

# Elasticity of Nuclear Continuum as a Principal Macrodynamical Promoter of Electric Dipole Pygmy Resonance

I.V. Molodtsova<sup>1</sup>, D.V. Podgainy<sup>1</sup>, Ş. Mişicu<sup>2</sup>, S.I. Bastrukov<sup>3</sup>, H.-K. Chang<sup>3</sup>

<sup>1</sup>e-mail: molod@theor.jinr.ru, Laboratory of Information Technologies, JINR, Dubna; <sup>2</sup>National Institute for Nuclear Physics, Bucharest, P. O. Box MG6, Romania; <sup>3</sup>Department of Physics and Institute of Astronomy, National Tsing Hua University, Hsinchu, 30013 Taiwan

**Abstract.** The macroscopic solid-mechanical continuum model of electric pygmy dipole resonance (PDR) is outlined, in which its origin is attributed to perturbation-induced effective decomposition of nucleus into two spherical domains – undisturbed inner region treated as a static core and dynamical layer undergoing elastic shear vibrations. The focus is placed on the imprinted in the core-layer model mechanism of emergence of the low-energy dipole electric resonant excitation as Goldstone soft mode of translation layer-against-core oscillations. The obtained analytic equations for the energy of  $E1$  vibrational soft mode and its excitation strength lead to the following estimates for the PDR energy centroid  $E_{PDR}(E1) = [31 \pm 1] A^{-1/3}$  MeV and the total excitation strength  $B_{PDR}(E1) = [1.85 \pm 0.05] 10^{-3} Z^2 A^{-2/3} e^2 \text{ fm}^2$  throughout the nuclear chart exhibiting fundamental character of this soft dipole mode of nuclear resonant response [1].

## Introduction

It is generally recognized today that macroscopic behavior of nucleus at excitation of giant resonances of multipole degree  $\ell \geq 2$  located lower than the compressional giant monopole and the giant dipole resonances bears strong resemblance to elastic shear (non-compressional) oscillations of a solid globe. Such an understanding has come into existence during the past three decades as a result of numerous investigations showing that solid-mechanical continuum model of nuclear giant-resonant excitations provides proper account of experimentally observed size effect – smooth variation of integral parameters of isoscalar giant resonances throughout the nuclear chart, such as centroid of energy, spread width and total excitation strength (e.g. [2] and references therein). This feature is generally thought of as exhibiting fundamental character of nuclear giant-resonant response. In this latter context, a great deal of current interest centers on the electric pygmy dipole resonance (PDR) which is observed by the Nuclear Resonance Fluorescence (NRF) technique as a concentration of electric dipole strength near the neutron threshold, that is, in the energy domain where nuclear resonance-like excitations exhibit features generic to shear oscillations of an elastic sphere.

It is the subject of paper [1] to investigate elastodynamic excitation mechanism of the electric PDR, that is, as owing its origin to elasticity of nuclear

material continuum. We focus on dipole overtone of the layer-against-core elastic oscillations by accentuating the imprinted in the model macroscopic mechanism of emergence of dipole vibrational excitation as Goldstone soft mode whose most conspicuous feature is that such a mode can exist if and only if elastic oscillations turn out trapped in the peripheral layer of finite depth, not in the entire volume of nucleus.

The frequency  $\omega(\ell)$  of quasi-static regime of nodeless spheroidal oscillations is uniquely computed by the Rayleigh's energy variational method. The experimentally measured energy centroid of isoscalar electric resonance  $E(E\ell)$  is identified with the energy of  $\ell$ -pole spheroidal oscillations with frequency  $\omega_s(\ell)$

$$E(E\ell) = \hbar\omega_s(\ell). \quad (1)$$

Bearing in mind that non-compressional oscillations of an ultra fine electrically charged spherical mass of nuclear continuum are accompanied by oscillations of the charge-current density  $\delta\mathbf{j} = \rho_e \dot{\mathbf{u}}_s$  (where  $\dot{\mathbf{u}}_s$  stands for the rate of displacements in spheroidal mode of nodeless elastic oscillations) and that the integral characteristics of corresponding vibrational states is the electric moment of charge-current density  $\mathcal{M}_j(E\ell)$ , the electric excitation strength of the  $\ell$ -pole nuclear response to the long-wavelength electromagnetic field can be evaluated by standard equation of the macroscopic electrodynamics of continuous media

