar^2}\, e^{-\pi/\epsilon}\left[1 + \tilde{k}^2\left(\frac{\pi\epsilon}{4} - \frac{3\epsilon}{2\pi} - \frac{\pi^2}{8} - \right.\right.$$

$$\left.\left. -\frac{3}{8\pi^2} + \frac{33\pi}{8\epsilon} + \frac{\pi^2}{8\epsilon^2}\right) + \cdots\right]. \quad (8)$$

In a constant-field limit $k \to 0$, this yields the intensities

$$n^{\mathrm{f}} = 2n^{\mathrm{b}} = \frac{(e|E|)^2}{4\pi^3\hbar^2}\, e^{-\pi/\epsilon} \quad (9)$$

again in a full agreement with the result of Schwinger [1]. The obtained formulas preserve the normal form of the spin-statistics relation, and reproduce also the known Schwinger results in a constant-field limit.

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