A Study of Pion-Nucleus Elastic and Inelastic Scattering Using Microscopic Optical Potential

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Our contribution briefly covers our investigations of pion-nucleus scattering in period 2012-2013.

1. Calculations of the π-nucleus differential cross sections of elastic scattering

It is customary to use two approaches in calculations of the pion-nucleus elastic scattering cross sections in the region of the (3,3) resonance energies T_{lab} ~100-300 MeV. The one is based on the Kisslinger optical potential (OP) that includes information on the density distribution of a target nucleus and also on the $s-, p-$ and d-phases of the pion-nucleon amplitude of scattering, and thus it depends on 7 and more parameters. This OP was applied in a number of works, but if one studies in the in-medium effect on the pion-nucleon amplitude then the problem arises to fit too many parameters. The other approach is based on the high-energy multiple scattering theory developed by Glauber and Sitenko, where the pion-nucleon amplitude and nuclear density distribution functions are also utilized. But here exist difficulties when accounting for the Coulomb and nuclear distortions of the incident and exit waves and also in calculations of rather complicated multiple scattering terms of phases and therefore applications are usually available only for light nuclei.

We apply [1, 2] a simple microscopic optical potential (OP) derived earlier (see *V.K. Lukyanov, E.V. Zemlyanaya, K.V. Lukyanov, Phys.At.Nucl., 2006, V.69, No.2, pp.240-254*) by comparisons of the eikonal phase of a potential and that obtained in the optical limit of the Glauber theory:

$$U_{opt}(r) = -\frac{(\hbar c)\beta_c}{(2\pi)^2} \sum_{N=p,n} \sigma_{\pi N}[i + \alpha_{\pi N}]\times$$

$$\times \int_0^\infty j_0(qr)\rho_N(q) f_{\pi N}(q) q^2 dq.$$

Here $\rho_N(q)$ is the form factor of a density distribution of bare nucleons in a nucleus, while $\sigma_{\pi N}$ and $\alpha_{\pi N}$ are the total πN cross sections and the ratio of real to imaginary part of the pion-nucleon amplitude at forward scattering. The πN-amplitude itself has the form

$$F_{\pi N}(q) = \frac{k}{4\pi} \sigma_{\pi N} [i + \alpha_{\pi N}] \times f_{\pi N}(q)$$

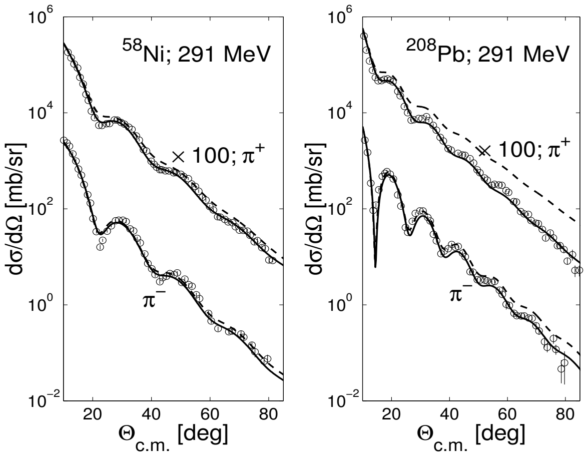

Figure 1: Comparisons of calculated differential cross sections of $\pi^{\pm}$-mesons scattered on nuclei ^{58}Ni, ^{208}Pb at T_{lab}=291 MeV to experimental data when using the in-medium parameters (see [1]). Dashed curves are for parameters of free πN amplitudes, solid lines are for the fitted parameters.

$$f_{\pi N} = e^{-\beta_{\pi N} q^2/2},$$

where $\sigma_{\pi N}$, $\alpha_{\pi N}$ and the slope parameter $\beta_{\pi N}$ can be obtained using results of the phase shift analysis of the pion scattering on protons and deuterons.

In applications of the πN-amplitude for calculations of the pion-nucleus scattering one can reduce the number of parameters by using isospin symmetry relations:

$$\sigma_{\pi^\pm n} = \sigma_{\pi^\mp p}, \alpha_{\pi^\pm n} = \alpha_{\pi^\mp p}, \beta_{\pi^\pm n} = \beta_{\pi^\mp p}.$$

Then the pion-nucleus OP takes the form

$$U_{opt}(r) = -\frac{(\hbar c)\beta_c}{(2\pi)^2}\sigma[i+\alpha]\cdot \int_0^\infty j_0(qr)\rho(q)f(q)q^2 dq,$$

where the average parameters are as follows

$$\sigma = \frac{1}{2}[\sigma_{\pi^+p} + \sigma_{\pi^-p}], \quad \alpha = \frac{1}{2}[\alpha_{\pi^+p} + \alpha_{\pi^-p}],$$

$$f(q) = e^{-\beta q^2/2}.$$

Then this OP is applied to solve the relativistic wave equation and to get the respective elastic scattering differential cross sections using DWUCK4