$$B(E\ell) = (2\ell + 1) \langle |\mathcal{M}_j(E\ell)|^2 \rangle, \quad (2)$$

$$\mathcal{M}_j(E\ell) = \frac{i}{\omega(\ell + 1)} \int \delta\mathbf{j} \cdot \nabla r^\ell P_\ell(\theta) dV.$$

The last two formulae provide computational basis of macroscopic treatment of nuclear giant-resonant excitations in terms of nodeless shear vibrations of charged elastic sphere.

## Dipole soft mode of elastic layer-against-core shear oscillations

To evaluate a fractional part of the nucleus volume involved in elastic shear vibrations, detected as giant isoscalar  $E\ell$  resonances, the nucleus response has been considered in the core-layer model presuming the perturbation-induced decomposition of nucleus into effective static core and dynamical layer

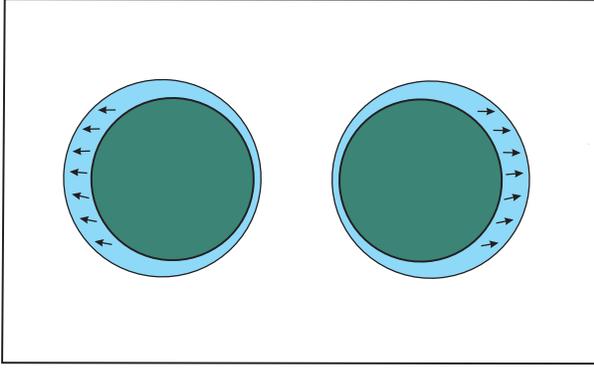


Figure 1: Artist view of nuclear elastic distortions in suggested macroscopic mechanism of isoscalar electric pygmy dipole resonance as elastic dipole soft mode. The excitation process is thought of as an effective decomposition of nucleus, induced by elastically scattered gamma-quanta of FNR technique, into two domains – undisturbed by perturbation internal spherical region treated, thereby, as a static core and peripheral dynamic layer undergoing differentially translational oscillations driven by restoring force of elastic shear stresses. The emergence of elastic force is attributed to resistivity to disruption of peripheral circular periodic orbit of incessant Fermi-motion of independent quasi-particles in the nuclear mean field of shell model.

undergoing nodeless shear oscillations which are controlled by elastic restoring force. The term effective means that such two-component, core-layer, image emerges solely in the process of excitation and, thereby, the very notion of core should be thought of as reflecting the dynamically inert central region of nucleus unaffected by perturbation and remaining at rest. To get better understanding dynamical peculiarities of elastodynamic mechanism of giant-resonant excitations, an analytic calculation of spectral equation for the frequency has been performed in the approximation of sharp edge and homogeneous material parameters, to wit, the density  $\rho$  and the shear modulus  $\mu$  of nuclear elastic Fermi-continuum. The depth of dynamical layer involved in elastic vibrations can be conveniently represented as  $\Delta R = R(1 - x)$ , where  $x = R_c/R$ , is the basic parameter of the core-layer model regulating dependence of the energy and the excitation strength of vibrational state upon the layer depth.

The resultant frequency spectrum  $\omega_s(\ell, x)$  reads

$$\omega_s^2(\ell, x) = \omega_0^2 \frac{2(2\ell + 1)}{(1 - x^{2\ell+1})} \times \frac{(\ell^2 - 1)(1 - x^{2\ell-1}) + \ell(\ell + 2)x^{2\ell-1}(1 - x^{2\ell+3})}{(\ell + 1) + \ell x^{2\ell+1}}. \quad (3)$$

