

# Calculations of the $\gamma$ -Vibrational States in Nuclei with $Z \sim 100$

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It is well known from many calculations of the structure of the  $\gamma$ -vibrational states in the well deformed axially symmetric nuclei [1] that the structure of the  $\gamma$ -phonons is mainly exhausted by a rather small number of the two-quasiparticle components. Due to this fact the energy of the  $\gamma$ -vibrational state can be strongly influenced by the presence of the low-energy two-quasiparticle state with  $K^\pi = 2^+$ . This happens if near the Fermi level are located two nearly lying single particle states having the same parities and the projections of the angular momentum on the axial symmetry axis whose sum or difference is equal to  $K=2$ . Such closely lying single particle states can be the members of the pseudospin doublet. Calculation the energies of the  $\gamma$ -vibrational states for the sequences of the isotopes of the elements with  $Z \sim 100$  and investigation the influence of the appearance of the pseudospin doublets near the Fermi level on the energies of the  $\gamma$ -vibrational states have been performed in [2].

The Quasiparticle-Phonon Model is used to calculate the energies of the  $\gamma$ -vibrational states. The Hamiltonian of the Quasiparticle-Phonon Model [1] contains the mean fields for protons and neutrons, monopole pairing and the multipole-multipole interaction, both isoscalar and isovector, acting in the particle-hole and the particle-particle channels

$$H = H_{sp} + V_{pair} + V_M^{ph} + V_M^{pp}. \quad (1)$$

Here  $H_{sp}$  is a one-body Hamiltonian,  $V_{pair}$  describes the monopole pairing,  $V_M^{ph}$  and  $V_M^{pp}$  are particle-hole separable multipole interaction and particle-particle multipole pairing interaction, respectively.

As the mean field term we have taken the Woods-Saxon potential

$$\begin{aligned} V_{sp}(\vec{r}) &= V_{WS}(\vec{r}) + V_{so}(\vec{r}), \\ V_{WS}(\vec{r}) &= -V_0 (1 + \exp[(r - R(\theta, \varphi))/a])^{-1} \\ V_{so}(\vec{r}) &= -\kappa(\vec{p} \times \sigma) \nabla V_{WS}(\vec{r}) \end{aligned} \quad (2)$$

with the parameters taken from [3]. They are presented in the Table 1. These parameters are quite close to those used in the calculations for the rare earth nuclei [3, 4]. There is some experimental information about the quadrupole deformation parameter  $\beta_2$ . The recent calculations [5] give the value

$\beta_2=0.26$  for nuclei with  $Z=96-104$  and  $N=148-156$ . The results of calculations presented in [6] demonstrate a stability of the quadrupole deformation for nuclei with  $Z=100-104$ . For these reasons we have used the same value of  $\beta_2=0.26$  for all considered nuclei.

The  $\gamma$ -vibrational states can be considered as the most collective vibrational excitations in many well deformed axially symmetric nuclei. They have been observed in many nuclei and are well understood theoretically. However, the experimental information on these excitations in nuclei with  $Z \sim 100$  is rather scarce. The  $\gamma$ -vibrational excitations have been observed only in <sup>246,248</sup>Cm, <sup>250,252</sup>Cf and <sup>254,256</sup>Fm.

The results of our calculations of the energies and the two-quasiparticle structure of the  $\gamma$ -vibrational states are presented in the Tables 2, 3 and 4 for three values of the interaction constant  $\kappa_{22}$  of the quadrupole-quadrupole forces in the particle-hole channel. The value of  $\kappa_{22} = 0.0174$  was fixed previously in the calculations for U isotopes. As it is seen from the Table 2 with this value of  $\kappa_{22}$  we obtain a good description of the known experimental data. The results for the other two values of  $\kappa_{22}$  are shown in order to get an idea of sensitivity of the energies of  $\gamma$ -phonons to variations of  $\kappa_{22}$ . The results presented in the Tables 2-4 are obtained without taking into account a mixing of the one-phonon and two-phonon states.

