Cosmological Models with Spinor Fields

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According to the inflationary scenario, it is believed that a scalar field known as inflaton is responsible for a rapid accelerated expansion of the early Universe. For the inflationary mechanism to work there must exist a weakly coupled scalar field which is initially at a false vacuum which leads to the inflation until the right vacuum value is obtained. This inflationary model solves the problem of flatness, isotropy of microwave background radiation and unwanted relics. Contrary to the prediction of the standard cosmological models, recent observations showed an accelerated mode of expansion of the present day Universe. Though the existence of an inflationary scenario is not of much concern, the question of where the scalar field comes from and why it undergoes such a peculiar phase transition from false to right vacuum still remains unanswered. This leads cosmologists to reconsider alternative possibilities. As one of the way out many specialists considered spinor field as an alternative source. Being related to almost all stable elementary particles such as proton, electron and neutrino, spinor field, especially Dirac spin-1/2 play a principal role at the microlevel. However, in cosmology, the role of spinor field was generally considered to be restricted. Only recently, after some remarkable works by different authors showing the important role that spinor fields play on the evolution of the Universe, the situation began to change. This change of attitude is directly related to some fundamental questions of modern cosmology: (i) problem of initial singularity; (ii) problem of isotropization and (iii) late time acceleration of the Universe. Moreover it was shown that the spinor field can successfully describe the describes perfect fluid from phantom to ekpyrotic matter.

In [1] exploiting the spinor description of perfect fluid and dark energy evolution of the Universe within the scope of Bianchi type I, III, V, VI_0, VI and isotropic FRW models was studied. It was shown that due to the non-zero non-diagonal components of the Einstein tensor the initial anisotropy in case of Bianchi type III, V, VI_0 and VI models does not die away, while anisotropic BI model becomes isotropic in course of time.

Further study in [2, 3, 4] shows that the nonlinear spinor field with the exception of FRW model admits non-zero non-diagonal components of energy momentum tensor. This fact imposes severe restriction either on the components of spinor field or on the metric functions thus giving rise two alternative possibilities.

To demonstrate it a Bianchi type-I spacetime filled with spinor field was studied. The BI spacetime given by the line element

$$ds^2 = dt^2 - a_1^2\,dx^2 - a_2^2\,dy^2 - a_3^2\,dz^2, \qquad (1)$$

with a_1, a_2 and a_3 being the functions of time only has only nonzero diagonal components of the Einstein tensor

$$\begin{aligned}
G_1^1 &= -\frac{\ddot a_2}{a_2} - \frac{\ddot a_3}{a_3} - \frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3}, & (2)\\
G_2^2 &= -\frac{\ddot a_3}{a_3} - \frac{\ddot a_1}{a_1} - \frac{\dot a_3}{a_3}\frac{\dot a_1}{a_1}, & (3)\\
G_3^3 &= -\frac{\ddot a_1}{a_1} - \frac{\ddot a_2}{a_2} - \frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2}, & (4)\\
G_1^1 &= -\frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2} - \frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3} - \frac{\dot a_3}{a_3}\frac{\dot a_1}{a_1}. & (5)
\end{aligned}$$

On the other hand a nonlinear spinor field given by

$$L_{\rm sp} = \frac{\imath}{2}\Big[\bar\psi\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma^\mu\psi\Big] - m_{\rm sp}\bar\psi\psi - F, \quad (6)$$

where the nonlinear term F describes the self-interaction of a spinor field and $\psi = \psi(t)$ in case of BI metric possesses the following components of energy momentum tensor

