The Electromagnetic Effects in K_{e4} Decay

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Abstract

The final-state interaction of pions in K_{e4} decay allows to obtain the value of the isospin and angular momentum zero $\pi\pi$ scattering length a_0^0. We take into account the electromagnetic interaction of pions and isospin-symmetry breaking effects caused by different masses of neutral and charged pions and estimate the impact of these effects on the procedure of scattering length extraction from K_{e4} decays.

Introduction

For many years the decay

$$K^{\pm} \to \pi^+\pi^- e^{\pm}\nu \tag{1}$$

was considered as the cleanest method to determine the isospin and angular momentum zero scattering length a_0^0 [1]. At present the value of a_0^0 is predicted by Chiral Perturbation Theory (ChPT) with high precision [2] and its measurement with relevant accuracy can provide useful constraints on the ChPT Lagrangian. The appearance of new precise experimental data [3-6] requires approaches, which can take into account the effects, neglected up to now, in extracting the scattering length from experimental data on K_{e4} decays.

The common way to get the scattering length a_0^0 from the decay probability is based on the classical works [7, 8]. The transition amplitude for decay (1) can be written as the product of the lepton and hadronic currents:

$$A = G_F \sin\theta_c \langle \pi^+\pi^- | J^{\mu}_{had} | K^+\rangle \langle e^+\nu_e | J^{lep}_{\mu} | 0\rangle / \sqrt{2}. \tag{2}$$

The leptonic part of this matrix element is known exactly, while the hadronic part can be described by four hadronic form factors[1] F, G, R, H [8]. By making the partial-wave expansion of the hadronic current with respect to the angular momentum of the dipion system, the hadronic form factors can be written in the following form:

$$\begin{aligned} F &= f_s \exp(i\delta_s(s)) + f_p \exp(i\delta_p(s)) \cos\theta_\pi; \\ G &= g_p \exp(i\delta_p(s)); \quad H = h_p \exp(i\delta_p(s)). \end{aligned} \tag{3}$$

[1]The form factor R is proportional to the electron mass and thus it cannot be extracted from K_{e4} decay.

Here, $s = M^2_{\pi\pi}$ is the square of dipion invariant mass; θ_π is the polar angle of the pion in the dipion rest frame measured with respect to the flight direction of dipion in the K-meson rest frame. The coefficients f_s, f_p, g_p, h_p can be parameterized as functions of pion momenta q in the dipion rest system and of the invariant mass of lepton pair $s_{e\nu}$ in the known way [9]. It is widely accepted that the s- and p- wave phases δ_s, δ_p coincide with the corresponding phases in elastic $\pi\pi$ scattering (Fermi—Watson theorem [10]) and can be related to the scattering lengths using the set of Roy equations [1].

Nevertheless, the different masses of charged and neutral pions lead to the isospin-symmetry breaking [11-14] and require the new approach to connect the phases with scattering lengths.

Another isospin symmetry breaking effect is the electromagnetic interaction in the dipion system [13, 15], which can have impact on the value of scattering length extracted from K_{e4} decay rates. In the present work we develop the approach that allows one to take into account the electromagnetic interaction in the dipion system and estimates its impact on the value of scattering lengths extracted from K_{e4} decay.

Isospin symmetry breaking due to pion mass difference

The s-wave phase shift δ_s has an impact only on axial form factor F, whereas the axial form factors G and vector form factor H depend only on p-wave phase shift δ_p. If to be limited to the account of s and p waves, the inelastic process $\pi^0\pi^0 \to \pi^+\pi^-$ and the reversed one are forbidden due to identity of neutral pions in $l = 1$ state. Thus, inelastic transitions can change only the first term in the form factor F, relevant to production of pions in s-wave.

In one-loop approximation of nonperturbative effective field theory (see, e.g., [16]) the decay amplitude relevant to dipion in the state with $I = l = 0$ reads:

$$T = T_1(1 + ik_c a_c(s)) + ik_n a_x(s) T_2. \tag{4}$$

Here, T_1, T_2 are the so-called “unperturbed” amplitudes [17] corresponding to the decays with charged and neutral dipions in the final state, respectively.