The results given in the Table 2 show that for all considered elements the energy of the  $\gamma$ -vibrational state takes its minimum in nuclei with the number of neutrons equal to  $N=156$ . To understand this fact let us analyze the quasiparticle structure of the  $\gamma$ -phonon. It is seen from the Tables 3 and 4 that if the number of neutrons approaches the value  $N=156$  the contribution of the two-quasiparticle component  $3/2[622] \otimes 1/2[620]$  to the norm of the  $\gamma$ -vibrational one phonon state becomes the largest one. At  $N=156$  and  $\beta_2 = 0.26$  the neutron Fermi

Table 1: Parameters of the Woods-Saxon potential used in the calculations.

A	isospin	$V_0$ , MeV	$r_0$ , fm	$a$ , fm	$\kappa$ , fm <sup>2</sup>
243	<i>n</i>	46.0	1.26	0.72	0.430
243	<i>p</i>	62.0	1.24	0.65	0.370

Table 2: The experimental and the calculated energies of the  $K^\pi = 2^+$   $\gamma$ -vibrational states. The energies are given in keV. The quadrupole interaction constant  $\kappa_{22}$  is given in fm<sup>2</sup>/MeV. This dimension of the interaction constant is determined by the use of the radial derivative of the Woods–Saxon potential as a formfactor of the multipole–multipole interaction. The experimental data are taken from [7].

Nucleus	$E(2_\gamma^+)_{exp}$	$E(2_\gamma^+)_{calc}$		
		$\kappa_{22}$		
		0.0174	0.0165	0.0150
<sup>246</sup> Cm	1124	1225	1432	1680
<sup>248</sup> Cm	1049	997	1229	1492
<sup>248</sup> Cf	–	1289	1478	1708
<sup>250</sup> Cf	1032	1079	1282	1517
<sup>252</sup> Cf	805	781	987	1207
<sup>254</sup> Cf	–	553	758	957
<sup>256</sup> Cf	–	612	834	1058
<sup>250</sup> Fm	–	1181	1354	1543
<sup>252</sup> Fm	–	1021	1225	1462
<sup>254</sup> Fm	694	735	933	1148
<sup>256</sup> Fm	682	510	713	909
<sup>258</sup> Fm	–	586	812	1042
<sup>252</sup> No	–	1261	1461	1703
<sup>254</sup> No	–	1065	1274	1520
<sup>256</sup> No	–	809	1018	1251
<sup>258</sup> No	–	531	725	914
<sup>260</sup> No	–	602	826	1052
<sup>254</sup> Rf	–	1291	1478	1626
<sup>256</sup> Rf	–	1077	1277	1497
<sup>258</sup> Rf	–	833	1049	1286
<sup>260</sup> Rf	–	539	731	917
<sup>262</sup> Rf	–	606	832	1057

level is located between the single particle states  $3/2^+[622]$  and  $1/2^+[620]$  and therefore the two-quasiparticle state consisting of these quasiparticles has the smallest energy compared to the other two-quasiparticle states. This energy is equal to 1.251 MeV. This explains why the energy of the  $\gamma$ -vibrational one phonon state has a minimum when the number of neutrons is equal to  $N=156$ . The other important neutron two-quasiparticle component  $7/2[613] \otimes 3/2[622]$  has at  $N=156$  the energy 1.343 MeV. However, our calculations have shown that the energy of this two-quasiparticle component has its minimum not at  $N=156$  but at  $N=154$ . The energies of the others neutron two-quasiparticle components are larger than 2 MeV if  $N=156$ .

