Bianchi Type-I String Cosmological Model with Viscous Fluid: Spinor Description

B. Saha[1], V.S. Rikhvitsky [1], M. Visinescu[2]

[1] Laboratory of Information Technologies, JINR, Dubna; e-mail: bijan@jinr.ru ; [2] Department of Theoretical Physics, National Institute for Physics and Nuclear Engineering Magurele, P. O. Box MG-6, RO-077125 Bucharest, Romania; e-mail: mvisin@theory.nipne.ro

Abstract. The role of a cosmic string and viscous fluid in the evolution of the Universe given by a Bianchi type I model is studied. In doing so the spinor description of cosmic string is exploited. The corresponding system is qualitatively analyzed.

Introduction

Since the observation of the current expansion of the Universe which has apparently accelerated in the recent past, the anomalies found in the cosmic microwave background (CMB) and the large structures observations it becomes obvious that a pure Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmology should be amended. Since Bianchi type one model is the strait-forward generalization of the FLRW model and asymptotically evolves into the FLRW one in presence of a perfect fluid, many authors consider it as one of the prime candidate for studying the possible effects of an anisotropy in the early universe on present-day observations. In this class of models it is possible to accommodate the presence of cosmic strings. In the last time the string cosmological models have been used in attempts to describe the early Universe and to investigate anisotropic dark energy component including a coupling between dark energy and a perfect fluid (dark matter) [1]. Cosmic strings are one dimensional topological defects associated with spontaneous symmetry breaking in gauge theories. Their presence in the early Universe can be justified in the frame of grand unified theories (GUT).

In some recent papers we have studied the evolution of an anisotropic universe in presence of a magnetic fluid and cosmic string [2, 3]. The object of this paper is to investigate a Bianchi type I string cosmological model in the presence of a magnetic flux and viscous fluid. The inclusion of the magnetic field is motivated by the observational cosmology and astrophysics indicating that many subsystems of the Universe possess magnetic fields (see e. g. the reviews [4, 5] and references therein).

In the following section we introduce a system of cosmic string and magnetic field in the Bianchi type I cosmology presenting some of its general features. In Section III we introduce a few plausible assumptions usually accepted in the literature and some exact solutions are produced. In the last section we present some conclusions and perspectives.

Fundamental Equations and general solutions

We consider the gravitational filed given by an anisotropic Bianchi type I (BI) metric

$$ds^2 = a_0^2(dx^0)^2 - a_1^2(dx^1)^2 - a_2^2(dx^2)^2 - a_3^2(dx^3)^2, \tag{1}$$

with $a_0 = 1$, $x^0 = ct$ and $c = 1$. The metric functions a_i $(i = 1, 2, 3)$ are the functions of time t only.

Einstein's gravitational field equation for the BI space-time has the form

$$\frac{\ddot a_2}{a_2} + \frac{\ddot a_3}{a_3} + \frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3} = \kappa T_1^1, \tag{2}$$

$$\frac{\ddot a_3}{a_3} + \frac{\ddot a_1}{a_1} + \frac{\dot a_3}{a_3}\frac{\dot a_1}{a_1} = \kappa T_2^2, \tag{3}$$

$$\frac{\ddot a_1}{a_1} + \frac{\ddot a_2}{a_2} + \frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2} = \kappa T_3^3, \tag{4}$$

$$\frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2} + \frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3} + \frac{\dot a_3}{a_3}\frac{\dot a_1}{a_1} = \kappa T_0^0. \tag{5}$$

Here κ is the Einstein gravitational constant and over-dot means differentiation with respect to t. The energy momentum tensor for a system of cosmic string and magnetic field in presence of a viscous fluid in a comoving coordinate is given by

$$T_\mu^\nu = (\rho + \varepsilon + p')u_\mu u^\nu - \lambda x_\mu x^\nu + E_\mu^\nu - p'\delta_\mu^\nu$$
$$+\eta g^{\nu\beta}[u_{\mu;\beta} + u_{\beta:\mu} - u_\mu u^\alpha u_{\beta;\alpha} - u_\beta u^\alpha u_{\mu;\alpha}], \tag{6}$$

with

$$p' = p - (\xi - \frac{2}{3}\eta)u^\mu_{;\mu}. \tag{7}$$

Here ε is the energy density, p - pressure, η and ξ are the coefficients of shear and bulk viscosity, respectively. Note that the bulk and shear viscosities, η and ξ, are both positively definite and are the functions of energy density

$$\eta = |A|\varepsilon^\alpha, \quad \xi = |B|\varepsilon^\beta. \tag{8}$$

The pressure p is connected to the energy density by means of a equation of state. In this report we consider the one describing a perfect fluid :

$$p = \zeta\varepsilon, \quad \zeta \in (0, 1]. \tag{9}$$

Note that here $\zeta \neq 0$, since for dust pressure, hence temperature is zero, that results in vanishing viscosity.

