Bayesian Analysis of Hybrid EoS Based on Astrophysical Observational Data

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Introduction

The most basic features of a neutron star (NS) are its radius and mass which so far have not been well determined simultaneously for a single object. In some cases masses are precisely measured like in the case of binary systems but radii are quite uncertain. In the other hand, for isolated neutron stars some radius and mass measurements exist but lack the necessary precision to inquire into their interiors. In fact, it has been conjectured that there exist a unique relation between the mass and radius relation for all neutron stars and their equation of state (EoS), that determines their internal composition [8]. For this reason, accurate observations of masses and radii are crucial to study cold dense nuclear matter expected to exist in neutron stars.

However, the present observable data allows to make probabilistic estimation of the internal structure of the star. In this report preliminary probabilistic estimation of the superdense stellar matter equation of state using Bayesian Analysis and modeling of relativistic configurations of neutron stars is shown. This analysis is important for research of existence the quark-gluon plasma in massive (around 2 sun masses) neutron stars.

NS structure

The microscopical properties of compact stars are modeled in the framework of general relativity, where the Einstein equations are solved for a static (non-rotating), spherical star resulting into the so called Tolman–Oppenheimer–Volkoff (TOV) equation equations [6]:

$$\begin{cases} \dfrac{dm}{dr} = C_1 \epsilon r^2 \\[2ex] \dfrac{dp}{dr} = -C_2 \dfrac{(\epsilon + p)(m + C_1 p r^3)}{r(r - 2C_2 m)} \end{cases} \tag{1}$$

as well as the equation for the baryon mass of the star:

$$\frac{dm_B}{dr} = C_1 n_B m_N \frac{r^2}{\sqrt{1 - 2C_2 m/r}} \tag{2}$$

with constants defined as:

$$\begin{cases} C_1 = 1.11269 \cdot 10^{-5} \dfrac{\mathrm{M}_\odot}{\mathrm{km}^3} \dfrac{\mathrm{fm}^3}{\mathrm{MeV}} \\[2ex] C_2 = 1.4766 \dfrac{\mathrm{km}}{\mathrm{M}_\odot} \end{cases} \tag{3}$$

These equations are integrated from the center of the star towards its surface, with the radius of the star R defined by the condition $p(r) = 0$ while the gravitational mass by $M = m(R)$. In a similar manner, the baryon mass is given by $M_B = m_B(R)$.

To complement the solution to the TOV equations the EoS is required. It is given by the relation $p = p(\epsilon)$ which carries information about the microscopic ingredients of the dense nuclear matter, as mentioned before. Thus, the above equations have to be solved simultaneously using the equation of state under the following boundary conditions at the star centre ($r = 0$):

$$\begin{cases} p(r \simeq 0) \simeq p(0) = p(\epsilon_c) \\ m(r) \simeq \dfrac{C_1}{3} \epsilon_c r^3 \end{cases} \tag{4}$$

where ϵ_c is the energy density at the centre of the neutron star, taken as an input. In this way, for a given value of ϵ_c the solution of the TOV equations are the $p(r)$ and $m(r)$ profiles and with them the parametric relation $M(R)$ as a function of ϵ_c can be obtained.

The chosen EoS

For this study we follow the AHP scheme [2] for the Equation of State (EoS):

$$p(\epsilon) = p^I(\epsilon)\Theta(\epsilon_c - \epsilon) + c^2_{QM}\epsilon\Theta(\epsilon - \epsilon_c - \Delta\epsilon), \tag{5}$$

where p^I is given by a pure hadronic EoS and p^{II} represents the high density nuclear matter introduced here as quark matter with c^2_{QM} as its squared speed of sound, as parametrized by Haensel et al. [9] which describes pretty well the superconducting NJL model derived in [4]. For the hadronic EoS we take the well known model of APR [1] that is in agreement with experimental data of densities about nuclear saturation. For this hadronic branch (I) all the relevant thermodynamical variables, energy density ϵ, pressure p, baryon density n and chemical potential μ are well defined and taken as input for determination of the hybrid (hadronic + quark matter) EoS. As a starting point in derivation of the high density EoS we introduce the pressure as function of energy density in the quark matter side:

