Nuclear Spin Scissors – New Type of Collective Motion

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Abstract The coupled dynamics of low lying modes and various giant resonances are studied with the help of the Wigner Function Moments method on the basis of Time Dependent Hartree-Fock equations in the harmonic oscillator model including spin-orbit potential plus quadrupole-quadrupole and spin-spin residual interactions [1]. New low lying spin dependent modes are analyzed. Special attention is paid to the spin scissors mode.

Introduction

The idea of the possible existence of the collective motion in deformed nuclei similar to the scissors motion continues to attract the attention of physicists who extend it on various kinds of objects, not necessary nuclei, (for example, magnetic traps, see the review by Heyde at al [2]) and invent new sorts of scissors, for example, the rotational oscillations of neutron skin against a proton-neutron core [3]. In recent papers [1, 4] the WFM method was applied for the first time to solve the TDHF equations including spin dynamics As a first step, only the spin-orbit interaction was included in the consideration [4], as the most important one among all possible spin dependent interactions because it enters into the mean field. The most remarkable result was the discovery of a new type of nuclear collective motion: rotational oscillations of "spin-up" nucleons with respect of "spin-down" nucleons (the spin scissors mode). It turns out that the experimentally observed group of peaks in the energy interval 2-4 MeV corresponds very likely to two different types of motion: the conventional (orbital) scissors mode and the spin scissors mode.

The aim of the work [1] is to get a qualitative understanding of the influence of the spin-spin force on the new states analyzed in [4], as, for instance, the spin scissors mode. As a matter of fact we will find that the spin-spin interaction does not change the general picture of the positions of excitations described in [4]. The most interesting result concerns the B(M1) values of both scissors modes – the spin-spin interaction strongly redistributes M1 strength in the favour of the spin scissors mode, that allows us to give a tentative explanation of recent experimental findings [5, 6].

TDHF equation and model Hamiltonian

We consider the TDHF equation in coordinate space keeping all spin indices:

$$i\hbar\langle\mathbf{r}|\dot{\hat{\rho}}|\mathbf{r}''\rangle^{ss"} = \sum_{s'}\int d^3r'\left(\langle\mathbf{r}|\hat{h}|\mathbf{r}'\rangle^{ss'}\langle\mathbf{r}'|\hat{\rho}|\mathbf{r}''\rangle^{s's"}\right.$$
$$\left. - \langle\mathbf{r}|\hat{\rho}|\mathbf{r}'\rangle^{ss'}\langle\mathbf{r}'|\hat{h}|\mathbf{r}''\rangle^{ss"}\right). \quad (1)$$

We do not specify the isospin indices in order to make the formulae more transparent.

¿From the technical point of view it is more convenient to work with the Wigner function $f(\mathbf{r},\mathbf{p})$ instead of density matrix $\langle\mathbf{r}|\hat{\rho}|\mathbf{r}'\rangle$. To this end we rewrite the expression (1) with the help of the Wigner transformation: By means of the Wigner transformation [7] the following set of equations for the Wigner functions $f(\mathbf{r},\mathbf{p})$ with specified spin indeces is derived:

$$i\hbar\dot{f}^+ = \frac{i\hbar}{2}\{h^+,f^+\} + \frac{i\hbar}{2}\{h^-,f^-\}$$
$$+i\hbar\{h^{\uparrow\downarrow},f^{\downarrow\uparrow}\} + i\hbar\{h^{\downarrow\uparrow},f^{\uparrow\downarrow}\},$$
$$i\hbar\dot{f}^- = \frac{i\hbar}{2}\{h^+,f^-\} + \frac{i\hbar}{2}\{h^-,f^+\} - 2h^{\downarrow\uparrow}f^{\uparrow\downarrow}$$
$$+2h^{\uparrow\downarrow}f^{\downarrow\uparrow} + \frac{\hbar^2}{4}\{\{h^{\downarrow\uparrow},f^{\uparrow\downarrow}\}\} - \frac{\hbar^2}{4}\{\{h^{\uparrow\downarrow},f^{\downarrow\uparrow}\}\},$$
$$i\hbar\dot{f}^{\uparrow\downarrow} = \frac{i\hbar}{2}\{h^{\uparrow\downarrow},f^+\} + \frac{i\hbar}{2}\{h^+,f^{\uparrow\downarrow}\} - h^{\uparrow\downarrow}f^-$$
$$+h^-f^{\uparrow\downarrow} + \frac{\hbar^2}{8}\{\{h^{\uparrow\downarrow},f^-\}\} - \frac{\hbar^2}{8}\{\{h^-,f^{\uparrow\downarrow}\}\},$$
$$i\hbar\dot{f}^{\downarrow\uparrow} = \frac{i\hbar}{2}\{h^{\downarrow\uparrow},f^+\} + \frac{i\hbar}{2}\{h^+,f^{\downarrow\uparrow}\} + h^{\downarrow\uparrow}f^-$$
$$-h^-f^{\downarrow\uparrow} - \frac{\hbar^2}{8}\{\{h^{\downarrow\uparrow},f^-\}\} + \frac{\hbar^2}{8}\{\{h^-,f^{\downarrow\uparrow}\}\}, (2)$$

where the functions h, f are the Wigner transforms of $\hat{h}$, $\hat{\rho}$ respectively, $\{f,h\}$ is the Poisson bracket of the functions $f(\mathbf{r},\mathbf{p})$ and $h(\mathbf{r},\mathbf{p})$ and $\{\{f,h\}\}$ is their double Poisson bracket, $f^{\pm} = f^{\uparrow\uparrow} \pm f^{\downarrow\downarrow}$, $h^{\pm} = h^{\uparrow\uparrow} \pm h^{\downarrow\downarrow}$. The conventional notation $\uparrow$ for $s = 1/2$ and $\downarrow$ for $s = -1/2$ is used.

The microscopic Hamiltonian of the model, harmonic oscillator with spin orbit potential plus separable quadrupole-quadrupole and spin-spin residual

interactions is given by (see [1, 4] for details)

$$H = \sum_{i=1}^{A} \left[\frac{\hat{\mathbf{p}}_i^2}{2m} + \frac{1}{2} m\omega^2 \mathbf{r}_i^2 - \eta \hat{\mathbf{l}}_i \hat{\mathbf{S}}_i \right] + H_{qq} + H_{ss}. \quad (3)$$

Equations of motion

Integrating the set of equations (2) over phase space with the weights

$$W = \{r \otimes p\}_{\lambda\mu}, \{r \otimes r\}_{\lambda\mu}, \{p \otimes p\}_{\lambda\mu}, \text{ and } 1, \quad (4)$$

where $\{r \otimes r\}_{\lambda\mu} = \sum_{\sigma,\nu} C_{1\sigma,1\nu}^{\lambda\mu} r_\sigma r_\nu$ – tensor product, $C_{1\sigma,1\nu}^{\lambda\mu}$ is the Clebsch-Gordan coefficient and r_1, r_0, r_{-1} are cyclic coordinates [8], one gets dynamic equations for the following collective variables:

$$L_{\lambda\mu}^{\tau\varsigma}(t) = \iint \frac{d\mathbf{r}\,d\mathbf{p}}{(2\pi\hbar)^3} \{r \otimes p\}_{\lambda\mu} f^{\tau\varsigma}(\mathbf{r}, \mathbf{p}, t),$$

$$R_{\lambda\mu}^{\tau\varsigma}(t) = \iint \frac{d\mathbf{r}\,d\mathbf{p}}{(2\pi\hbar)^3} \{r \otimes r\}_{\lambda\mu} f^{\tau\varsigma}(\mathbf{r}, \mathbf{p}, t),$$