In (6) ρ is the rest energy density of strings with massive particles attached to them and can be expressed as $\rho = \rho_p + \lambda$, where ρ_p is the rest energy density of the particles attached to the strings and λ is the tension density of the system of strings [6, 7, 8], which may be positive or negative. Here u_i is the four velocity and x_i is the direction of the string, obeying the relation

$$u_i u^i = -x_i x^i = 1, \quad u_i x^i = 0. \tag{10}$$

In (6) $E_{\mu\nu}$ is the electromagnetic field given by Lichnerowich [9]

$$E_\mu^\nu = \bar{\mu}\Big[|h|^2\Big(u_\mu u^\nu - \frac{1}{2}\delta_\mu^\nu\Big) - h_\mu h^\nu\Big]. \tag{11}$$

Here $\bar{\mu}$ is a constant characteristic of the medium and called the magnetic permeability. Typically $\bar{\mu}$ differs from unity only by a few parts in 10^5 ($\bar{\mu} > 1$ for paramagnetic substances and $\bar{\mu} < 1$ for diamagnetic). We choose the incident magnetic field to be in the direction of x-axis so that the magnetic flux vector has only one nontrivial component, namely $h_1 \neq 0$. Under this assumption for E_μ^ν one finds the following non-trivial components

$$E_0^0 = E_1^1 = -E_2^2 = -E_3^3 = \frac{\mathcal{I}^2}{2\bar{\mu} a_2^2 a_3^2}. \tag{12}$$

Taking the string along x^1 direction and using co-moving coordinates we have the following components of energy momentum tensor [10]:

$$T_0^0 = \rho + \frac{\mathcal{I}^2}{2\bar{\mu}}\frac{a_1^2}{\tau^2} + \varepsilon, \tag{13}$$

$$T_1^1 = \lambda + \frac{\mathcal{I}^2}{2\bar{\mu}}\frac{a_1^2}{\tau^2} - p' + 2\eta\frac{\dot{a}_1}{a_1}, \tag{14}$$

$$T_2^2 = -\frac{\mathcal{I}^2}{2\bar{\mu}}\frac{a_1^2}{\tau^2} - p' + 2\eta\frac{\dot{a}_2}{a_2}, \tag{15}$$

$$T_3^3 = -\frac{\mathcal{I}^2}{2\bar{\mu}}\frac{a_1^2}{\tau^2} - p' + 2\eta\frac{\dot{a}_3}{a_3}, \tag{16}$$

where we used the definition

$$\tau = a_1 a_2 a_3. \tag{17}$$

It is indeed the volume scale of the BI space-time, i.e., $\tau = \sqrt{-g}$ [11].

In view of $T_2^2 = T_3^3$ from (3), (4) one finds

$$a_2 = a_3 D\Bigg[\exp\Bigg(X\int\frac{e^{-2\kappa\int\eta dt'}}{\tau}dt\Bigg)\Bigg], \tag{18}$$

with D and X being integration constants. Due to anisotropy of the source filed, in order to solve the remaining Einstein equation we have to impose some additional conditions. Here we give two different conditions. It can be shown that the metric functions can be expressed in terms of τ. So let us first derive the equation for τ. Summation of Einstein Eqs. (2), (3), (4) and 3 times (5) gives

$$\frac{\ddot{\tau}}{\tau} = \frac{1}{2}\kappa\Big(3\rho + 3\varepsilon + \lambda - 3p + 3\xi\frac{\dot{\tau}}{\tau} + \frac{\mathcal{I}^2}{\bar{\mu}}\frac{a_1^2}{\tau^2}\Big). \tag{19}$$

Let us demand the energy-momentum to be conserved, i.e., $T_{\mu;\nu}^\nu = 0$, which in our case takes the form

$$\frac{1}{\tau}\frac{d}{dt}\Big(\tau T_0^0\Big) - \frac{\dot{a}_1}{a_1}T_1^1 - \frac{\dot{a}_2}{a_2}T_2^2 - \frac{\dot{a}_3}{a_3}T_3^3 = 0. \tag{20}$$

After a little manipulation from (20) one obtains

$$\Big[\dot{\rho} + \frac{\dot{\tau}}{\tau}\rho - \frac{\dot{a}_1}{a_1}\lambda\Big] + \Big[\dot{\varepsilon} + \Big(\varepsilon + p - \xi\frac{\dot{\tau}}{\tau}\Big)\frac{\dot{\tau}}{\tau} + 4\eta\Big(\kappa\varepsilon - \frac{\dot{\tau}^2}{3\tau^2}\Big) + 4\eta\kappa\Big(\rho + \frac{\mathcal{I}^2}{\bar{\mu}}\frac{a_1^2}{\tau^2}\Big)\Big] = 0. \tag{21}$$