$$p^{II}(\epsilon) = c_{QM}^2\epsilon - B(1 + c_{QM}^2), \quad (6)$$

with $B(1 + c_{QM}^2)$ playing the role of a bag constant. To determine the remaining thermodynamical quantities n^{II} and μ^{II} of the quark matter side we use the following relations:

$$n = \int_{\bar{\epsilon}}^{\epsilon} \frac{\epsilon'}{\epsilon' + p(\epsilon')} \quad (7)$$

$$\mu = \frac{\epsilon + p}{n} \quad (8)$$

where $\bar{\epsilon}$ is the quark matter energy density right after the phase transition following the jump $\Delta\epsilon$ as density increases. Therefore one arrives at the following formula

$$n^{II}(\epsilon) = \frac{p_c + \bar{\epsilon}}{p_c + \epsilon_c} n_c \left(\frac{\epsilon - B}{\bar{\epsilon} - B}\right)^{\frac{1}{1+c_{QM}^2}}, \quad (9)$$

obtained by enforcing conditions of equal pressure and chemical potential at the transition (Gibbs conditions):

$$\mu_c = \frac{p_c + \bar{\epsilon_c}}{\bar{n}} = \frac{p_c + \epsilon_c}{n_c}. \quad (10)$$

The free parametes of the model are the transition density ϵ_c, the energy density jump $\Delta\epsilon \equiv \gamma\epsilon_c$ and c_{QM}^2, the quark matter speed of sound squared. The resulting EoS in the plane pressure versus density is depicted in figure 1 for a given set of input parameters.

BA Formulation and Formalization

We define the vector of free parameters $\overrightarrow{\pi}\left(\epsilon, \gamma, c_s^2\right)$, where ϵ is the critical value of energy density at phase transition (PhT), $\gamma = \epsilon/\epsilon_c$ is a ratio of the energy jump on PhT to the critical one, and c_s^2 is square of speed of sound in quark matter. These parameters are define the equation of state with phase transition from nuclear to quark matter. The nuclear equation of state can be taken APR [1].

These parameters were sampled:

$$\pi_i = \overrightarrow{\pi}\left(\epsilon_k, \gamma_l, c_{s\,m}^2\right), \quad (11)$$

where $i = 0 \ldots N-1$ (here $N = N_1 \times N_2 \times N_3$) as $i = N_1 \times N_2 \times k + N_2 \times l + m$ and $k = 0 \ldots N_1 - 1$, $l = 0 \ldots N_2 - 1$, $m = 0 \ldots N_3 - 1$, here N_1, N_2 and N_3 number of parameters ϵ_k, γ_l and $c_{s\,m}^2$ respectively.

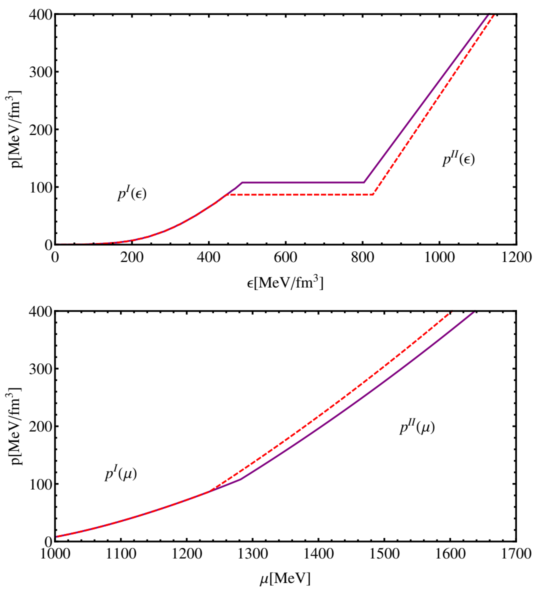

Figure 1: Hybrid EoS scheme for two different sets of parameters $\left(\epsilon, \gamma, c_s^2\right)$.

Using equation of state (EoS) one can calculate the neutron star construction by solving TOV equations. Then it is possible to use different neutron star observations to check possibility of EoS. We use three constraints: mass constraint [3], radius constraint [5] and constraint of ratio between gravitational mass and baryon mass [7].

