On the Continuum Limit of SU(2) Landau Gauge Gluodynamics

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The systematic investigation of $SU(2)$ gluon and ghost Landau gauge propagators on large lattices was continued [1] in order to receive from first principles information on the behavior of these propagators and of the running coupling in the continuum limit for all momenta q including the infrared (IR) region. As in previous investigations [4] for solving the Gribov problem we assume the Landau gauge functional to be driven as close as possible to its global extremum. Employing the standard Wilson plaquette action we have studied gluon (D) and ghost (G) propagators for lattice sizes $L^4, L = 40, 56, 80, 96, 112$ The gluon propagator was computed using $N_{MC}(\beta, L)$ of order 10^2 independent Monte Carlo (MC) configurations generated with the given set of parameters, while the ghost propagator was calculated only on a smaller subsets of $N_{ghost}(\beta, L)$ MC configurations.

We employed very long simulated annealing (SA) runs followed by overrelaxation (OR) to obtain gauge copies with a gauge fixing functional close to its global extremum for each MC configuration.

We show the results for the bare gluon propagator (Fig. 1) and the bare ghost dressing function (Fig. 2) for fixed physical volume $(aL)^4$ but varying lattice scale a. For the gluon propagator we have drawn also curves obtained from fits with the 6-parameter formula proposed in [5]

$$D(q) = C\frac{q^4 + A^2q^2 + B}{q^6 + Eq^4 + Fq^2 + G^2} \qquad (1)$$

We found that the resulting fit curves nicely capture the IR turnover of the gluon propagator. The χ^2/dof values are close to unity in most cases.

To obtain the renormalized gluon propagator $D_{ren}(q, \mu) = \tilde{Z}(a, \mu)D(q, a)$ we apply the normalization condition $D_{ren}(\mu, \mu) = 1/\mu^2$. Since the fit formula of Eq. (1) nicely works throughout the whole momentum region we can use it to carry out the renormalization at any μ. From Fig. 3 and Fig. 4 we can then conclude also for the renormalized gluon propagator $D_{ren}(q^2)$ and the renormalized ghost dressing function $J_{ren}(q^2)$ found for various lattice spacings $a(\beta)$ to be compatible with the so-called *decoupling solution* of Dyson-Schwinger or functional renormalization group equations (see [7]). The numerical values, however, of $D_{ren}(q^2)$ and $J_{ren}(q^2)$ in the limit $q \to 0$ appear to be β- or a-dependent. From these plots one can see that the convergence of the renormalized lattice propagators / dressing functions in the deep IR momentum

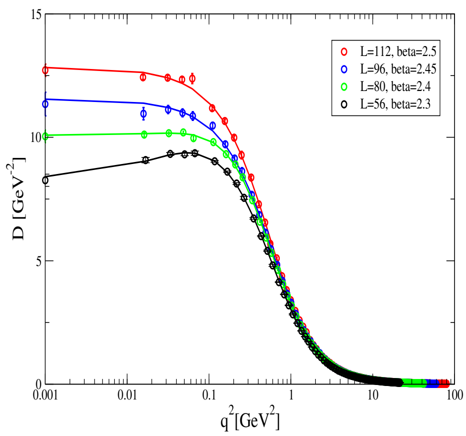

Figure 1: The unrenormalized gluon propagator $D(q^2)$ and its fits according to Eq. (1) for approximately equal physical volume ($(aL)^4 \simeq (9.6\text{fm})^4$) but different β-values

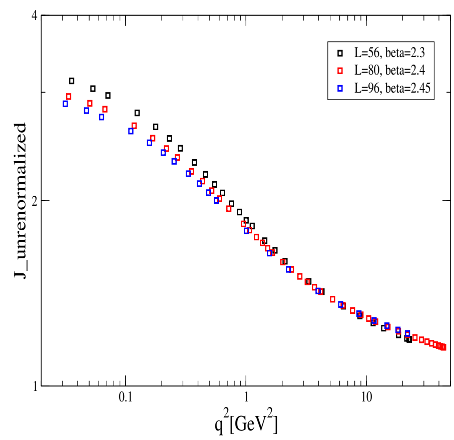

Figure 2: The unrenormalized ghost dressing function $J(q^2)$ for approximately equal physical volume ($(aL)^4 \simeq (9.6\text{fm})^4$) but different β-values.