To verify further the effect of the neutron two-quasiparticle component  $3/2[622] \otimes 1/2[620]$  on the neutron number dependence of the energies of the  $\gamma$ -vibrational states we have shifted artificially the energy of the neutron single particle state  $3/2[622]$  up,

i.e. decreased its binding, by 0.6 MeV and 1.2 MeV. As the result there appear two and three others neutron single particle states, respectively, between the  $1/2[620]$  and  $3/2[622]$  neutron single particle states at  $\beta_2=0.26$ . This shift of the energy of the single particle state changes the neutron number dependence of the energy of the  $\gamma$ -vibrational state in such a way that the minimum at  $N=156$  disappears and  $E(2_\gamma^+)$  decreases continuously if  $N$  increases from  $N=150$  to  $N=160$ . The absolute value of  $E(2_\gamma^+)$  is increased because of this shift of the energy of the single particle neutron state  $3/2^+[622]$  for all considered isotopes. This fact indicate on the strong neutron number dependence of the  $E(2_\gamma^+)$  and its absolute value on a relative position of the neutron single particle states  $1/2[620]$  and  $3/2[622]$ .

The single particle neutron states  $3/2^+[622]$  and  $1/2^+[620]$  are the members of the pseudospin doublet with the quantum numbers  $[\tilde{N}\tilde{n}_3\tilde{\Lambda}]=[521]$ . The connection of the pseudospin quantum numbers to the Nilsson asymptotic quantum numbers  $[Nn_z\Lambda_{1,2}]$  is the following:  $\tilde{N} = N-1$ ,  $\tilde{n}_3 = n_3$ ,  $\tilde{\Lambda} = \frac{1}{2}(\Lambda_1 + \Lambda_2)$ . Our discussion above have shown that the difference between the single particle energies of these states influences on the neutron number dependence and the absolute energy value of the  $\gamma$ -vibrational state. Small splitting of the  $1/2[620]$  and  $3/2[622]$  single particle state will mean that the pseudospin symmetry is approximately preserved. Thus, the experimental observation (or nonobservation) of the minimum of the energy of the  $\gamma$ -vibrational one phonon state when the number of neutrons is equal to  $N=156$  is important for studying manifestation of the pseudospin symmetry in very heavy exotic nuclei. The value of  $E(2_\gamma^+)$  at  $N=156$  gives an information on the splitting of the pseudospin doublet  $[\tilde{N}\tilde{n}_3\tilde{\Lambda}]=[521]$ .

To get the feeling of the effect of the mixing of the one-phonon and the two-phonon states we have performed the calculations for the Cf isotopes with and without the mixing. The results of the calculations have shown that the mixing of the one-phonon and the two-phonon states decreases the energies of the  $E(2_\gamma^+)$  states approximately by 50 keV. However, this mixing does not influence on the neutron number dependence of the  $E(2_\gamma^+)$  and keeps the minimum at  $N=156$ . Approximately 98% of the norm of the state vectors corresponding to the  $\gamma$ -vibrational states obtained in the calculations including the mixing effect are provided with the one-phonon component.

In conclusion, basing on the Quasiparticle–Phonon Model we have calculated the energies and the two-quasiparticle structure of the  $\gamma$ -vibrational states. The results of calculations show that in the isotopes of Cm, Cf, Fm, No, and Rf the energies of the one-phonon  $\gamma$ -vibrational states have a minimum if the number of neutrons is equal to  $N=156$ .

Table 3: The quasiparticle structure of the calculated  $K^\pi = 2^+$   $\gamma$ -vibrational states of  $^{246,248}\text{Cm}$ ,  $^{248-256}\text{Cf}$ , and  $^{250-258}\text{Fm}$ . The quantum numbers of the most important two-quasiparticle components and their contribution (in %) to the norm of the  $\gamma$ -phonon are shown. The quadrupole interaction constant  $\kappa_{22}$  is given in  $\text{fm}^2/\text{MeV}$ .