It follows when the core radius  $R_c \rightarrow 0$  and, hence parameter,  $x = (R_c/R) \rightarrow 0$ , a limiting case when the entire volume of nucleus sets in oscillations, the last spectral formula takes the form

$$\omega_s(\ell) = \omega_0 [2(2\ell + 1)(\ell - 1)]^{1/2}, \quad \omega_0 = \frac{c_t}{R} \quad (4)$$

showing that the lowest overtone is of quadrupole,  $\ell = 2$ , degree;  $c_t = [\mu/\rho]^{1/2}$  is the speed of transverse shear wave in the bulk nuclear matter. However, when  $x \neq 0$  the lowest overtone, as is easily seen, is of dipole,  $\ell = 1$ , degree. In this latter case a peripheral layer executes elastic differentially translational shear oscillations relative to static core, as pictured in Fig.1.

The energy of dipole vibrational state is given by

$$E(E1, x) = \omega_0 \left[ \frac{9x(1 - x^5)}{(1 - x^3)(1 + x^3/2)} \right]^{1/2}. \quad (5)$$

When  $x = 0$  the coefficient of vibrational rigidity vanishes and the total absorbed energy goes in kinetic energy of the center-of-mass motion. This simple argument shows that the dipole excitation in question can exist as vibrational mode when perturbation sets in differentially translational fluctuations solely peripheral nuclear layer of finite depth leaving the central spherical region of nucleus unaltered. Such behavior is typical for the Goldstone soft modes whose most conspicuous feature is that the mode disappears (the frequency turns into zero), when one of parameters of vibrating system tends to zero:  $\omega(\ell = 1, x) \rightarrow 0$ , when  $x \rightarrow 0$ .

The total dipole strength of electromagnetic response computed as squared dipole moment of the charge-current density fluctuations excited in the surface finite-depth layer is given by

$$B(E1, x) = \frac{\tilde{\rho}_e^2 \hbar R^3}{\rho 2\omega_0} \left[ \frac{(1 - x^3)^3}{x(1 - x^5)(1 + x^3/2)} \right]^{1/2} \quad (6)$$

By  $\tilde{\rho}_e$  we denote the charge density of the peripheral layer which in the model of homogeneous layer can be defined as  $\tilde{\rho}_e = \gamma \rho_e$  where  $\rho_e = (Z/A)en$  stands for the average charge density of nucleus as a whole with  $n$  being the average particle density of nucleons. Physically, the parameter of relative charge density,  $\gamma < 1$ , takes into account the neutron-dominated content of nuclear matter in the surface layer of nucleus.

The considered macroscopic mechanism of emergence of dipole vibrational mode can be regarded as having universal character (generic to all stable nuclei of nuclear chart) if the input parameters of the model, namely, the speed of transverse shear wave  $c_t$ , geometrical parameter  $x$ , and fractional charge density would have one and the same values for all nuclei. If so, from the obtained equations for the energy centroid and excitation strength it follows that the integral characteristics smoothly vary with mass number  $A$  and this variation is given by typical for the giant resonances estimates:  $E(E1, x) = \kappa_E(x) A^{-1/3}$  and  $B(E1, x) = \kappa_B(x) Z^2 A^{-4/3}$ , respectively, where  $\kappa_E(x)$  and  $\kappa_B(x)$  are constants and the link between atomic number  $Z$  and mass

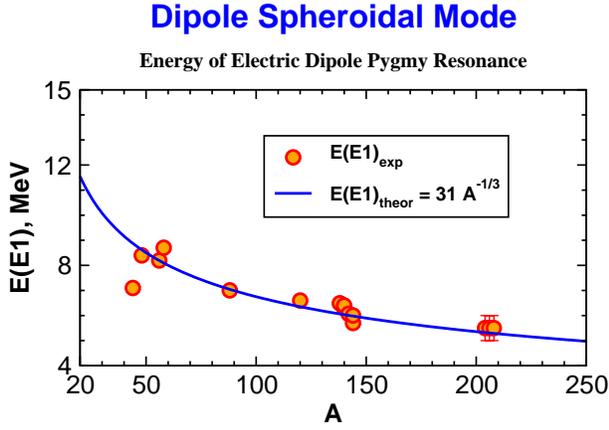


Figure 2: Theoretically computed energy of electric dipole soft mode of elastic translational oscillations of layer against core in juxtaposition with experimental data on energy cen-