The goal is to find the set of most probable π_i basing on given constraints using Bayesian Analysis (BA). For initializing BA we propose that *a priori* each vector of parameter π_i has probability equal one: $P(\pi_i) = 1$ for $\forall i$.

Mass Constraint

We propose that error of measurement is normal distributed $\mathcal{N}(\mu_A, \sigma_A^2)$, where $\mu_A = 2.01$ M$_\odot$ and $\sigma_A = 0.04$ M$_\odot$, it is measurements of massive PSR J0348+0432 [3]. Using this assumption we can calculate conditional probability of event E_A that mass of neutron star corresponds to measurement:

$$P\left(E_A \left|\pi_i\right.\right) = \Phi(M_i, \mu_A, \sigma_A), \quad (12)$$

where M_i - maximal mass constructed by π_i and $\Phi(x, \mu, \sigma)$ is the cumulative distribution function for the normal distribution:

Radius Constraint

Radius measurement gives $\mu_B = 15.5$ km and $\sigma_B = 1.5$ km, data for PSR J0437-4715 [5]. Now it is possible to calculate conditional probability of event E_B that radius of neutron star corresponds to the given measurement:

$$P\left(E_B\left|\pi_i\right.\right) = \Phi(R_i, \mu_B, \sigma_B), \tag{13}$$

$M_G M_B$ Ratio Constraint

This constraint gives region in the $M_G M_B$ plane. We need to estimate probability of closing point $M_i = (M_{Gi}, M_{Bi})$ to point $\mu = (\mu_G, \mu_B)$. The mean values $\mu_G = 1.249$, $\mu_B = 1.36$ and standard deviations $\sigma_{M_G} = 0.001$, $\sigma_{M_B} = 0.002$ are given in [7]. Needed probability can be calculated by following formula:

$$P\left(E_K\left|\pi_i\right.\right) = \left[\Phi\left(\xi_G\right) - \Phi\left(-\xi_G\right)\right] \cdot \left[\Phi\left(\xi_B\right) - \Phi\left(-\xi_B\right)\right], \tag{14}$$

where $\Phi\left(x\right) = \Phi\left(x, 0, 1\right)$, $\xi_G = \frac{\sigma_{M_G}}{d_{M_G}}$ and $\xi_B = \frac{\sigma_{M_B}}{d_{M_B}}$, d_{M_G} and d_{M_B} are absolute values of components of vector $\mathbf{d} = \mu - \mathbf{M}_i$, here $\mu = \left(\mu_G, \mu_B\right)^T$ given int [7] and $\mathbf{M}_i = \left(M_{Gi}, M_{Bi}\right)^T$ is solution of TOV using i^{th} vector of EoS parameters π_i. Note that formula (14) does not correspond to multivariate normal distribution.

Calculation of a posteriori Probabilities

Note, that these measurements are independent on each other. That means that we can calculate complete conditional probability of event that contracted by π_i object corresponds to all measurements:

$$P\left(E\left|\pi_i\right.\right) = P\left(E_A\left|\pi_i\right.\right) \times P\left(E_B\left|\pi_i\right.\right) \times P\left(E_K\left|\pi_i\right.\right). \tag{15}$$

Now, we can calculate probability of π_i using Bayes' theorem:

$$P\left(\pi_i\left|E\right.\right) = \frac{P\left(E\left|\pi_i\right.\right) P\left(\pi_i\right)}{\sum\limits_{j=0}^{N-1} P\left(E\left|\pi_j\right.\right) P\left(\pi_j\right)}. \tag{16}$$

Conclusion and Discussion

We a play the scheme of BA for probabilistic estimation of the EoS given by vector parameter π. Varying parameters ϵ, γ, c_s^2 in intervals $[400..10^3] MeV/fm^{-3}$, [0..1] and [0.3..1] respectively we explore the calculations $N = 10^3$ and the results presented in 2. Mass-radius relation is shown on top figure and pressure in depending on energy density is given on bottom one. Results are shown for different sets of neutron star configurations corresponding to different sets of EoS parameters. The thickness of the lines is chosen to be proportional to the probability value of parameter vector π. We show that the chosen constraints are not enough to distinguish two cases one with the existent of third family of twins (two stars with same masses and different radii) from the case where only neutron star family is possible. This result can be also seeing from the bottom figure of 2 where shown EoS plotted with different thickness of the lines corresponding to the probability value of parameter vector π. It is easy to see that two branches of EoS with and without the phase transition to quark matter have approximately same probability. Nevertheless, if phase transition to quark matter is possible then our constraint requires that the phase transition should a care for energy densities higher then $900 MeV/fm^{-3}$ with density jump up to $10^3 MeV/fm^{-3}$. Our conclusion could be that the current state of knowledge of observable data which we use does not allow us to be sure about of

