Renormdynamics, Fractal calculus and Quanputers

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In the report [1], for high energy multiparticle processes, n dimensional counting was invented,

$$[n]=1,\ [\sigma_n]=-1, \sigma=\Sigma_n\sigma_n,\ [\sigma]=0,\ [<n>]=1. \quad (1)$$

By the n dimensional counting, well known KNO scaling relation

$$<n>\frac{\sigma_n}{\sigma}=\Psi(\frac{n}{<n>}), \quad (2)$$

and similar relation for inclusive distributions [2]

$$<n(p)>\frac{d\sigma_n/dp}{d\sigma/dp}=\Psi(\frac{n}{<n(p)>}), \quad (3)$$

were obtained. Renormdynamic motion equations for the multiparticle observable quantities were invented and there solutions given. Using these equations, an explicit form of the Ψ function was constructed.

In the lecture notes [3] the statistical methods of description of the extended particle systems based on the density of the states function at high energy were developed. At high energy and temperature, we have classical statistical description

$$Z(\beta)=\int\frac{d^Dxd^Dp}{(2\pi)^D}e^{-\beta H(p,x)} \\ =(\frac{m}{2\pi\beta})^{D/2}\int d^Dxe^{-\beta V(x)} \quad (4)$$

For potential of the form

$$V(x) = 0,\ 0\leq|x|\leq a, \\ = b\cdot ln\frac{|x|}{a},\ |x|>a, \quad (5)$$

we have

$$Z(\beta)=(\frac{m}{2\pi\beta})^{D/2}\frac{\Omega_D}{D}\frac{a^D\beta b}{\beta b-D} \quad (6)$$

So the temperature of the system is restricted by condition

$$T=\beta^{-1}<b/D\equiv T_H \quad (7)$$

Statistical energy of the system is

$$E=E_c=-\frac{\partial lnZ(\beta)}{\partial\beta} \\ =\frac{D/2-1}{\beta}+\frac{1}{\beta-D/b} \quad (8)$$

The case when $E=0$ corresponds to a self supporting, non expending, no collapsing state of the system at the temperature

$$\beta_N=\frac{2}{b}(\frac{D}{2}-1)=\frac{D}{b}-\frac{2}{b}<\beta_H \\ T_N=\frac{b}{D-2}=\frac{T_H}{1-2/D}>T_H=\frac{b}{D} \quad (9)$$

The normal temperature is positive, corresponds to the stable state, for $D>2$. It decreases when dimension increase, from infinity to zero. So it is easy (easier)to create higher dimensional normal states. The volume of the system is

$$V_c=\frac{\int dxdpVe^{-\beta H}}{\int dxdpe^{-\beta H}} \\ =\frac{\Omega_D}{D}a^D\frac{b\beta-D}{2(b\beta-2D)},\ V_N=\frac{V(a)}{D+2} \quad (10)$$

The volume has positive value, for $T>T_2=b/D=T_H$ or $T<T_1=b/2D$, $T_N>T_2$. This normal state can not be reached by rising continually temperature of the system. In the corresponding realistic models, e.g. for a heavy nucleus, we can obtain such a state in high energy collisions.

Fractal calculus (FC) invented in [4]

$$D_{0,x}^{-\alpha}f=\frac{|x|^\alpha}{\Gamma(\alpha)}\int_0^1|1-t|^{\alpha-1}f(xt)dt \\ =\frac{|x|^\alpha}{\Gamma(\alpha)}B(\alpha,\ \partial x)f(x)=|x|^\alpha\frac{\Gamma(\partial x)}{\Gamma(\alpha+\partial x)}f(x), \\ f(xt)=t^{x\frac{d}{dx}}f(x). \quad (11)$$

were used [5] in string theory calculations and extended hypergeometric functions.

For (symmetrized, 4-tachyon) Veneziano amplitude we have

$$B_s(\alpha,\beta) = B(\alpha,\beta)+B(\beta,\gamma)+B(\gamma,\alpha) \\ = \int_{-\infty}^{\infty}dx|1-x|^{\alpha-1}|x|^{\beta-1}, \\ \alpha+\beta+\gamma=1 \quad (12)$$

For the p-adic Veneziano amplitude we take

$$B_p(\alpha,\beta)=\int_{Q_p}dx|1-x|_p^{\alpha-1}|x|_p^{\beta-1}=\frac{\Gamma_p(\alpha)\Gamma_p(\beta)}{\Gamma_p(\alpha+\beta)} \quad (13)$$

Now we obtain the N-tachyon amplitude using fractal calculus.