$k_n = \sqrt{s-4m_0^2}/2$, $k_c = \sqrt{s-4m_c^2}/2$ are the pion momenta, correspondingly, in the $\pi^0\pi^0$ and $\pi^+\pi^-$ systems with the same invariant mass $s = M^2_{\pi\pi}$. The real functions $a_c(s)$, $a_x(s)$ are relevant to elastic scattering $\pi^+\pi^- \to \pi^+\pi^-$ and charge-exchange reaction $\pi^0\pi^0 \to \pi^+\pi^-$, correspondingly.

In the case of isospin symmetry a_c,a_x can be expressed through the s-wave "amplitudes" with certain isospin $a_0(s)$, $a_2(s)$, which at threshold are equal to relevant scattering lengths a_0^0, a_0^2. In the case of isospin symmetry breaking we adopt the relations followed from ChPT [16]:

$$\begin{aligned} a_c(s) &= [2a_0(s)+a_2(s)](1+\eta)/3; & (5)\\ a_x(s) &= \sqrt{2}(a_0(s)-a_2)(1+\eta/3)/3; & (6)\\ \eta &= (m_c^2-m_0^2)/m_c^2. & (7) \end{aligned}$$

In the isospin symmetry limit ($k_c = k_n = k$; $\eta = 0$) a simple relation takes place between the "unperturbed" amplitudes: $T_1 = \sqrt{2}T_2$, which follows from the rule $\Delta I = 1/2$ for semi-leptonic decays. In this limit it is easy to obtain:

$$T = T_1(1+ika_0(s)) = T_1\sqrt{1+k^2a_0(s)^2}\exp i\delta_0^0. \quad (8)$$

This equation is nothing else than the Fermi—Watson theorem [10] for the $\pi\pi$ interaction in the final states.

In the general case using the expressions (4)-(7) and relations between the s-wave "amplitudes" and relevant phases

$$\begin{aligned} \tan\delta_s(s) &= k_ca_c(s);\ \tan\delta_0^0 = k_ca_0(s);\\ \tan\delta_0^2 &= k_ca_2(s), & (9) \end{aligned}$$

after a bit algebra it is easy to obtain:

$$\begin{aligned} \delta_s &= \arctan(A_s\tan\delta_0^0+B_s\tan\delta_0^2);\\ A_s &= [2(1+\eta)+\lambda(1+\eta/3)]/3;\\ B_s &= [(1+\eta)-\lambda(1+\eta/3)]/3;\\ \lambda &= k_n/k_c. & (10) \end{aligned}$$

Another isospin-breaking effect, which can be important in the procedure of the scattering length extraction from the experimental data on K_{e4} decay, is the Coulomb interaction between the charged pions [13, 15]. The widely spread wisdom is that in order to take this effect into account it is sufficient to multiply the square of matrix element (2) by Gamov factor

$$\begin{aligned} G &= 2\pi\xi/[1-\exp(-2\pi\xi)];\ \xi = \alpha/\upsilon;\\ \upsilon &= \sqrt{1-4\beta}/[1-2\beta];\ \beta = 2k_c/\sqrt{s}. & (11) \end{aligned}$$

Here, υ is the relative velocity in the dipion system, and $\alpha = e^2/4\pi$ is the fine structure constant.

Later on we show, that besides this multiplier, the electromagnetic interaction between pions also changes the expression (10) for the strong phase and adds the proper Coulomb phase.

Electromagnetic interaction in $\pi\pi$ system

In order to take into account the electromagnetic interactions between pions, we take an advantage of the trick successfully used in [19]. To switch on the electromagnetic interaction, we replace the charged pion momenta k_c in (9) by a logarithmic derivative of the pion wave function in the Coulomb potential at the boundary of the strong field r_0:

$$ik_c \to \tau = \left.\frac{d\log[G_0(kr)+iF_0(kr)]}{dr}\right|_{r=r_0}. \quad (12)$$

Here, F_0, G_0 are the regular and irregular solutions of the Coulomb problem.

