Spinor Description of Perfect Fluid and Dark Energy

B. Saha

e-mail: bijan@jinr.ru, Laboratory of Information Technologies, JINR, Dubna

Abstract. Different types of perfect fluid obeying the barotropic equation of states, as well as dark energy such as quintessence, Chaplygin gas and phantom matter are modeled by nonlinear spinor field. The influence of the corresponding source field on the evolution of the Universe has been studied within the framework of FRW and Bianchi type-I cosmological models.

Introduction

The role of nonlinear spinor field in the evolution of an anisotropic universe were studied in recent past by a number of authors [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. It was shown that a suitable choice of nonlinearity (i) accelerates the isotropization process, (ii) gives rise to a singularity-free Universe and (iii) generates late time acceleration. Naturally arose the question whether it is possible to describe the common sources such as perfect fluid and dark energy by means of a spinor field. Positive answer to this question was given in a number of recent papers [11, 12, 13, 14]. In this report we review the results obtained in the papers mentioned above.

Spinor field Lagrangian and all that

We consider the nonlinear spinor field given by the Lagrangian

$$L_{\rm sp} = \frac{i}{2}\left[\bar\psi\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma^\mu\psi\right] - m\bar\psi\psi + F, \quad (1)$$

where F is some arbitrary functions of invariants $I = S^2$ or $J = P^2$, constructed from the bilinear spinor forms $S = \bar\psi\psi$ and $P = i\bar\psi\gamma^5\psi$. We consider the case when F is some arbitrary functions of S. We also consider the case when the spinor is the functions of t only. In this case the nontrivial components of energy-momentum tensor have the form

$$\begin{aligned} T_0^0 &= mS - F, & (2a)\\ T_1^1 = T_2^2 = T_3^3 &= S\frac{dF}{dS} - F. & (2b)\end{aligned}$$

Let us now simulate the perfect fluid with a nonlinear spinor field.

Perfect fluid with the barotropic equation of state

First we consider a perfect fluid obeying the barotropic equation of state

$$p = W\varepsilon, \quad (3)$$

where W is a constant. Depending on the value of W (3) describes perfect fluid from phantom to ekpyrotic matter, namely

$$\begin{aligned}
W &= 0, && \text{(dust)}, & (4a)\\
W &= 1/3, && \text{(radiation)}, & (4b)\\
W &\in (1/3, 1), && \text{(hard Universe)}, & (4c)\\
W &= 1, && \text{(stiff matter)}, & (4d)\\
W &\in (-1/3, -1), && \text{(quintessence)}, & (4e)\\
W &= -1, && \text{(cosmological constant)}, & (4f)\\
W &< -1, && \text{(phantom matter)}, & (4g)\\
W &> 1, && \text{(ekpyrotic matter)}. & (4h)
\end{aligned}$$

Most recently the relation (3) is exploited to generate a quintessence in order to explain the accelerated expansion of the Universe [15, 16].

Taking into account that the energy density $\varepsilon = T_0^0$ and pressure $p = -T_1^1 = -T_2^2 = -T_3^3$, inserting (2a) and (2b) into (3) one finds [11, 12, 13, 14]

$$S\frac{dF}{dS} - (1+W)F + mWS = 0, \quad (5)$$

with the solution

$$F = \lambda S^{1+W} + mS, \quad (6)$$

with λ being an integration constant. The nonnegativity of energy density implies that λ is a negative constant

In account of it the spinor field Lagrangian now reads

$$L_{\rm sp} = \frac{i}{2}\left[\bar\psi\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma^\mu\psi\right] + \lambda S^{1+W}, \quad (7)$$

Thus a massless spinor field with the Lagrangian (7) describes perfect fluid from phantom to ekpyrotic matter.

Spinor model of a Chaplygin gas

An alternative model for the dark energy density was used by Kamenshchik *et al.* [17], where the authors suggested the use of some perfect fluid but obeying "exotic" equation of state. This type of matter is known as *Chaplygin gas.* Let us now generate a Chaplygin gas by means of a spinor field. A Chaplygin gas is usually described by a equation of state

$$p = -A/\varepsilon^\gamma. \quad (8)$$

Inserting (2a) and (2b) into (8) one finds [14]

$$\frac{(-F)^\gamma d(-F)}{(-F)^{1+\gamma} - A} = \frac{dS}{S}, \quad (9)$$

with the solution

$$-F = \left(A + \lambda S^{1+\gamma}\right)^{1/(1+\gamma)}. \quad (10)$$

Thus the spinor field Lagrangian corresponding to a Chaplygin gas reads

$$L_{\rm sp} = \frac{i}{2}\Bigl[\bar\psi\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma^\mu\psi\Bigr] - \bigl(A + \lambda S^{1+\gamma}\bigr)^{1/(1+\gamma)}. \tag{11}$$

Setting $\gamma = 1$ we find the result obtained in [12].