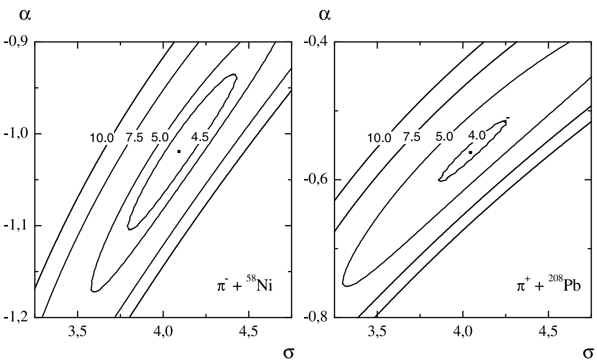

Figure 2: Fitting result for σ and α for pion scattering on ^{58}Ni and ^{208}Pb at T_{lab}=291 MeV. The numbers on lines show the respective χ^2/point values.

code. The subsequent comparison of them with experimental data provides a way to obtain the respective "in-medium" parameters σ and α. In the following we leave the parameter β to be the same as for scattering on free protons.

Calculations were made for differential elastic scattering cross sections of $\pi^{\pm}$-mesons on nuclei ^{28}Si,^{58}Ni,^{208}Pb at kinetic energy T_{lab}=291 MeV. Analyzing the obtained results one can conclude that the usage of free πN amplitudes does not allow one to get successful agreement. The good agreement can be achieved if one fit the two parameters σ and α of three to the data to demonstrate the typical transformation of the free πN amplitude to the in-medium amplitude of scattering in nuclear matter (see Fig.2). As to the cross section parameters, one can note that at 291 MeV the πN system takes place in a boundary of existence of the 33-resonance region and thus the obtained fitted parameters are fairly close to the free one. Therefore the actual task is to move investigations into lower region of energies to study effects of "in-medium" factors on the mechanism of scattering.

2. In-medium effect on the πN amplitude from analysis of π-nucleus data

Our purpose is to fit the parameters σ, α and β of the πN scattering amplitude and thus to get the best agreement of the calculated πA differential cross sections with the respective experimental data. To this end we minimize the function

$$\chi^2 = f(\sigma, \alpha, \beta) = \sum_i \frac{(y_i - \hat{y}_i(\sigma, \alpha, \beta))^2}{s_i^2},$$

where $y_i = \log[\frac{d\sigma}{d\Omega}]_i$ and $\hat{y}_i = \log[\frac{d\sigma}{d\Omega}(\sigma, \alpha, \beta)]_i$ are logarithms of, respectively, experimental and theoretical differential cross sections; s_i is an error of i-th experimental point. The fitting technique is based on the asynchronous differential evolution algorithm [3] known as effective approach to obtain a global minimum of multi-parameter functions. One can note that if the value of the other minimum occurs closely to the global one then one should to invoke an additional (say, physical) information to select one of two sets of parameters.

The calculations of differential cross sections of elastic scattering of $\pi^{\pm}$-mesons on the target nuclei ^{28}Si,^{58}Ni,^{40}Ca, and ^{208}Pb were made for energies 130, 162, 180, 226, 230, and 291 MeV, in the region of the (3,3)-resonance energy. The theoretical curves were calculated using the microscopic optical potentials presented in the previous section. The best-fit parameters σ, α, β of the in-medium pion scattering amplitude were obtained by fitting to the respective experimental data and provide a reasonable agreement with experimental data, (see Fig.3 and respective figures in [4, 5, 6]). This allows one to make estimations of "in-medium" effect on πN amplitude. Fig.4 shows the averaged values $X = (X_{\pi^+} + X_{\pi^-})/2$ where X means one of the parameters σ, α, β. The comparisons are done of behaviors of the averaged "free" $\pi^{\pm}N$-scattering parameters with the best-fit "in-medium" parameters (we mark them by "eff") in their dependency on the collision energy T^{lab}. Note, the bell-like forms of σ^{free} and $\sigma^{eff}(T^{lab})$ have maximum at the same T^{lab}. The dark gray (blue) domain of in-medium σ^{eff} is located below the light gray (yellow) σ^{free} region in about 30%. This means that the "in-medium" $\pi^{\pm}N$-interaction becomes weaker as compared to that for "free" $\pi^{\pm}N$-scattering. It might be due to the Pauli-blocking effect, when incident pions interact with nucleons bounded in nuclear shells.