$$P_{\lambda\mu}^{\tau\varsigma}(t) = \iint \frac{d\mathbf{r}\,d\mathbf{p}}{(2\pi\hbar)^3} \{p \otimes p\}_{\lambda\mu} f^{\tau\varsigma}(\mathbf{r}, \mathbf{p}, t),$$

$$F^{\tau\varsigma}(t) = \iint \frac{d\mathbf{r}\,d\mathbf{p}}{(2\pi\hbar)^3} f^{\tau\varsigma}(\mathbf{r}, \mathbf{p}, t), \quad (5)$$

where $\tau = n, p$ – isospin indices, $\varsigma = +, -, \uparrow\downarrow, \downarrow\uparrow$. By analogy with isoscalar $f^n + f^p$ and isovector $f^n - f^p$ functions we call the functions $f^{\tau+}$ and $f^{\tau-}$ and the corresponding collective variables $X_{\lambda\mu}^{\tau+}(t)$ and $X_{\lambda\mu}^{\tau-}(t)$ as spin-scalar and spin-vector ones.

We are interested in the scissors mode, the excitation with $K^\pi = 1^+$, therefore, we only need the part of dynamic equations with $\mu = 1$. These equations are nonlinear due to quadrupole-quadrupole and spin-spin interactions and will be solved in a small amplitude approximation.

It is convenient to rewrite the dynamical equations in terms of isovector and isoscalar variables $X_{\lambda\mu} = X_{\lambda\mu}^n + X_{\lambda\mu}^p$, $\bar{X}_{\lambda\mu} = X_{\lambda\mu}^n - X_{\lambda\mu}^p$. Then the equations for the neutron and proton systems are transformed into isovector and isoscalar ones. Supposing that all equilibrium characteristics of the proton system are equal to that of the neutron system one decouples isovector and isoscalar equations. We get the closed set of 19 equations [1].

Discussion and interpretation of the results

Imposing the time evolution via $e^{i\Omega t}$ for all variables one transforms dynamical equations into a set of algebraic equations. Eigenfrequencies are found as the zeros of its determinant. Excitation probabilities are calculated with the help of the linear response theory. The used spin-spin interaction is

taken from the paper [9]. The results of calculations without spin-spin interaction (variant I) are compared with those performed with two sets of constants K_s, q (variants II, III). The strength of the spin-orbit interaction is taken from [10].

One can see from table 1 that the spin-spin interaction does not change the qualitative picture of the positions of excitations described in [4]. It pushes all levels up proportionally to its strength (20-30% in the case II and 40-60% in the case III) without changing their order. The most interesting result concerns the relative B(M1) values of two low lying scissors modes, namely the spin scissors $(1, 1)^-$ and the conventional (orbital) scissors $(1, 1)^+$.

The spin-spin interaction strongly redistributes M1 strength in the favour of the spin scissors mode. We tentatively want to link this fact to the recent experimental finding in isotopes of Th and Pa [5]. The authors have studied deuteron and ^{3}He-induced reactions on ^{232}Th and found in the residual nuclei ^{231,232,233}Th and ^{232,233}Pa "an unexpectedly strong integrated strength of $B(M1) = 11 - 15\ \mu_N^2$ in the $E_\gamma = 1.0 - 3.5$ MeV region". The $B(M1)$ force in most nuclei in [5] shows evident splitting into two Lorentzians.

Quite similar results were obtained one year earlier by A. S. Adekola et al [6]. They have studied the (γ, γ') reaction on ^{232}Th and found two groups of levels with energy centroids $E_1 = 2.1$ MeV and $E_2 = 2.9$ MeV having $B(M1)_1 = 2.52\mu_N^2$ and $B(M1)_2 = 1.74\mu_N^2$. The splitting $E_2 - E_1 = 0.8$ MeV and the ratio $B(M1)_1/B(M1)_2 = 1.45$ are in good agreement with the results of [5] and with our calculations.