Nucleus	Structure (in %)					
	$\kappa_{22}=0.0174$		$\kappa_{22}=0.0165$		$\kappa_{22}=0.0150$	
$^{246}\text{Cm}$	nn 7/2[624] $\otimes$ 3/2[622]	23	nn 7/2[624] $\otimes$ 3/2[622]	27	nn 7/2[624] $\otimes$ 3/2[622]	35
	nn 5/2[622] $\otimes$ 1/2[620]	20	nn 5/2[622] $\otimes$ 1/2[620]	22	nn 5/2[622] $\otimes$ 1/2[620]	25
	pp 3/2[651] $\otimes$ 1/2[660]	7	pp 3/2[651] $\otimes$ 1/2[660]	7	pp 3/2[651] $\otimes$ 1/2[660]	6
$^{246}\text{Cm}$	nn 7/2[624] $\otimes$ 3/2[622]	30	nn 7/2[624] $\otimes$ 3/2[622]	36	nn 7/2[624] $\otimes$ 3/2[622]	51
	nn 3/2[622] $\otimes$ 1/2[620]	12	nn 3/2[622] $\otimes$ 1/2[620]	12	nn 3/2[622] $\otimes$ 1/2[620]	12
	nn 5/2[622] $\otimes$ 1/2[620]	12	nn 5/2[622] $\otimes$ 1/2[620]	12	nn 5/2[622] $\otimes$ 1/2[620]	10
	pp 3/2[651] $\otimes$ 1/2[660]	5				
$^{248}\text{Cf}$	nn 7/2[624] $\otimes$ 3/2[622]	24	nn 7/2[624] $\otimes$ 3/2[622]	29	nn 7/2[624] $\otimes$ 3/2[622]	37
	nn 5/2[622] $\otimes$ 1/2[620]	20	nn 5/2[622] $\otimes$ 1/2[620]	23	nn 5/2[622] $\otimes$ 1/2[620]	26
	nn 3/2[622] $\otimes$ 1/2[620]	6	nn 3/2[622] $\otimes$ 1/2[620]	6	nn 3/2[622] $\otimes$ 1/2[620]	5
	pp 3/2[521] $\otimes$ 1/2[521]	8	pp 3/2[521] $\otimes$ 1/2[521]	8	pp 3/2[521] $\otimes$ 1/2[521]	8
$^{250}\text{Cf}$	nn 7/2[624] $\otimes$ 3/2[622]	31	nn 7/2[624] $\otimes$ 3/2[622]	38	nn 7/2[624] $\otimes$ 3/2[622]	52
	nn 3/2[622] $\otimes$ 1/2[620]	12	nn 3/2[622] $\otimes$ 1/2[620]	13	nn 3/2[622] $\otimes$ 1/2[620]	11
	nn 5/2[622] $\otimes$ 1/2[620]	11	nn 5/2[622] $\otimes$ 1/2[620]	11	nn 5/2[622] $\otimes$ 1/2[620]	10
	pp 3/2[521] $\otimes$ 1/2[521]	7	pp 3/2[521] $\otimes$ 1/2[521]	7	pp 3/2[521] $\otimes$ 1/2[521]	6
$^{252}\text{Cf}$	nn 3/2[622] $\otimes$ 1/2[620]	39	nn 3/2[622] $\otimes$ 1/2[620]	48	nn 3/2[622] $\otimes$ 1/2[620]	63
	nn 7/2[624] $\otimes$ 3/2[622]	16	nn 7/2[624] $\otimes$ 3/2[622]	16	nn 7/2[624] $\otimes$ 3/2[622]	13
	nn 7/2[613] $\otimes$ 3/2[611]	6	nn 7/2[613] $\otimes$ 3/2[611]	5		
	nn 5/2[622] $\otimes$ 1/2[620]	6				
$^{254}\text{Cf}$	nn 3/2[622] $\otimes$ 1/2[620]	55	nn 3/2[622] $\otimes$ 1/2[620]	66	nn 3/2[622] $\otimes$ 1/2[620]	81
	nn 7/2[624] $\otimes$ 3/2[622]	8	nn 7/2[624] $\otimes$ 3/2[622]	6		