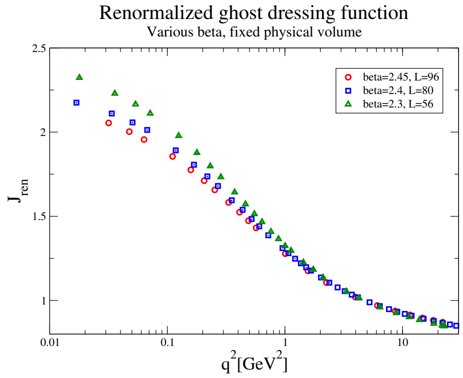

Figure 4: Renormalized ghost propagator $J_{ren}(q^2)$ at $\mu = 2.2\ GeV]$

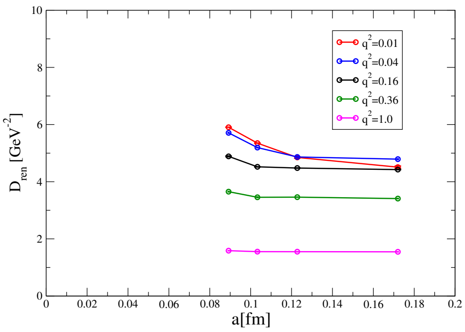

Figure 5: Renormalized gluon propagator $D_{ren}(q^2)$ vs lattice spacing a at $\mu = 2.2\ GeV]$

range to the respective continuum counterpart, that should be observed for decreasing $a(\beta)$, is rather slow. For their direct numerical study near the continuum limit one has to use rather large β-values which consequently requires simulations on unrealistically huge lattices, which are not accessible today even on most powerful parallel supercomputers. Instead, we can try to make contact with the continuum limit by extrapolating $D_{ren}(q^2, a)$ to the zero-a limit as done e.g. in [3] for $SU(3)$ and non-zero temperature. In Fig. 5 we plot the a-dependence of lattice $D_{ren}(q^2)$ for several selected values of q^2. We see the lower the momentum is the less well-defined the convergence for $a \to 0$ becomes. For getting reliable numerical values of $SU(2)$ gluon and ghost propagators in the continuum limit more work is needed. Although the ghost dressing function $J(q^2)$ has been computed only for a subset of MC configurations, it provided useful quantitative information, see Fig. 4. Even a single MC configuration, e.g.for $L = 96$, already seems to yield a first estimate (a fast decrease of the statistical fluctuations of $J(q^2)$ with increasing L was first observed for the $SU(3)$ case in [6]). Our analysis shows that in the deep IR region $D_{ren}(q^2)$ increases with β, and $J_{ren}(q^2)$ decreases.

We have checked whether the differences of propagator values in the deep IR could be compensated by other systematic effects. From Fig. 6 one can see that finite-volume effects are small (at least for $\beta = 2.3$) if the linear physical size is $a(\beta)L \simeq 9.6$ fm or even larger. What concerns Gribov copy artifacts at $\beta = 2.4$ and $L = 80$ we have compared the results of two sets of SA+OR gauge fixing simulations: (i) one gauge copy fixing with 9600 SA sweeps ("SA1 schedule") ($N_{MC} = 314$) and (ii) "best of two copies" gauge fixing with 12000 SA sweeps each ("SA2 schedule") ($N_{MC} = 187$). For more details see, e.g., Ref. [4]. SA1 and SA2 results obtained

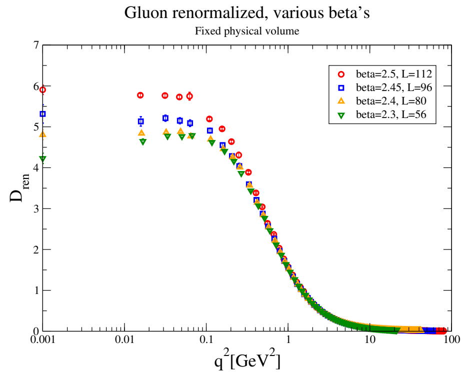

Figure 3: Renormalized ghost propagator $J_{ren}(q^2)$ at $\mu = 2.2\ GeV]$

for unrenormalized gluon propagators are plotted in Fig. 7 . We have found that the differences between these cases are much smaller than the magnitude of Gribov copy effects measured in [2] as difference between results of one-copy SA+OR and one-copy OR gauge-fixing procedures. Our analysis shows that further “improvement” of SA schedules could not change $D_{ren}(q^2)$ essentially and hence notice-

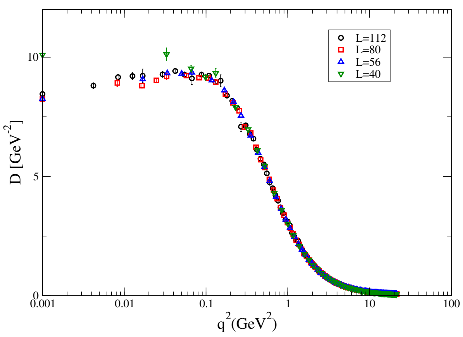

Figure 6: Check of systematic finite-volume errors. The unrenormalized $D(q^2)$ computed for $\beta = 2.3$ and various L.