Spinor description of a modified quintessence
It should be noted that one of the problems that face models with dark energy is that of eternal acceleration. In order to get rid of that problem quintessence with a modified equation of state was proposed which is given by [18]

$$p = -W(\varepsilon - \varepsilon_{\rm cr}), \quad W \in (0,\,1), \tag{12}$$

Here $\varepsilon_{\rm cr}$ some critical energy density. Setting $\varepsilon_{\rm cr} = 0$ one obtains ordinary quintessence.

Inserting $\varepsilon = T_0^0$ and $p = -T_1^1$ into (12) we find

$$F = -\eta S^{1-W} + mS + \frac{W}{1-W}\varepsilon_{\rm cr}, \tag{13}$$

with η being a positive constant.

We see that a nonlinear spinor field with specific type of nonlinearity can substitute perfect fluid and dark energy, thus give rise to a variety of evolution scenario of the Universe.

Examples

Let us now study the evolution of the Universe filled with spinor field within the framework of an anisotropic or isotropic model.

Bianchi type-I cosmological model
It was shown that the metric functions $a_i(t)$ of the B-I cosmological model given by the interval

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 dy^2 - a_3^2 dz^2, \tag{14}$$

has the form [2]

$$\begin{aligned} a_i &= D_i \tau^{1/3} \exp\Bigl(X_i \int \frac{dt}{\tau}\Bigr), \qquad (15)\\ \prod_{i=1}^3 D_i &= 1, \quad \sum_{i=1}^3 X_i = 0, \end{aligned}$$

where the volume-scale τ is defined as $\tau = a_1a_2a_3$ and determined by the equation

$$\ddot\tau = \frac{3}{2}\kappa\Bigl(T_1^1 + T_0^0\Bigr)\tau. \tag{16}$$

In the case concerned it can be shown that S is inverse proportional to τ, i.e.,

$$S = \frac{C_0}{\tau}, \quad C_0 = {\rm const.} \tag{17}$$

Inserting T_0^0 and T_1^1 into (16) one finds

$$\int \frac{d\tau}{\sqrt{3\kappa\nu C_0^{1+W}\tau^{1-W} + C_1}} = t + t_0, \tag{18}$$

$$\int \frac{d\tau}{\sqrt{C_1 + 3\kappa\tau\bigl(A\tau^{1+\gamma} + \lambda C_0^{1+\gamma}\bigr)^{1/(1+\gamma)}}} = t + t_0, \tag{19}$$

$$\int \frac{d\tau}{\sqrt{3\kappa\bigl[\eta C_0^{1-W}\tau^{1+W} - \frac{W\varepsilon_{\rm cr}}{1-W}\tau^2\bigr] + C_1}} = t + t_0, \tag{20}$$

for barotropic fluid, Chaplygin gas and modified quintessence, respectively. Here C_1 and t_0 are the integration constants.

FRW cosmological model
Einstein equations corresponding to the FRW model read

$$2\frac{\ddot a}{a} + \frac{\dot a^2}{a^2} = \kappa T_1^1 \tag{21a}$$

$$3\frac{\dot a^2}{a^2} = \kappa T_0^0. \tag{21b}$$

In order to find the solution that satisfies both (21a) and (21b) we rewrite (21a) in view of (21b) in the following form:

$$\ddot a = \frac{\kappa}{6}\Bigl(3T_1^1 - T_0^0\Bigr)a. \tag{22}$$

It can be shown that in this case S is related to a in the following way From the spinor field equation in this case we find

$$S = \frac{C_0}{a^3}, \quad C_0 = {\rm const.} \tag{23}$$

Inserting T_0^0 and T_1^1 into (16) one finds

$$\int \frac{da}{\sqrt{(\kappa/3)\nu C_0^{1+W} a^{-(1+3W)} + E_1}} = t + t_0, \tag{24}$$