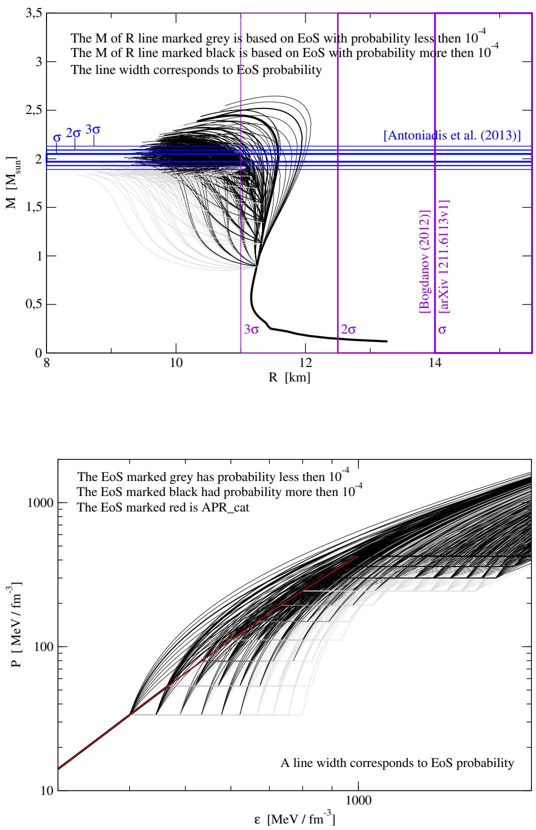

Figure 2: Mass-radius relation (on top) and pressure vs energy density (on bottom) for different sets of NS configurations corresponding to different sets of EoS parameters, the thickness of the lines is chosen to be proportional to the probability value of parameter vector π.

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References

[1] A. Akmal, V.R. Pandharipande, and D.G. Ravenhall. *Equation of state of nucleon matter and neutron star structure.* Physical Review C (1998), vol. 58, no. 3, pp. 1803-1838.

[2] M. G. Alford, S. Han and M. Prakash. *Generic conditions for stable hybrid stars.* arXiv:1302.4732 (2013), 11 p.

[3] J. Antoniadis et al. *A Massive Pulsar in a Compact Relativistic Binary.* Science (2013), vol. 340, no. 6131; doi: 10.1126/science.1233232.

[4] D. Blaschke, D.E. Alvarez-Castillo, S. Benic, G. Contrera, R. Lastowiecki. *Nonlocal PNJL models and heavy hybrid stars.* arXiv:1302.6275 (2013), 8 p.

[5] Slavko Bogdanov. *The Nearest Millisecond Pulsar Revisited with XMM-Newton: Improved Mass-radius Constraints for PSR J0437-4715.* The Astrophysical Journal (2013), vol. 762, iss. 2, pp. 96.

[6] N.K. Glendenning. *Compact stars: nuclear physics, particle physics, and general relativity.* New York: Springer, 2000.

[7] F.S. Kitaura, H.-Th. Janka and W. Hillebrandt. *Explosions of O-Ne-Mg cores, the Crab supernova, and subluminous type II-P supernovae.* Astronomy and Astrophysics (2006), vol. 450, no. 1, pp. 345-350.

[8] L. Lindblom. *Limits on the Gravitational Redshift from Neutron Stars.* The Astrophysical Journal (1984), vol. 278, 364-368.

[9] J.L. Zdunik and P. Haensel. *Maximum mass of neutron stars and strange neutron-star cores.* Astronomy and Astrophysics (2013), vol. 551, no. A61.