$$\begin{aligned}
T_0^0 &= m_{\rm sp}S + F(S), & (7)\\
T_1^1 &= T_2^2 = T_3^3 = F(S) - SF_S, & (8)\\
T_2^1 &= -\frac{\imath}{4}\frac{a_2}{a_1}\left(\frac{\dot a_1}{a_1} - \frac{\dot a_2}{a_2}\right)\bar\psi\bar\gamma^1\bar\gamma^2\bar\gamma^0\psi, & (9)\\
T_3^2 &= -\frac{\imath}{4}\frac{a_3}{a_2}\left(\frac{\dot a_2}{a_2} - \frac{\dot a_3}{a_3}\right)\bar\psi\bar\gamma^2\bar\gamma^3\bar\gamma^0\psi, & (10)\\
T_3^1 &= -\frac{\imath}{4}\frac{a_3}{a_1}\left(\frac{\dot a_3}{a_3} - \frac{\dot a_1}{a_1}\right)\bar\psi\bar\gamma^3\bar\gamma^1\bar\gamma^0\psi. & (11)
\end{aligned}$$

On account of it Einstein field equations take the form

$$\begin{aligned}
\frac{\ddot a_2}{a_2} + \frac{\ddot a_3}{a_3} + \frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3} &= \kappa\big(F(S) - SF_S\big), & (12)\\
\frac{\ddot a_3}{a_3} + \frac{\ddot a_1}{a_1} + \frac{\dot a_3}{a_3}\frac{\dot a_1}{a_1} &= \kappa\big(F(S) - SF_S\big), & (13)\\
\frac{\ddot a_1}{a_1} + \frac{\ddot a_2}{a_2} + \frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2} &= \kappa\big(F(S) - SF_S\big), & (14)\\
\frac{\dot a_1}{a_1}\frac{\dot a_2}{a_2} + \frac{\dot a_2}{a_2}\frac{\dot a_3}{a_3} + \frac{\dot a_3}{a_3}\frac{\dot a_1}{a_1} &= \kappa\big(m_{\rm sp}S + F(S)\big), & (15)
\end{aligned}$$

together with the additional constrains

$$\left(\frac{\dot a_1}{a_1} - \frac{\dot a_2}{a_2}\right)\bar\psi\bar\gamma^1\bar\gamma^2\bar\gamma^0\psi = 0, \tag{16}$$

$$\left(\frac{\dot a_2}{a_2} - \frac{\dot a_3}{a_3}\right)\bar\psi\bar\gamma^2\bar\gamma^3\bar\gamma^0\psi = 0, \tag{17}$$

$$\left(\frac{\dot a_3}{a_3} - \frac{\dot a_1}{a_1}\right)\bar\psi\bar\gamma^3\bar\gamma^1\bar\gamma^0\psi = 0. \tag{18}$$

Let us first consider the case when the non-diagonal components of the energy momentum tensor impose restrictions on the spinor field. In this case from **??** we obtain

$$\bar\psi\bar\gamma^1\bar\gamma^2\bar\gamma^0\psi = \bar\psi\bar\gamma^3\bar\gamma^1\bar\gamma^0\psi = \bar\psi\bar\gamma^2\bar\gamma^3\bar\gamma^0\psi = 0. \tag{19}$$

In case if the metric functions are allowed to be unaltered, the restrictions are imposed on the components of the spinor field. In this case the initially anisotropic spacetime becomes isotropic asymptotically.

As far as second possibility is concerned in this case from **??** follows

$$\frac{\dot a_1}{a_1} - \frac{\dot a_2}{a_2} = \frac{\dot a_2}{a_2} - \frac{\dot a_3}{a_3} = \frac{\dot a_3}{a_3} - \frac{\dot a_1}{a_1} = 0, \tag{20}$$

which can be rewritten as

$$\frac{\dot a_1}{a_1} = \frac{\dot a_2}{a_2} = \frac{\dot a_3}{a_3} \equiv \frac{\dot a}{a}. \tag{21}$$

As a result the Einstein equation now can be given by

$$2\frac{\ddot a}{a} + \frac{\dot a^2}{a^2} = \kappa T_1^1, \tag{22}$$

$$3\frac{\dot a^2}{a^2} = \kappa T_0^0. \tag{23}$$

which in fact describes a FRW spacetime. Thus we see that in the at hand case the spacetime becomes isotropic from the very beginning.

References

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