	nn 7/2[613] $\otimes$ 3/2[611]	6				
$^{256}\text{Cf}$	nn 3/2[622] $\otimes$ 1/2[620]	47	nn 3/2[622] $\otimes$ 1/2[620]	59	nn 3/2[622] $\otimes$ 1/2[620]	76
	nn 7/2[613] $\otimes$ 3/2[611]	7	nn 7/2[613] $\otimes$ 3/2[611]	6		
	nn 9/2[615] $\otimes$ 5/2[613]	7	nn 9/2[615] $\otimes$ 5/2[613]	6		
$^{250}\text{Fm}$	nn 7/2[624] $\otimes$ 3/2[622]	15	nn 7/2[624] $\otimes$ 3/2[622]	15	nn 7/2[624] $\otimes$ 3/2[622]	10
	nn 5/2[622] $\otimes$ 1/2[620]	14	nn 5/2[622] $\otimes$ 1/2[620]	14	nn 5/2[622] $\otimes$ 1/2[620]	10
	pp 3/2[521] $\otimes$ 1/2[521]	33	pp 3/2[521] $\otimes$ 1/2[521]	40	pp 3/2[521] $\otimes$ 1/2[521]	41
					pp 7/2[514] $\otimes$ 3/2[521]	23
$^{252}\text{Fm}$	nn 7/2[624] $\otimes$ 3/2[622]	23	nn 7/2[624] $\otimes$ 3/2[622]	26	nn 7/2[624] $\otimes$ 3/2[622]	31
	nn 3/2[622] $\otimes$ 1/2[620]	10	nn 3/2[622] $\otimes$ 1/2[620]	10	nn 3/2[622] $\otimes$ 1/2[620]	9
	nn 5/2[622] $\otimes$ 1/2[620]	10	nn 5/2[622] $\otimes$ 1/2[620]	10	nn 5/2[622] $\otimes$ 1/2[620]	8
	pp 3/2[521] $\otimes$ 1/2[521]	22	pp 3/2[521] $\otimes$ 1/2[521]	25	pp 3/2[521] $\otimes$ 1/2[521]	31
$^{254}\text{Fm}$	nn 3/2[622] $\otimes$ 1/2[620]	37	nn 3/2[622] $\otimes$ 1/2[620]	45	nn 3/2[622] $\otimes$ 1/2[620]	58
	nn 7/2[624] $\otimes$ 3/2[622]	15	nn 7/2[624] $\otimes$ 3/2[622]	15	nn 7/2[624] $\otimes$ 3/2[622]	12
	nn 7/2[613] $\otimes$ 3/2[611]	6				
	pp 3/2[521] $\otimes$ 1/2[521]	11	pp 3/2[521] $\otimes$ 1/2[521]	11	pp 3/2[521] $\otimes$ 1/2[521]	9
$^{256}\text{Fm}$	nn 3/2[622] $\otimes$ 1/2[620]	54	nn 3/2[622] $\otimes$ 1/2[620]	65	nn 3/2[622] $\otimes$ 1/2[620]	80
	nn 7/2[624] $\otimes$ 3/2[622]	8	nn 7/2[624] $\otimes$ 3/2[622]	6		
	nn 7/2[613] $\otimes$ 3/2[611]	6				
	pp 3/2[521] $\otimes$ 1/2[521]	6	pp 3/2[521] $\otimes$ 1/2[521]	5		
$^{258}\text{Fm}$	nn 3/2[622] $\otimes$ 1/2[620]	44	nn 3/2[622] $\otimes$ 1/2[620]	55	nn 3/2[622] $\otimes$ 1/2[620]	73
	nn 7/2[613] $\otimes$ 3/2[611]	7	nn 7/2[613] $\otimes$ 3/2[611]	6		
	nn 9/2[615] $\otimes$ 5/2[613]	7	nn 9/2[615] $\otimes$ 5/2[613]	6		
	pp 3/2[521] $\otimes$ 1/2[521]	8	pp 3/2[521] $\otimes$ 1/2[521]	7	pp 3/2[521] $\otimes$ 1/2[521]	5

