Quark Degrees of Freedom inside Nuclei

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Nuclear physicists developed three distinct models to describe the structure of nuclei. These models, namely, the shell (independent particle) model, liquid-drop and cluster models, are based on different assumptions about the phase state of the nucleus. The dilemma of nuclear structure theory is that these mutually exclusive models work surprisingly well for qualitative and quantitative explanation of certain limited data sets, but each model is utterly inappropriate for application to other data sets. We now understand, however, that the problem of the nuclear matter cannot be solved in a consistent way, if it is based on conception of binary nucleon–nucleon interactions only. Three-body forces, introduced to improve the situation, do not provide a solution because the three-body problem, even in classical physics, still contains unanswered questions.

The majority of the physics community believes that the fundamental theory of the strong interactions is Quantum Chromodynamics (QCD). However, the description of the dynamical structure of hadrons and, especially, nuclei in the framework of QCD has thus far remained an unsolved problem. Moreover, quark degrees of freedom manifest themselves, as conventionally accepted, at high-momentum transfer and high densities and temperature. Hence, the most important problem of nuclear physics concerns the role of quarks in forming nuclear structure: how are nucleons bound inside nuclei and do quarks manifest themselves explicitly in the ground-state nuclei?

Quark structure of nuclei is analyzed in the framework of the Strongly Correlated Quark Model (SCQM) of a nucleon structure [1, 2, 3]. Proposed by one of the authors (G.M.), this semiclassical model demonstrates the interconnection between constituent and current quark models. According to the model quark and antiquark in mesons and three quarks in baryons oscillate around the origin in correlated motion. Derived interquark potential explicitly demonstrates that relativistic (current) quark configurations are located at the origin of oscillation and constituent (nonrelativistic) quarks are at maximal displacements, respectively. Putting aside the mass and charge differences of valence quarks one can say that inside nucleons three quarks, surrounded by condensate of sea quarks and gluons, oscillate along the bisectrices of an equilateral triangle with spins perpendicular to the plane of oscillation. It turns out that these oscillations are nonlinear and the SCQM implies the breather solution of sine-Gordon equation [3].

The parameters of the model, namely, the maximum displacement, $x_{\max}$, and the parameters of the gaussian function, $\sigma_{x,y,z}$, for hadronic matter distribution around VQ are chosen to be $x_{\max} = 0.64$ fm, $\sigma_{x,y} = 0.24$ fm, $\sigma_z = 0.12$ fm. They are adjusted by comparison to calculated and experimental values of the total and differential cross sections for pp and $\overline{p}p$ collisions [2]. The mass of the constituent quark at maximum displacement is taken as $M_{Q(\overline{Q})}(x_{\max}) = \frac{1}{3}\left(\frac{m_\Delta + m_N}{2}\right) \approx 360$ MeV, where m_Δ and m_N are masses of the delta isobar and nucleon correspondingly. The current mass of the valence quark is taken to be 5 MeV. Because of plane oscillations of VQs and the flattened shape the hadronic matter distribution around them, the 3-quark system, representing baryons, is a non-spherical, oblate object. Its dimension perpendicular to the plane of VQs oscillations is flattened. This feature of nucleons plays an important role in the structure of nuclei.

The correlations among quarks have noteworthy implications for the construction of any nucleus. Here it is seen that quark correlations emerging from color binding between quarks in different nucleons is the basic mechanism underlying the nuclear force. With regards to the spin and flavor alignment of adjacent quarks, we should take into account the fact that the multiquark states of 6, 9, 9, and 12 quarks in deuteron, ^{3}H, ^{3}He and ^{4}He belong to the completely antisymmetric representation of the $SU(12)$ group which contains the direct product $SU(2)_{flavor} \otimes SU(2)_{spin} \otimes SU(3)_{color}$. That is, up to 12 quarks can occupy the s state. Some quark configurations in the above multiquark systems built according to the group representations correspond to, so-called, "hidden color" states as these can not be represented in term of the free (color-singlet) nucleons. We restrict the multiquark configurations by those which result in the color-singlet nucleons composing the nuclei. In that way, binding between nucleons occurs when two quarks at linkage, being in antisymmetric color state have different isospins (antisymmetric) and parallel spins (symmetric). This is a basic rule for the construction of any multiquark or multinucleon system.