Concluding remarks

In the work [1] we continued the investigation of spin modes [4] with the Wigner Function Method. The inclusion of spin-spin interaction does not change qualitatively the described picture concerning spin modes found in [4]. It pushes all levels up without changing their order. However, it strongly redistributes M1 strength between the conventional and spin scissors mode in the favour of the last one. We mentioned the recently appeared experimental works [5, 6], where for the two groups of low lying magnetic states a stronger B(M1) transition for the lower group with respect to the higher one was found. Our theory can naturally predict such a scenario with a non vanishing spin-spin force. It would indeed be very exciting, if the results of [5, 6] had already discovered the isovector spin scissors mode.

In the light of the above results, the study of spin excitations with pairing included, will be the natural continuation of this work. Pairing is important for a quantitative description of the orbital scissors

Table 1: Isovector energies and excitation probabilities of ^{164}Er. Deformation parameter $\delta = 0.25$, spin-orbit constant $\eta = 0.36$ MeV. Spin-spin interaction constants are: I – $K_s = 0$ MeV; II – $K_s = 92$ MeV, $q = -0.8$; III – $K_s = 200$ MeV, $q = -0.5$. Quantum numbers (including indices $\varsigma = +, -, \uparrow\downarrow, \downarrow\uparrow$) of variables responsible for the generation of the present level are shown in the first column. For example: $(1,1)^-$ – spin scissors, $(1,1)^+$ – conventional scissors.

$(\lambda,\mu)^\sigma$	E_{iv}, MeV			$B(M1)$, μ_N^2			$B(E2)$, B_W		
	I	II	III	I	II	III	I	II	III
$(1,1)^-$	1.61	2.02	2.34	3.54	5.44	7.91	0.12	0.36	0.82
$(1,1)^+$	2.18	2.45	2.76	5.33	4.48	2.98	1.02	1.23	1.26
$(0,0)^{\downarrow\uparrow}$	12.80	16.81	20.02	0.01	0.01	0.04	0.04	0.13	0.52
$(2,1)^-$	14.50	18.52	21.90	0.01	0.02	0.34	0.03	0.13	4.29
$(2,2)^{\uparrow\downarrow}$	16.18	20.61	24.56	0.02	0.23	0.03	0.18	3.09	0.44
$(2,0)^{\downarrow\uparrow}$	16.20	22.65	27.67	0	0.03	0	0	0.39	0.02
$(2,1)^+$	20.59	21.49	22.42	2.78	2.19	1.77	35.45	30.47	27.43

mode. The same is expected for the novel spin scissors mode discussed here.

References

[1] E. B. Balbutsev, I.V. Molodtsova and P. Schuck, Phys. Rev. C **88** (2013) 014306.

[2] K. Heyde, P. von Neuman-Cosel and A. Richter, Rev. Mod. Phys. **82** (2010) 2365.

[3] D. Pena Arteaga and P. Ring, *A new scissors mode on the skin of deformed neutron rich nuclei*, 2009, arXiv:0912.0908v1 [nucl-th].

[4] E. B. Balbutsev, I.V. Molodtsova and P. Schuck, Nucl. Phys. A **872** (2011) 42.

[5] M. Guttormsen, *et al*, Phys. Rev. Lett. **109** (2012) 162503.

[6] A. S. Adekola, *et al*, Phys. Rev. C **83** (2011) 034615.

[7] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer, Berlin, 1980).

[8] D.A. Varshalovich, A.N. Moskalev and V.K. Khersonskii, *Quantum Theory of Angular Momentum* (Nauka, Leningrad, 1975; World Scientific, Singapore, 1988).

[9] P. Sarriguren, E. Moya de Guerra and R. Nojarov, Phys. Rev. C **54** (1996) 690; Z. Phys. A **357** (1997) 143.

[10] V. G. Soloviev, *Theory of complex nuclei* (Pergamon Press, Oxford, 1976).