Table 4: The quasiparticle structure of the calculated  $K^\pi = 2^+$   $\gamma$ -vibrational states of  $^{252-260}\text{No}$  and  $^{254-262}\text{Rf}$ . The quantum numbers of the most important two-quasiparticle components and their contribution (in %) to the norm of the  $\gamma$ -phonon are shown. The quadrupole interaction constant  $\kappa_{22}$  is given in  $\text{fm}^2/\text{MeV}$ .

Nucleus	Structure (in %)					
	$\kappa_{22}=0.0174$		$\kappa_{22}=0.0165$		$\kappa_{22}=0.0150$	
$^{252}\text{No}$	nn 5/2[622] $\otimes$ 1/2[620]	17	nn 5/2[622] $\otimes$ 1/2[620]	18	nn 5/2[622] $\otimes$ 1/2[620]	19
	nn 7/2[624] $\otimes$ 3/2[622]	17	nn 7/2[624] $\otimes$ 3/2[622]	18	nn 7/2[624] $\otimes$ 3/2[622]	19
	nn 3/2[622] $\otimes$ 1/2[620]	5	nn 3/2[622] $\otimes$ 1/2[620]	5		
	pp 3/2[521] $\otimes$ 1/2[521]	21	pp 3/2[521] $\otimes$ 1/2[521]	25	pp 3/2[521] $\otimes$ 1/2[521]	33
$^{254}\text{No}$	nn 7/2[624] $\otimes$ 3/2[622]	25	nn 7/2[624] $\otimes$ 3/2[622]	29	nn 7/2[624] $\otimes$ 3/2[622]	37
	nn 3/2[622] $\otimes$ 1/2[620]	11	nn 3/2[622] $\otimes$ 1/2[620]	11	nn 3/2[622] $\otimes$ 1/2[620]	11
	nn 5/2[622] $\otimes$ 1/2[620]	11	nn 5/2[622] $\otimes$ 1/2[620]	10	nn 5/2[622] $\otimes$ 1/2[620]	9
	pp 3/2[521] $\otimes$ 1/2[521]	16	pp 3/2[521] $\otimes$ 1/2[521]	18	pp 3/2[521] $\otimes$ 1/2[521]	21
$^{256}\text{No}$	nn 3/2[622] $\otimes$ 1/2[620]	34	nn 3/2[622] $\otimes$ 1/2[620]	41	nn 3/2[622] $\otimes$ 1/2[620]	55
	nn 7/2[624] $\otimes$ 3/2[622]	16	nn 7/2[624] $\otimes$ 3/2[622]	16	nn 7/2[624] $\otimes$ 3/2[622]	14
	nn 7/2[613] $\otimes$ 3/2[611]	6	nn 7/2[613] $\otimes$ 3/2[611]	5		
	nn 5/2[622] $\otimes$ 1/2[620]	6	nn 5/2[622] $\otimes$ 1/2[620]	5		
	pp 3/2[521] $\otimes$ 1/2[521]	10	pp 3/2[521] $\otimes$ 1/2[521]	9	pp 3/2[521] $\otimes$ 1/2[521]	8
$^{258}\text{No}$	nn 3/2[622] $\otimes$ 1/2[620]	56	nn 3/2[622] $\otimes$ 1/2[620]	66	nn 3/2[622] $\otimes$ 1/2[620]	81
	nn 7/2[624] $\otimes$ 3/2[622]	7	nn 7/2[624] $\otimes$ 3/2[622]	6		
	nn 7/2[613] $\otimes$ 3/2[611]	6				
	pp 3/2[521] $\otimes$ 1/2[521]	5				
$^{260}\text{No}$	nn 3/2[622] $\otimes$ 1/2[620]	46	nn 3/2[622] $\otimes$ 1/2[620]	57	nn 3/2[622] $\otimes$ 1/2[620]	75
	nn 7/2[613] $\otimes$ 3/2[611]	7	nn 7/2[613] $\otimes$ 3/2[611]	6		
	nn 9/2[615] $\otimes$ 5/2[613]	7	nn 9/2[615] $\otimes$ 5/2[613]	6		