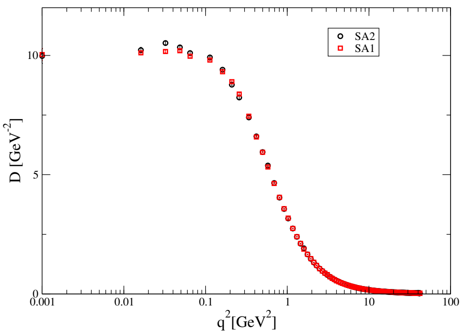

Figure 7: Check of Gribov copy effect. $D(q^2)$ for 2 different SA schedules at $\beta = 2.4$ and $L = 80$.

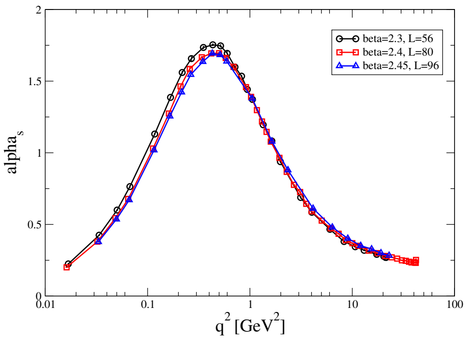

Figure 8: Running coupling for $\beta = 2.3$, $\beta = 2.4$ and $\beta = 2.45$

able differences of $D_{ren}(q^2)$ values in the deep IR region found for different β-values certainly cannot be accounted for by the Gribov copy effect.

With the bare gluon ($Z(q^2)$) and ghost ($J(q^2)$) dressing functions at hand one can easily compute the running coupling

$$\alpha_s(q^2) \;=\; \frac{g_0^2}{4\pi}\; J^2(q^2)\; Z(q^2).$$

The dependence of the resulting curves of $\alpha_s(q^2)$ on β or a seems to be rather weak even in the deep IR momentum region (see Fig. 8).

We conclude that naive multiplicative renormalizability for the $SU(2)$ Landau gauge gluon and ghost propagators gets violated in the deep IR region. Due to the slow convergence of gluon and ghost renormalized propagators their continuum counterparts may strongly differ in the deep IR momentum region from what we have obtained here in lattice simulations with admissible values of L and $\beta = 4/g_0^2$.

Simulations have been done on the MVS100K supercomputer of the Joint Supercomputer Centre (JSCC, Moscow).

References

[1] I.L. Bogolubsky, E.-M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck, *On the continuum limit of Landau gauge gluon and ghost propagators in SU(2) lattice gauge gluodynamics.* PoS Confinement X (2012) 285-289; arXiv:1303.3423 [hep-lat]

[2] I.L. Bogolubsky, E.-M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck *The Landau gauge gluon propagator in 4D SU(2) lattice gauge theory revisited: Gribov copies and scaling properties.* PoS LAT2009 (2009) 237-242 ; arXiv:0912.2249 [hep-lat].

[3] R. Aouane, V. Bornyakov, E.-M. Ilgenfritz, V. Mitrjushkin, M.Müller-Preussker, and A. Sternbeck *Landau gauge gluon and ghost propagators at finite temperature from quenched lattice QCD.* Phys. Rev. D**85**, (2012) 034501-034512; arXiv:1108.1735 [hep-lat].

[4] I.L. Bogolubsky, E.-M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck *Lattice gluodynamics computation of Landau-gauge Green's functions in the deep infrared.* Phys. Lett. B**676** (2009) 69-73; arXiv: 0901.0736 [hep-lat].

[5] A. Cucchieri, D. Dudal, T. Mendes and N. Vandersickel *Modeling the gluon propagator in Landau gauge: Lattice estimates of pole masses and dimension-two condensates.* Phys. Rev. D**84**, (2012) 094513-27, arXiv:1111.2327 [hep-lat].

[6] I.L. Bogolubsky, E.-M. Ilgenfritz, M. Müller-Preussker, A. Sternbeck *The Landau gauge gluon and ghost propagators in 4D SU(3) gluodynamics in large lattice volumes.* PoS LAT2007 (2007), 290-296; arXiv:0710.1968 [hep-lat].

[7] C.S. Fischer, A. Maas and J.M. Pawlowski *On the infrared behavior of Landau gauge Yang-Mills theory.* Annals Phys. **324**,(2009) 2408-2437; arXiv:0810.1987[hep-lat].