Noting that three quarks inside nucleons are totally antisymmetric in the color space and two quarks from different nucleons at linkage are in the antisymmetric color state ($\overline{3}$) having different flavors and parallel spins, one can construct all light and medium nuclei. The three-nucleon system is formed by the binding of two quarks of each nucleon with quarks of two other nucleons according to the above rules. Three-nucleon nuclei, namely ^{3}H and ^{3}He, represent triangular configurations with three quarks at free ends. Completion of a four–nucleon system, ^{4}He, from a three-nucleon one,

Table 1: Correlation of the experimental values of binding energy with the number of quark loops and unbound quark ends.

Nucleus	Binding energy/bond, MeV	Quark loops	Unbound quark ends
^{2}H	2.22	0	4
^{3}H	2.83	1	3
^{3}He	2.57	1	3
^{4}He	7.07	4	0

occurs by binding the free quark ends in ^{3}H (^{3}He) with the three quarks of an additional proton (neutron) again in accordance with the above rules. Here we should make one remark. As seen from the Table, the binding energy per one bond is minimal for the deuteron and maximal for the ^{4}He nucleus. This variability is due to the number of quark or color loops and the number of unbound quarks ends. Quark or color loops are created by the quark ends of three nucleons, as in ^{3}H and ^{3}He. The more color loops the larger is the binding energy. On the other hand the more unbound quark ends the less is the binding energy. The maximal binding energy of ^{4}He is due to the presence of four color loops, binding all quark ends of the four nucleons. Exotic isotopes of ^{4}He, ^{6}He and ^{8}He are (loosely) bound systems due to the presence of color loops created by dineutron bounds with the protons of core the ^{4}He nucleus. Hence, only the dineutron configuration in ^{6}He nucleus is realized but not the cigar-like one. Removal of one of the neutrons composing a dineutron destroys the color loop and the other neutron becomes unbound. The ^{8}He nucleus is the last bound state helium isotope and there is no possibility for a ^{10}He bound state because no more color loops can be created.

Starting from the structure of the ^{4}He, it can be shown that all nuclei possess 3D-crystal-like structure (Fig. 1).

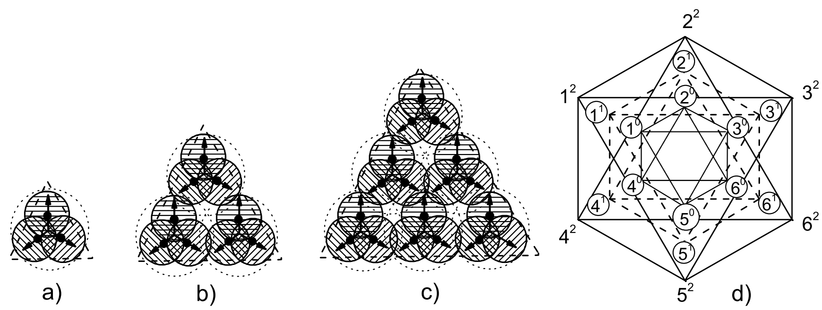

Figure 1: Quark composition of octahedron faces for building of s - , p - and d - shells, respectively a), b), c). d) Three nested octahedra corresponding to these three shells