	pp 3/2[521] $\otimes$ 1/2[521]	5				
$^{254}\text{Rf}$	nn 7/2[624] $\otimes$ 3/2[622]	21	nn 7/2[624] $\otimes$ 3/2[622]	21	nn 7/2[624] $\otimes$ 3/2[622]	82
	nn 5/2[622] $\otimes$ 1/2[620]	19	nn 5/2[622] $\otimes$ 1/2[620]	19		
	nn 3/2[622] $\otimes$ 1/2[620]	6				
	pp 3/2[521] $\otimes$ 1/2[521]	8	pp 3/2[521] $\otimes$ 1/2[521]	17		
	pp 5/2[512] $\otimes$ 1/2[521]	8	pp 5/2[52] $\otimes$ 1/2[521]	8		
$^{256}\text{Rf}$	nn 7/2[624] $\otimes$ 3/2[622]	31	nn 7/2[624] $\otimes$ 3/2[622]	36	nn 7/2[624] $\otimes$ 3/2[622]	39
	nn 3/2[622] $\otimes$ 1/2[620]	12	nn 3/2[622] $\otimes$ 1/2[620]	12	nn 3/2[622] $\otimes$ 1/2[620]	9
	nn 5/2[622] $\otimes$ 1/2[620]	12	nn 5/2[622] $\otimes$ 1/2[620]	11	nn 5/2[622] $\otimes$ 1/2[620]	8
	pp 3/2[521] $\otimes$ 1/2[521]	6	pp 3/2[521] $\otimes$ 1/2[521]	6	pp 3/2[521] $\otimes$ 1/2[521]	24
$^{258}\text{Rf}$	nn 3/2[622] $\otimes$ 1/2[620]	35	nn 3/2[622] $\otimes$ 1/2[620]	43	nn 3/2[622] $\otimes$ 1/2[620]	57
	nn 7/2[624] $\otimes$ 3/2[622]	16	nn 7/2[624] $\otimes$ 3/2[622]	17	nn 7/2[624] $\otimes$ 3/2[622]	15
	nn 7/2[613] $\otimes$ 3/2[611]	6	nn 7/2[613] $\otimes$ 3/2[611]	5		
	nn 5/2[622] $\otimes$ 1/2[620]	6	nn 5/2[622] $\otimes$ 1/2[620]	5		
$^{260}\text{Rf}$	nn 3/2[622] $\otimes$ 1/2[620]	57	nn 3/2[622] $\otimes$ 1/2[620]	68	nn 3/2[622] $\otimes$ 1/2[620]	82
	nn 7/2[624] $\otimes$ 3/2[622]	8	nn 7/2[624] $\otimes$ 3/2[622]	6		
	nn 7/2[613] $\otimes$ 3/2[611]	6				
$^{262}\text{Rf}$	nn 3/2[622] $\otimes$ 1/2[620]	47	nn 3/2[622] $\otimes$ 1/2[620]	59	nn 3/2[622] $\otimes$ 1/2[620]	77
	nn 7/2[613] $\otimes$ 3/2[611]	7	nn 7/2[613] $\otimes$ 3/2[611]	6		
	nn 9/2[615] $\otimes$ 5/2[613]	7	nn 9/2[615] $\otimes$ 5/2[613]	6		
	nn 7/2[624] $\otimes$ 3/2[622]	6				

In addition, a contribution of the two-quasiparticle component  $3/2^+[622] \otimes 1/2^+[620]$  to the norm of the  $\gamma$ -vibrational one-phonon state becomes the largest one at  $N=156$ . The single particle states  $3/2^+[622]$  and  $1/2^+[620]$  are the members of the pseudo-spin doublet. Thus, the experimental information on the energies of the  $\gamma$ -vibrational states in nuclei with  $Z \sim 100$  can be used to determine a splitting of the  $[521]$  pseudospin doublet.

### References

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