Parallel Algorithm for Heat Conduction Problem

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To model the heat transfer in the created object (Figure 1) with a composite design, a model with cylindrical symmetry has been developed. An algorithm for numerical solving initial-boundary value problem for the heat equation in cylindrical coordinates has been developed. A parallel algorithm has been implemented [1].

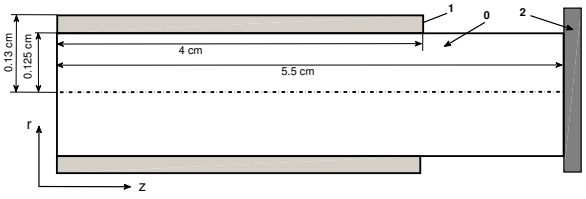

Figure 1: Schematic view of the object slice. The slice of the object: 0 – cooler (copper), 1 – heat source (graphite), 2 – liquid helium temperature terminal with $T = 4.2\,^{\circ}\mathrm{K}$

This object is considered to be a cryogenic cell pulse (in a millisecond range) feeding the working gases into the working space of some source of highly charged ions. It is considered that working gases are initially frozen at the surface of the cell since its temperature is $T = 4.2\,^{\circ}\mathrm{K}$. According to the ion source requirements, the surface of this cell has to be heated up to temperature around $T = 45 \div 60\,^{\circ}\mathrm{K}$ that provides an evaporation of a definite amount of the working gas molecules from the cell surface and their further penetration into the working space of ion source.

One has to maintain the cell surface temperature at the level of around $T = 45 \div 60\,^{\circ}\mathrm{K}$ during few milliseconds and then surface temperature must drop back down during about 10 milliseconds. Then this procedure has to be repeated many times with a frequency of about $1 \div 10$ Hz.

Due to the cylindrical symmetry of the object the heat conductivity in the object can be simulated by 2D model (instead of 3D one) using cylindrical coordinate system (r, z).

The heat transfer into the object can be described by the following couple of parabolic partial differential equations with temperature depended coefficients [2]:

$$\rho_m C_{V\,m}(T)\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_m(T)\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\lambda_m(T)\frac{\partial T}{\partial z}\right) + X(T), \quad (1)$$

where $(r, z) \in G$ and $t > 0$. Note that G is not rectangular area because of complex object design.

The object involves different materials in construction with different densities and thermal coefficients, thus the index m is introduced to (1) for each material ($m = 0$ – copper, $m = 1$ – graphite). The coefficients $C_{V\,m}$ and λ_m are specific heat capacity and thermal conductivity, respectively.

The initial condition is given in the form

$$T(r, z, t = 0) = T_0, \quad (2)$$

where $T_0 \equiv 4.2\,^{\circ}\mathrm{K}$. The boundary conditions are taken from the assumption that there is no heat flow across the surface of the object anywhere except the right boundary which is connected with liquid helium. The conditions can be defined as follows:

$$\frac{\partial T}{\partial \vec{n}} = 0, \quad (3)$$

where $(r, z) \in \Gamma \backslash \{r, z = z_{\max}\}$ and

$$T(r, z) = T_0, \quad (4)$$

where $(r, z) \in \{r, z = z_{\max}\}$. The condition of the interface between the materials is considered as ideal.

It is well known that thermal conductivity and specific heat for many materials could vary up to the order of magnitude as temperature varies from $T = 4.2\,^{\circ}\mathrm{K}$ to $T = 60\,^{\circ}\mathrm{K}$. For the chosen copper and graphite the corresponding data are obtained from the [3], and then these dependencies are approximated by the least-squares method using analytical functions.

Source dependence was approximated by function

$$X(T) = \begin{cases} \chi(T)\frac{I^2}{S} & \textit{in graphite,} \\ & \textit{when source is on} \\ 0 & \textit{in copper, always} \end{cases} \quad (5)$$

where $\chi(T) = 1.8/\sqrt{T}$ and constant $I^2/S = 10^6$.

In this formula I means electric current in the graphite slice along z direction, and S is graphite cross section in the direction orthogonal to the current propagation.

The cylindrical symmetry of the problem reduces the simulation time but it is very computational

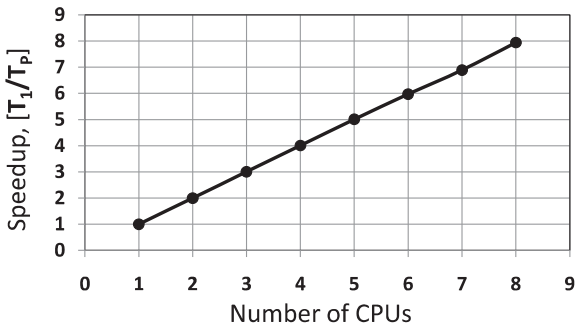

Figure 2: Speedup versus number of CPUs

intensive and, therefore, slow. For example, it takes two weeks to simulate a heat transfer for 10 ms time on Intel Xeon X5450 only for one configuration with given materials.

In practice it is required to simulate and study hundreds of models. Therefore, simulations have to be speeded up. For this a fast parallel algorithm for simulations was developed.

Parallelization was made using a block approach [1]. In this approach each process calculates its own block of data on current time step. Then neighboring processes exchange the corresponding boundary vectors for the calculation on next time step.

The speedup results are shown in Figure 2, where a simulation using parallel algorithm has been done on the system with two Intel Xeon E5405 (quad-core). The developed algorithm has an excellent linear speedup factor for the number of CPUs up to 8. It is a big success that speedup has the same order that the number of CPUs using this simple approach.

The parallel computations allow one to achieve a necessary speedup factor of the algorithm. Thus, it can be applied for systematic studies to find an optimal design and materials of the object in practical needs.

Acknowledgments

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References

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