$$\int \frac{da}{\sqrt{(\kappa/3)\Bigl[\nu C_0^{1-W} a^{3W-1} - \frac{W\varepsilon_{\rm cr}}{1-W}a^2 + E_1\Bigr]}} = t + t_0, \tag{25}$$

for barotropic fluid and modified quintessence, respectively. Here E_1 and t_0 are the integration constants. Corresponding equation for Chaplygin gas takes the form

$$\ddot a = \frac{\kappa}{6}\frac{2Aa^{3(1+\gamma)} - \lambda C_0^{1+\gamma}}{a^2\Bigl(Aa^{3(1+\gamma)} + \lambda C_0^{1+\gamma}\Bigr)^{\gamma/(1+\gamma)}}. \tag{26}$$

In the Figs. 1 and 2 we have plotted the evolution of the Universe defined by the nonlinear spinor field corresponding to perfect fluid and dark energy. In the Fig. 3 we have illustrated the dynamics of energy density and pressure of a modified quintessence. In the Fig. 4 the evolution of the Universe defined by the nonlinear spinor field corresponding to a modified quintessence has been

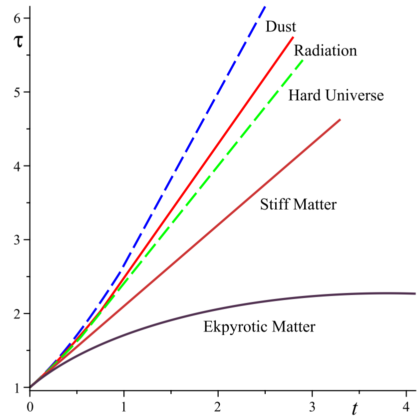

Figure 1: Evolution of the Universe filled with perfect fluid

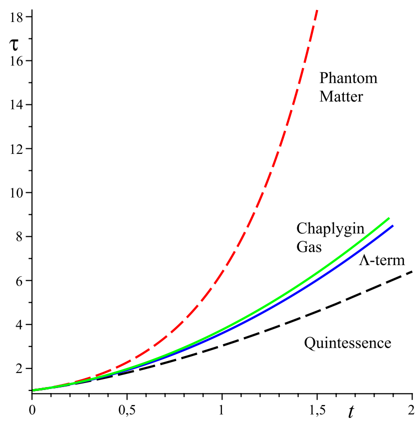

Figure 2: Evolution of the Universe filled with dark energy

presented. As one sees, in the case considered, acceleration alternates with declaration. In this case the Universe can be either singular (that ends in Big Crunch) or regular.

In the Figs. 1, 2, 3 and 4 we stick to the BI cosmological model. Behavior of a in case of a FRW model almost coincides with that of in BI model.

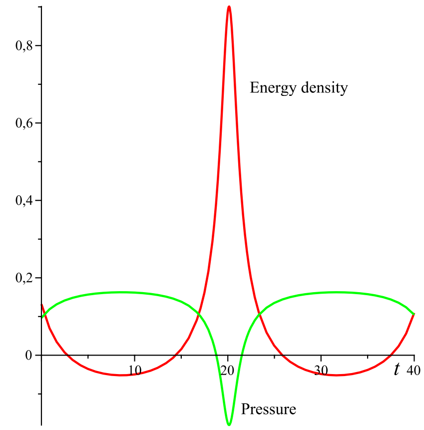

Figure 3: Dynamics of energy density and pressure for a modified quintessence

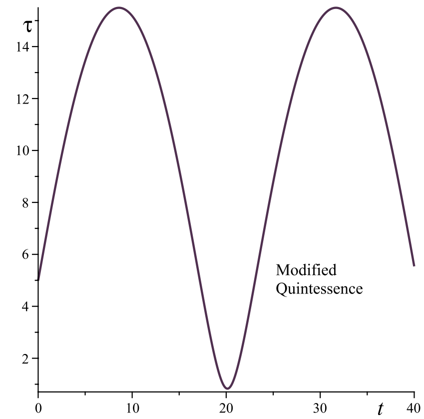

Figure 4: Evolution of the Universe filled with a modified quintessence

Conclusion

Exploiting the equation of states we have described different types of perfect fluid and dark energy in terms of nonlinear spinor field. Corresponding Lagrangian has been worked out. The influence of the corresponding source field on the evolution of the Universe has been studied within the framework of FRW and Bianchi type-I cosmological models.

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