For the closed trajectory of the particle passing through N points, we have

$$A(x_1,x_2,...,x_N)=\int dt\int dt_1...\int dt_N\delta(t-\Sigma t_n)$$
$$v(x_1,t_1;x_2,t_2)v(x_2,t_2;x_3,t_3)...v(x_N,t_N;x_1,t_1)$$
$$=\int dx(t)\Pi(\int dt_n\delta(x^\mu(t_n)-x_n^\mu))exp(-S[x(t)])$$
$$=\int \Pi(dk_n^\mu\chi(k_nx_n))\tilde{A}(k)exp(-S), \qquad (14)$$

where

$$\tilde{A}(k)=\int dxV(k_1)V(k_2)...V(k_N)exp(-S),$$
$$V(k_n)=\int dt\chi(-k_nx(t)) \qquad (15)$$

-vertex function.

Motion equation

$$D^\alpha x^\mu - i\Sigma k_n^\mu\delta(t-t_n)=0, \qquad (16)$$

in the momentum representation

$$|u|^\alpha\tilde{x}^\mu(u)-i\Sigma_nk_n^\mu\chi(-ut_n)=0 \qquad (17)$$

have the solution

$$\tilde{x}^\mu(u)=i\Sigma k_n^\mu\frac{\chi(-ut_n)}{|u|^\alpha},\quad u\neq 0, \qquad (18)$$

the constraint

$$\Sigma_nk_n=0, \qquad (19)$$

and the zero mod $\tilde{x}_n^\mu(0)$, which is arbitrary. Integration in (14) with respect to this zero mod gives the constraint (19). On the solution of the equation (16)

$$x^\mu(t) = iD_t^{-\alpha}\Sigma_nk_n^\mu\delta(t-t_n) = \frac{i}{\Gamma(\alpha)}\Sigma_nk_n^\mu|t-t_n|^{\alpha-1}, \qquad (20)$$

the action takes value

$$S=-\frac{1}{\Gamma(\alpha)}\Sigma_{n<m}k_nk_m|t_n-t_m|^{\alpha-1},$$
$$\tilde{A}(k)=\int\Pi_{n=1}^N dt_n exp(-S) \qquad (21)$$

In the limit, $\alpha\to 1$, for p-adic case we obtain

$$x^\mu(t)=-i\frac{p-1}{p\ lnp}\Sigma_nk_n^\mu ln|t-t_n|,$$
$$S[x(t)]=\frac{p-1}{p\ lnp}\Sigma_{n<m}k_nk_m\ ln|t_n-t_m|,$$
$$\tilde{A}(k)=\int\Pi_{n=1}^N dt_n\Pi_{n<m}|t_n-t_m|^{\frac{p-1}{p\ lnp}k_nk_m} \qquad (22)$$

Now in the limit $p\to 1$ we obtain the proper expressions of the real case

$$x^\mu(t)=-i\Sigma_nk_n^\mu ln|t-t_n|,$$
$$S[x(t)]=\Sigma_{n<m}k_nk_m\ ln|t_n-t_m|,$$
$$\tilde{A}(k)=\int\Pi_{n=1}^N dt_n\Pi_{n<m}|t_n-t_m|^{k_nk_m}. \qquad (23)$$

For extended hypergeometric functions we obtained

$$F(\alpha_1,...\alpha_p;\beta_1,...\beta_q;x)=$$
$$\frac{\Gamma(\alpha_1+x\partial)\cdots\Gamma(\alpha_p+x\partial)}{\Gamma(\alpha_1)\cdots\Gamma(\alpha_p)}\frac{\Gamma(\beta_1)\cdots\Gamma(\beta_q)}{\Gamma(\beta_1+x\partial)\cdots\Gamma(\beta_q+x\partial)}e^x$$
$$=\sum_{n\geq 0}\frac{(\alpha_1)_n\cdots(\alpha_p)_n}{(\beta_1)_n\cdots(\beta_q)_nn!}x^n=\frac{(\alpha_1)_\delta\cdots(\alpha_p)_\delta}{(\beta_1)_\delta\cdots(\beta_q)_\delta}e^x; \qquad (24)$$