Indeed, pairs of flattened protons and neutrons are located on opposite faces of an octahedron with a common vertex. In this geometrical configuration four nucleons are in an s state that corresponds to the first s shell of the shell model. Next, the p shell can be represented as a larger octahedron with two ^{3}He triangles instead of protons and two ^{3}H triangles instead of neutrons. The triangles are located parallel to empty faces of the ^{4}He octahedron, the free quark ends of these triangles are coupled as in the ^{4}He octahedron. This octahedron with the nested ^{4}He octahedron represents the nucleus of ^{16}O. The next shell with principal number $n = 2$ is constructed in the same manner, extending triangles beforehand by adding a row of three protons to the row of two neutrons in ^{3}H and a row of three neutrons to the row of two protons in ^{3}He. Again, these triangles are located in couples on opposite faces of an octahedron parallel to unoccupied faces of the nested p octahedron. Construction of the next shells is performed in the same manner by extending triangles with new rows of neutrons and protons. The nuclei built in accordance with the model are found to exhibit symmetries that are isomorphic with the independent particle description (shell model) of nucleon states. The model reproduces not only n shells but shell/subshell structure implied by the wave equation of the shell model, at least for $n \leq 2$. For larger nuclei, however, one factor comes to play an increasingly important role – the Coulomb repulsion among protons. This is the reason why nuclei with Z > 20 require excess neutrons. At these values of Z the Coulomb repulsion force acting on an additional proton at a specific position decreases the depth of additional minimum of the quark potential.

Inside nuclei constructed in this way the nucleons aggregate into a face-centered cubic (FCC) lattice with alternating spin and isospin layers. It turns out that this arrangement is the basis of the FCC-lattice model of the nuclear structure [4, 5, 6, 7], developed about 30 years ago. For finite nuclei the FCC arrangement appears as a tetrahedron (^{4}He) and truncated tetrahedrons (for larger nuclei). According to the FCC the nucleon principal number, n, is a function of the nucleon's distance from the center of the lattice – leading to approximately spherical shells for each consecutive n eigenvalue:

$$n = (|x| + |y| + |z| - 3)/2, \quad (1)$$

where x, y, z are odd integers (Fig. 2). The first shell (s shell) contains four nucleons with coordinates 111, -1-11, 1-1-1, -11-1. The second shell (p shell): 12 nucleons 31-1, 3-11, -311, -3-1-1, 1-31, -131, 13-1, -1-3-1, -113, 11-3, 1-13, -1-1-3 and so on… The total angular momentum value of a nucleon in the lattice

$$j = (|x| + |y| - 1)/2$$

is defined in terms of the distance of the nucleon from the spin axis of the system – leading to roughly cylindrical j subshels within each n shell. The azimuthal quantum number

$$m = |x|/2$$

is a function of the nucleon's distance from a central plane through the lattice. We have thus arrived at the result that the FCC structure brings together shell, liquid-drop and cluster characteristics, as found in the conventional models, within a single theoretical framework. Unique among the various lattice models, the FCC reproduces the entire sequence of allowed nucleon states as found in the shell model.

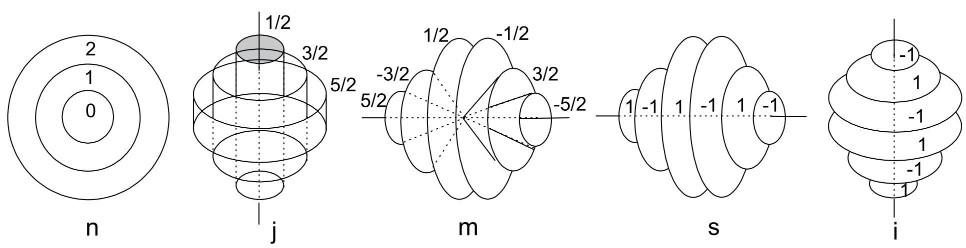

Figure 2: The eigenvalue symmetries in the FCC lattice for the first three shells

Summing up we would like to emphasize that trinucleon configurations play an important role in forming bound multinucleon systems. Moreover, according to our approach, the formation of "quark loops" is a basic element of the binding of the nuclei, both stable and exotic. Namely, the formation of trinucleon configurations is responsible for the even-even effect of binding energies because only two additional protons (neutrons) can form a quark loop with one nuclear neutron (proton). Further, pairs of protons and pairs of neutrons can form virtual alpha clusters inside the nucleus. These mechanisms are in agreement with the observed pairing effect of nuclear binding energies. Finally, all nuclei, even those with closed shells, are non-axially deformed.

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