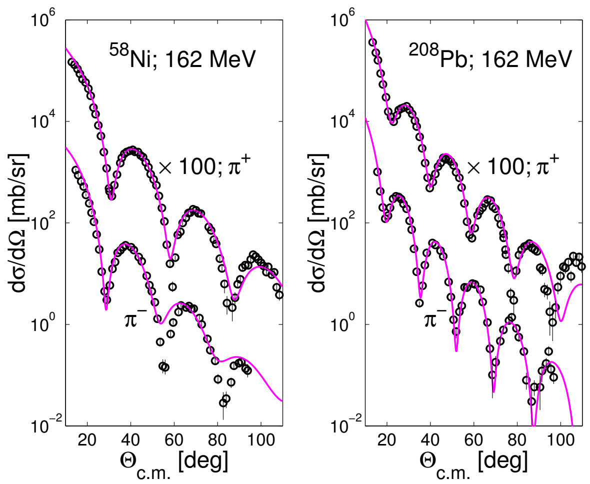

Figure 3: the same as in Fig. 1 but for T^{lab} = 162 MeV with experimental data. The best-fit "in-medium" parameters σ, α, and β are given in the [4, 6].

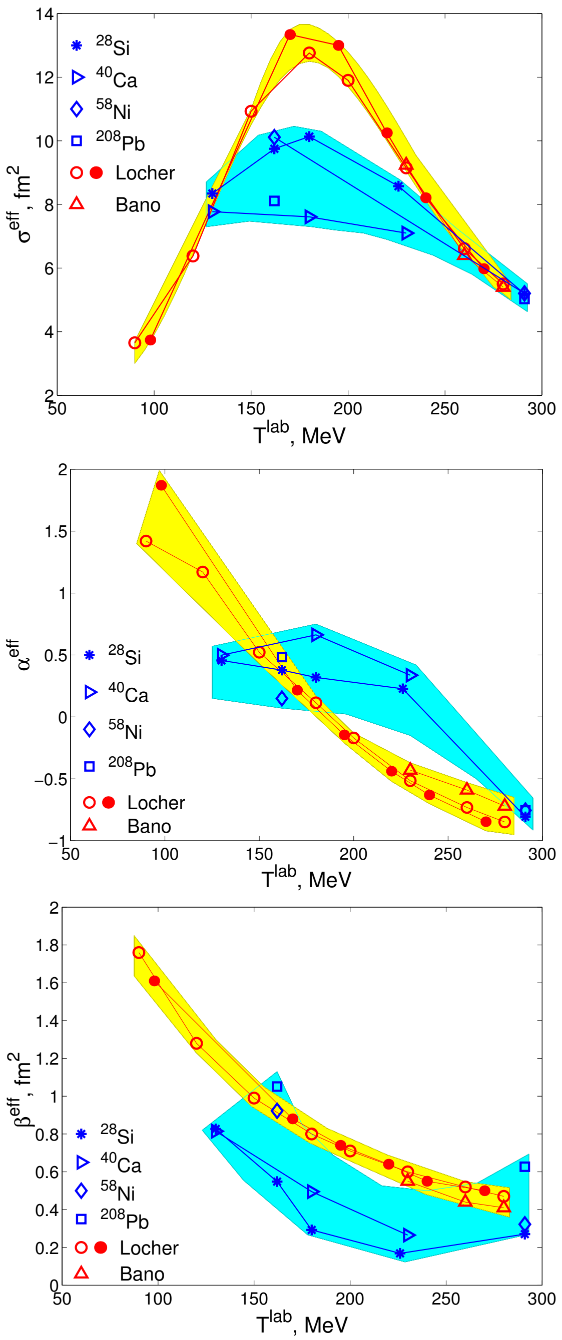

Figure 4: (Color online) Light gray (yellow): "free" $\pi^{\pm}N$-scattering parameters. Dark gray (blue): the best fit values $X^{eff} = (X_{\pi^+} + X_{\pi^-})/2$; $X = \sigma, \alpha, \beta$. Circles and triangles mark data from *Locher M.P. e.a. Nucl. Phys. B 27 (1971) 598* and *Bano N., Ahmad I. J. Phys. G 5 (1979) 39*

"In-medium" $\alpha^{eff}(T^{lab})$ behavior indicates that the real part of the forward scattering πN amplitude retains to be positive at energies higher than the maximum of the 33-resonance energy $T^{lab} > T^{lab}_{res} \simeq 170$ MeV while the α^{free} changes the sign in this region. This means that when scattering in nuclear matter, the refraction of pions increases as compared to the "free" scattering, and with the follow increase of the energy it becomes the same as for the free pions. It is seen in the figure for α, where the dark gray (blue) and light gray (yellow) regions become closer at $T^{lab} > 250$ MeV.