In the region $kr_0 \ll 1$, where the electromagnetic effects are significant, this expression can be simplified:

$$\begin{aligned} \tau &= ik-\alpha m\left[\log(-2ikr_0)+2\gamma+\psi(1-i\xi)\right]=\\ &= Re\ \tau + i\ Im\ \tau;\\ Re\ \tau &= -\alpha m\left[\log(2kr_0)+2\gamma+Re\,\psi(1-i\xi)\right];\\ Im\ \tau &= \pi k\xi\exp(\pi\xi)/\sinh\pi\xi. & (13) \end{aligned}$$

Here, $\gamma = 0.5772$ is Euler constant, and $\psi(z) = d\log\Gamma(z)/dz$ is digamma function.

Using the above relations one can express the modified phase for $\pi^+\pi^-$ state ($I = l = 0$) through the known [1] phases δ_0^0, δ_0^2. Representing the modified s-wave phase as a sum of strong δ_{str} and electromagnetic δ_{em} terms, we obtain:

$$\begin{aligned} \tilde{\delta_s} &= \delta_{str}+\delta_{em};\ \delta_{em} = \arctan(\alpha/\upsilon);\\ \delta_{str} &= \arctan(A_{em}\tan\delta_0^0+B_{em}\tan\delta_0^2);\\ A_{em} &= [2G(1+\eta)+\lambda(1+\eta/3)]/3;\\ B_{em} &= [G(1+\eta)-\lambda(1+\eta/3)]/3. & (14) \end{aligned}$$

Let us note that, whereas the electromagnetic phase δ_{em} has a common textbook form [20], the strong phase is essentially modified by electromagnetic effects (the Gamov factor G in δ_{str}) as well as by isospin symmetry breaking effects provided by pion mass difference.

Using the same approach, one can show that the modified p-wave phase reads:

$$\tilde{\delta_p} = \arctan\left(G(1+\alpha^2/\beta^2)\tan\delta_1^1\right). \quad (15)$$

Setting $a_0^0 = 0.225{m_c}^{-1}$; $a_0^2 = -0.03706{m_c}^{-1}$ and using the relevant phases δ_0^0, δ_1^1 from Appendix

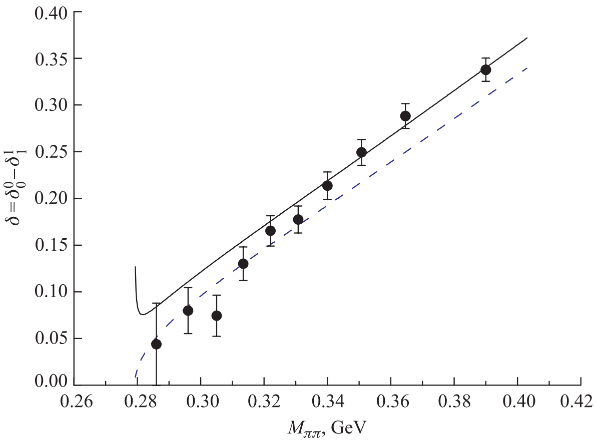

Figure 1: The dependence of phase difference $\delta = \delta_s - \delta_p$ (rad) on dipion invariant mass $M_{\pi\pi}$ (Gev) in the exact isospin symmetry case (dashed curve) and with all isospin symmetry breaking corrections taken into account (solid curve)

D of [1], we calculated the modified phase differences $\delta = \tilde{\delta}_s - \tilde{\delta}_p$ as a function of the invariant mass of dipion $M_{\pi\pi}$.

The dashed line in the figure corresponds to exact isospin symmetry limit $m_0 = m_c$; $\alpha = 0$. The solid line gives the dependence of modified phase difference accounting for all isospin breaking effects. The experimental data are from [4]. The above considered isospin breaking effects change remarkably δ and can have impact on the values of scattering lengths extracted from experimental data.

In the table we cite δ as a function of dipion invariant mass $M_{\pi\pi}$ in respect to different isospin-breaking corrections. In the last column of the table we cited the correction from $\pi^0\eta$ intermediate state [14]. The origin of this isospin-breaking correction is due to the different masses of quarks ($m_u \neq m_d$) and it is estimated using the expression (52) from work [14].