We consider the case when cosmic string is weakly connected to the viscous fluid, hence (21) can be separated

$$\dot{\rho} + \frac{\dot{\tau}}{\tau}\rho - \frac{\dot{a}_1}{a_1}\lambda = 0, \tag{22}$$

$$\dot{\varepsilon} + \Big(\varepsilon + p - \xi\frac{\dot{\tau}}{\tau}\Big)\frac{\dot{\tau}}{\tau} + 4\eta\Big(\kappa\varepsilon - \frac{\dot{\tau}^2}{3\tau^2}\Big) + 4\eta\kappa\Big(\rho + \frac{\mathcal{I}^2}{\bar{\mu}}\frac{a_1^2}{\tau^2}\Big) = 0. \tag{23}$$

In doing so, let us first write the equation of state for cosmic string. In the literature there exists a number of relations between ρ and λ, the simplest one being a proportionality relation:

$$\rho = \alpha\lambda \tag{24}$$

with the most usual choices of the constant α

$$\alpha = \begin{cases} 1 & \text{geometric string} \\ 1+\omega & \omega \geq 0,\ p\,\text{string or Takabayasi string} \\ -1 & \text{Reddy string}. \end{cases} \tag{25}$$

Using the spinor description the cosmic string then can be given by the Lagrangian [12, 13]

$$L_{\rm sp} = \frac{i}{2}\Big[\bar{\psi}\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar{\psi}\gamma^\mu\psi\Big] - \nu S^{(\alpha+1)/\alpha}, \tag{26}$$

where the spinor field depends on t only. Then from (26) one finds

$$\rho = \nu S^{(\alpha+1)/\alpha}, \tag{27}$$

$$\lambda = -\frac{\nu}{\alpha}S^{(\alpha+1)/\alpha}. \tag{28}$$

Inserting (27) and (28) into (22) we find

$$a_1 = \frac{A_1}{\tau}, \tag{29}$$

A_1 being a constant of integration.

Taking into account that in in our case $S = C_0/\tau$, the Eqs. (19) and (21) can be rewritten as [14, 15, 16, 17]

$$\dot{\tau} = 3H\tau, \tag{30a}$$

$$\dot{H} = \frac{\kappa}{2}\Big(3\xi H - \omega\Big) - \Big(3H^2 - \kappa\varepsilon\Big) + \frac{\kappa}{2}\Big(\frac{\nu}{\tau^{(\alpha+1)/\alpha}} - \frac{\nu}{3\alpha\tau^{(\alpha+1)/\alpha}} + \frac{\mathcal{I}^2}{2\bar{\mu}}\frac{A_1^2}{\tau^4}\Big) \tag{30b}$$

$$\dot{\varepsilon} = 3H\Big(3\xi H - \omega\Big) + 4\eta\Big(3H^2 - \kappa\varepsilon\Big) - 4\eta\kappa\Big(\frac{\nu}{\tau^{(\alpha+1)/\alpha}} + \frac{\mathcal{I}^2}{2\bar{\mu}}\frac{A_1^2}{\tau^4}\Big) \tag{30c}$$

where for simplicity we set $C_0 = 1$ and define $\omega = \varepsilon + p$. The system (30) has been analyzed qualitatively and corresponding behavior of H, τ and ε has been illustrated in Fig. (1).

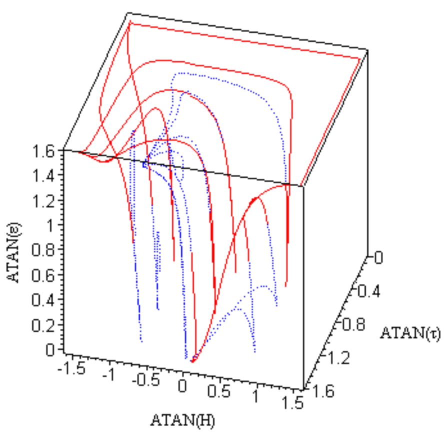

Figure 1: 3D view in the H, τ and ε space

Conclusions

In the present paper we investigated in the frame of Bianchi type I models a string cosmological model in the presence of a magnetic field and viscous fluid. In doing so we exploited the spinor description of cosmic string. Qualitative analysis of the system is carried out.

Acknowledgments

This work is supported in part by a joint Romanian-LIT, JINR, Dubna Research Project, theme no. 05-6-1060-2005/2010.

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