For the summary, in all the cases it was gained that the nuclear medium has a pronounced effect on the the pion-nucleon amplitude of scattering in the region of the (3,3)-resonance energies. As a result the energy dependence of all three parameters of the in-medium πN-amplitude differs noticeably from their behavior when scattering of pions on free nucleons.

3. Calculations of the pion-nucleus inelastic cross sections

Basing on the obtained above potentials we derive the transition optical potentials (TOP) $U_{inel}(\mathbf{r}, \xi)$ depended on the collective variables of nuclei. We construct it using the derivative of the target nucleus density distribution under the folding integral of a microscopic OP [7]. So if one gets the elastic and transition potentials, then the respective inelastic scattering cross sections can be calculated utilizing the DWUCK4 program.

The proposed scheme is distinguished from usually used models by that it operates with the main characteristic of a target nucleus, the density distribution function, while the other models are based from the beginning on an optical potential of elastic scattering.

To construct TOP we deform the surfaces of a target nucleus densities $\rho(\mathbf{r})$ by exchanging the r by $\mathbf{r}$ as follows

$$\mathbf{r} \Rightarrow r + \delta^{(\lambda)}(\mathbf{r}),$$

$$\delta^{(\lambda)}(\mathbf{r}) = -r(r/R)^{\lambda-2} \sum_{\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\hat{r}),$$

where $\lambda = 2, 3$, and $\alpha_{\lambda\mu}$ are variables of the collective motion of a nucleus (the angles of rotations or dynamic deviations relative the static "r").

Substituting this in the density we obtain and after some transformations the final equations for densities

$$\rho(\mathbf{r}) = \rho(r) + \rho_{\lambda}(r) \sum_{\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\hat{r}),$$

$$\rho_{\lambda}(r) = -r \frac{d\rho(r)}{dr} (r/R)^{\lambda-2},$$

and potential

$$U(\mathbf{r}) = U_{opt}(r) + U^{(\lambda)}(\mathbf{r}),$$

$$U^{(\lambda)}(\mathbf{r}) = U_{\lambda}(r) \sum_{\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\hat{r}),$$

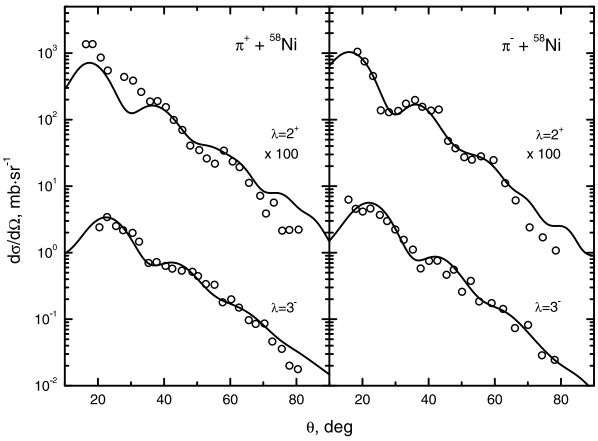

Figure 5: Comparisons of the calculated inelastic scattering cross sections of π-mesons on ^{58}Ni at T$_{lab}$= 291 MeV with experimental data, parameters see in [7].

$$U_{opt}(r) = -\frac{\hbar v}{(2\pi)^2}\sigma\,(i+\alpha)\int j_0(qr)\rho(q)f(q)\,q^2dq,$$

$$U_\lambda(r) = -\frac{\hbar v}{(2\pi)^2}\sigma\,(i+\alpha)\int j_\lambda(qr)\rho_\lambda(q)f(q)q^2dq.$$

Here $U_{opt}(r)$ provides elastic scattering calculations while the $U_\lambda(r)$ is the TOP used for calculations of inelastic scattering cross sections with excitations of the 2^+ and 3^- collective states of nuclei.

This scheme does not contain free parameters except the static or dynamic deformation parameters β_λ, that characterize the structure of their rotational or vibration excited states of nuclei. This is the only fitted parameter within the inelastic scattering calculation, all other parameters σ, α and β were fixed by elastic scattering calculations.

It is seen at Fig.5 that as a whole our calculations are in a good agreement to experimental data. The deformation parameters β_λ were obtained by adjusting the absolute values of cross sections to the data while the forms of theoretical curves are not distorted in this procedure. The obtained deformation parameters are presented in [7].

4. Summary

The main conclusions of our study are following.

1. The usage of free πN amplitudes does not allow one to get successful agrement of the calculated pion-nucleus differential cross sections to the respective experimental data caused by "in-medium" effect.

2. The "in-medium" parameters for the amplitude of elastic pion-nucleus scattering were obtained for two- and three parameters fit for the wide energy range in (3,3)-resonance region.

3. The transitional optical potential obtained for calculations of inelastic scattering cross sections gives the reasonable good agreement with experimental data without any free parameters except the deformation parameter β_λ

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