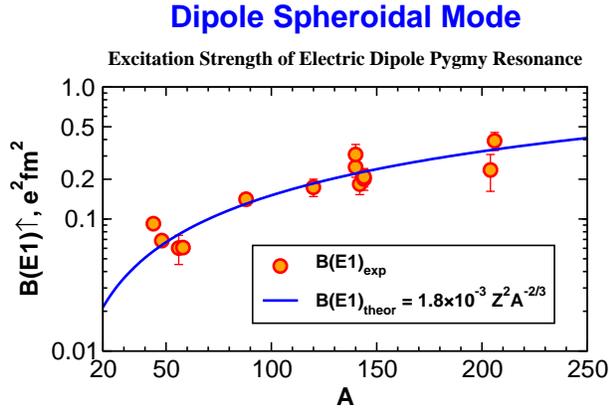


Figure 3: The strength of low-energy E1 electromagnetic nuclear resonant response. Symbols - experimental data and line - is E1 excitation strength of the dipole electric soft mode of elastic oscillations computed as integral electric dipole moment of charge-density current in this vibration state.

number  $A$  is given by the well-known empirical formula:  $Z = A[2 + 0.015A^{2/3}]^{-1}$ . Bearing this in mind (and that the elastodynamic excitation mechanism provide proper account of isoscalar giant resonances with  $\ell \geq 2$ , as discussed below) it is tempting to consider the available experimental data on the low-energy electric PDR in the context of above predictions of the core-layer model for the energy and excitation strength of the dipole soft mode. By varying parameter  $x$  so as to attain best agreement with systematics of data on the energy centroid of PDR as a function of mass number, we get  $x = 0.33$ . Having fixed this parameter and applying the obtained formula for the excitation strength to data on total excitation strength one finds that parameter of relative charge density is given by  $\gamma = 6.6 \cdot 10^{-2}$ . The net outcome is summarized by the following estimates

$$E_{PDR}(E1) = [31 \pm 1] A^{-1/3} \text{ MeV},$$

$$B_{PDR}(E1) = [1.85 \pm 0.05] 10^{-3} Z^2 A^{-2/3} e^2 \text{ fm}^2,$$

showing that the electric PDR is fundamental resonant mode of nuclear response generic to all stable nuclei of nuclear chart, as it is demonstrated in Fig.2 and Fig.3 where computed energy centroid and total excitation strength of elastic dipole soft vibrational mode are plotted in juxtaposition with experimental data for the electric PDR borrowed from (see references in [1]).

The presented line of argument shows in the model of elastic-like nuclear continuum the electric PDR emerges as a soft dipole mode of oscillating irrotational, potential, field of material displacements locked in the finite-depth surface layer. These two signatures distinguish the low-energy electric PDR from electric toroid-dipole resonant (TDR) mode centered at  $E_{TDR}(E1) \sim 70 A^{-1/3}$  MeV. The substantial difference of this later mode, considered in [3] on the same physical footing, that is, as driven by the above restoring force of shear elastic distortions, is that in the TDR mode the nucleus responds by oscillations excited in the whole volume of spherical nucleus and field of material displacement in which has substantially rotational, vortical, character with the torus-like shape of the elastic flow lines.

It is noteworthy that based on arguments similar to expanded above, in Ref. [4] it was shown that magnetic dipole resonance (MDR), experimentally detected by NRF technique, can be interpreted as manifest of perturbation-induced core-layer decomposition of nucleus with concomitant differentially rotational, torsional, elastic oscillations of peripheral layer relative to static core. To this end, it worth emphasizing that macroscopic elastodynamic treatment of low-frequency dipole nuclear resonant excitations provides a remarkable way of unifying understanding of the electric pygmy dipole resonance and magnetic dipole resonance as soft modes of differentially translational (PDR) and differentially rotational (MDR) elastic oscillations of the finite-depth layer against static core, respectively, the oscillations driven by one and the same restoring force of shear deformations.

## References

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