Table 1: The impact of considered corrections on phase difference $\delta = \delta_s - \delta_p$ (rad): 1) standard case [1] with $a_0^0 = 0.225{m_c}^{-1}$, $a_0^2 = -0.03706{m_c}^{-1}$; the cases with 2) charge exchange process $\lambda = k_n/k_c$; 3) parameter η (7); 4) electromagnetic interaction; 5) the additional Coulomb phase; 6) correction [14] due to different quarks masses

$M_{\pi\pi}$	1	2	3	4	5	6
0.285	**0.048**	**0.059**	**0.061**	**0.063**	**0.082**	**0.083**
0.300	**0.096**	**0.103**	**0.108**	**0.110**	**0.122**	**0.123**
0.315	**0.134**	**0.140**	**0.147**	**0.149**	**0.159**	**0.145**
0.330	**0.170**	**0.175**	**0.184**	**0.186**	**0.195**	**0.197**
0.345	**0.205**	**0.210**	**0.220**	**0.223**	**0.231**	**0.234**
0.360	**0.239**	**0.244**	**0.256**	**0.259**	**0.267**	**0.270**
0.375	**0.274**	**0279**	**0.292**	**0.296**	**0.304**	**0.308**
0.390	**0.309**	**0.314**	**0.328**	**0.333**	**0.340**	**0.361**

Conclusions

The isospin symmetry breaking corrections considered above increase the phase difference δ. Their contribution is maximal near the threshold, but they are essential even far from it. The K_{e4} decay amplitude in the real world with isospin symmetry breaking depends on two scattering lengths a_0^0, a_0^2, unlike the common approach. The proposed approach allows one to extract the values of scattering lengths with higher accuracy than in standard approximation.

References

[1] B. Ananthanarayan, G. Colangelo, J. Gasser, and H. Leutwyler, Phys. Rep., 353 (2001) 207.

[2] G. Colangelo, J. Gasser, and H. Leutwyler, Nucl. Phys., B603 (2001) 125.

[3] S. Pislak et al., Phys. Rev. Lett., 87 (2001) 221801; D67 (2003) 072004.

[4] L. Masetti, in: Proc. Intern. Conf. *"Heavy Quarks and Leptons"* Munich, October, 2006.

[5] B. Bloch-Devaux, in: Proc. *Kaon Intern. Conf.*, Frascati, May, 2007.

[6] J. Batley et al., Eur. Phys. J., C54 (2008) 411.

[7] N. Cabibbo, and A. Maksymowicz, Phys. Rev., B438 (1965) 137.

[8] A. Pais, and S. Treiman, Phys. Rev., 168 (1968) 1858.

[9] G. Amoros, and J. Bijnens, J. Phys., G25 (1999) 1607.

[10] K. Watson, Phys. Rev., 88 (1952) 1163.

[11] J. Gasser, and A. Rusetsky, *Isospin violations in K_{e4} decays.* Internal note. March, 2007.

[12] S. Gevorkyan, A. Sissakian, H. Torosyan, A. Tarasov, and 0. Voskresenskaya, Preprint JINR E2-2008-201, Dubna, 2008 [Accepted for Publication in Physics of Atomic Nuclei, 73].

[13] J. Gasser, in: Proc. *Kaon Intern. Conf.*, Frascati, May, 2007 [hep-ph/0710.3048].

[14] G. Colangelo, J. Gasser, and A. Rusetsky, Eur. Phys. J., C59 (2009) 777 [hep-ph/0811.0775].

[15] S. Gevorkyan, A. Sissakian, H. Torosyan, A. Tarasov, and 0. Voskresenskaya, Preprint JINR E2-2008-202, Dubna, 2008 [Accepted for Publication in Physics of Atomic Nuclei, 73].

[16] G. Colangelo, J. Gasser, B. Kubis, and A. Rusetsky, Phys. Lett., B638 (2006) 187 .

[17] N. Cabibbo, Phys. Rev. Lett., 93 (2004) 121801.

[18] N. Cabibbo, and G. Isidori, JHEP, 0503 (2005) 021.

[19] S. Gevorkyan, A. Tarasov, and 0. Voskresenskaya, Phys. Lett., B649 (2007) 159.

[20] L.D. Landau, and E.M. Lifshitz, *Quantum Mechanics*, Nauka Publication, Moscow, 1971.