$$F_A(a;b_1,...,b_n;c_1,...,c_n;z_1,...,z_n)$$
$$=\frac{(a)_{\delta_1+...+\delta_n}(b_1)_{\delta_1}...(b_n)_{\delta_n}}{(c_1)_{\delta_1}...(c_n)_{\delta_n}}\ e^{z_1+...+z_n}$$
$$=\frac{(a)_{\delta_1+...+\delta_n}}{(a_1)_{\delta_1}...(a_n)_{\delta_n}}\ F(a_1,b_1;c_1;z_1)...F(a_n,b_n;c_n;z_n)$$
$$=\Sigma_{m\geq 0}\frac{(a)_{m_1+...+m_n}(b_1)_{m_1}...(b_n)_{m_n}}{(c_1)_{m_1}...(c_n)_{m_n}}\frac{z_1^{m_1}}{m_1!}...\frac{z_n^{m_n}}{m_n!},$$
$$|z_1|+...+|z_n|<1; \qquad (25)$$

and similar formulas for F_B,F_C,F_D Lauricella functions ($LF_n, n=A,B,C,D.$)

For Lomidze hypergeometric functions [6]

$$LB_n(x,r)=det[x_j^{i-1}\int_{x_{j-1}/x_j}^1 duu^{i+r_0-2}(1-u)^{r_j-1}$$
$$\cdot\prod_{k=1,k\neq j}^n(\frac{x_ju-x_k}{x_j-x_k})^{r_k-1}],$$
$$0=x_0<x_1<x_2<...<x_n,\ n\geq 1,\ r_k\in C, \qquad (26)$$

by dirrect calculations the following formula was proved [7]

$$LB_n(x,r)=detV_n(x)B_n(r), V_n(x)=[x_j^{i-1}],$$
$$B_n(r)=\frac{\Gamma(r_0)\Gamma(r_1)...\Gamma(r_n)}{\Gamma(r_o+r_1+...+r_n)} \qquad (27)$$

Contemporary digital computer and its logical elements can be considered as a spatial type of discrete dynamical systems [8]

$$S_n(k+1)=\Phi_n(S(k)), \qquad (28)$$

where

$$S_n(k),\quad 1\leq n\leq N(k), \qquad (29)$$

is the state vector of the system at the discrete time step k. Vector S may describe the state and Φ transition rule of some Cellular Automata The systems of the type (28) appears in applied mathematics as an explicit finite difference scheme approximation of the equations of the physics.

Definition: *We assume that the system (28) is time-reversible if we can define the reverse dynamical system*

$$S_n(k)=\Phi_n^{-1}(S(k+1)). \qquad (30)$$

In this case the following matrix

$$M_{nm}=\frac{\partial\Phi_n(S(k))}{\partial S_m(k)}, \qquad (31)$$

is regular, i.e. has an inverse. If the matrix is not regular, this is the case, for example, when $N(k+1) \neq N(k)$, we have an irreversible dynamical system (usual digital computers and/or corresponding irreversible gates). Let us consider an extension of the dynamical system (28) given by the following action function

$$A = \sum_{kn} l_n(k)(S_n(k+1) - \Phi_n(S(k))) \tag{32}$$

and corresponding motion equations

$$\begin{aligned} S_n(k+1) &= \Phi_n(S(k)) = \frac{\partial H}{\partial l_n(k)}, \\ l_n(k-1) &= l_m(k)\frac{\partial \Phi_m(S(k))}{\partial S_n(k)} \\ &= l_m(k)M_{mn}(S(k)) = \frac{\partial H}{\partial S_n(k)}, \end{aligned} \tag{33}$$

where

$$H = \sum_{kn} l_n(k)\Phi_n(S(k)), \tag{34}$$

is discrete Hamiltonian. In the regular case, we put the system (33) in an explicit form

$$\begin{aligned} S_n(k+1) &= \Phi_n(S(k)), \\ l_n(k+1) &= l_m(k)M^{-1}_{mn}(S(k+1)). \end{aligned} \tag{35}$$

¿From this system it is obvious that, when the initial value $l_n(k_0)$ is given, the evolution of vector $l(k)$ is defined by evolution of the state vector $S(k)$. The equation of motion for $l_n(k)$ - Elenka is linear and has an important property that a linear superpositions of the solutions are also solutions.

Statement: *Any time-reversible dynamical system (e.g. a time-reversible computer) can be extended by corresponding linear dynamical system (quantum - like processor) which is controlled by the dynamical system and has a huge computational power,*

In the reports [9], [10], regular method of construction of the time reversible dynamical systems and their linear extensions - Quanputers were presented.

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