On Possibility of Pion Bose-Einstein Condensation in Compressed and Cooled Few-Nucleon Systems

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Abstract. Arguments for registering at JINR synchrophasotron a quantum phenomenon similar to theoretically predicted, but still unobserved experimentally, Hawking effect are given. Here an analogue of the Hawking radiation is a generalized coherent state corresponding to the dynamical symmetry group $SU(1,1)$, which describes Bose-Einstein condensate of pions in a strong external field.

Introduction. Recently a proposal of QCD investigation at high density and low temperature, complementary to the high-energy heavy nuclear collisions, was suggested [1, 2]. The proposal is based on the fact that a large number of nucleons in the interaction region is not necessary for the phase transition to occur, and only a change of the vacuum state should be initiated in a nuclear system, which may contain a small number of nucleons. Here I report on a possible observation of a phenomenon of this type at JINR, relying on papers [3, 4, 5, 6, 7].

In [4, 5] quasi-resonant peaks in two-proton effective mass distribution from reactions np→pp$\pi^- m\pi^0$ and np→ pp $\pi^+\pi^-\pi^- m\pi^0$, $m = 0, 1$ were observed. Recently it has been shown [6, 7] that dibaryons with nearly the same masses were detected in np system from reaction D+D→X+D in a more earlier paper by A.M. Baldin et al. [3]. This fact increases significantly the reliability of existence of the dibaryons like those and worths special studying.

With the assumption that some of dibaryons were unrecognized in the experiments [4, 5], it is possible to approximate the mass spectrum within rather small, at 1 – 2 MeV/c^2 level, experimental errors by the formula,

$$M_n = M_{NN} + 10.08\, n, \tag{1}$$

where $n = 0, 1, 2, ..., 40$, all values are defined in MeV, M_{NN} is equal to the value of mass of two protons [7]. The relation (1) can also explain the data on dibaryon production in n-p system [3] if to change M_{NN} with the deuteron value of mass.

Equidistance of the spectrum gives a hint to associate it with some kind of oscillator and to consider a picture of quarks coupled by gluon strings [8]. However, careful study has shown that a possible answer is far from it. Consideration of the quantum oscillator's wave functions based on the value of constituent quark mass and the distance of 10.08 MeV between levels revealed that such an oscillator should have out-of-tolerance dimensions. For example, even the ground state of such an oscillator has a size of about 10 Fm, and the state $\psi_{20}(x)$, lying only in the middle of the spectrum, has an enormous length of about 50 Fm. Therefore, another idea should be employed.

Actually, it was difficult to find an explanation better than to associate the spectrum with the production of pion pairs, strongly bound to compressed nucleon matter by a deep potential $-U_0$. The parity conservation requires pions to be produced in pairs (see below). Therefore, a value of energy of a single pion

$$E = \sqrt{p^2 + m^2 - U_0} \tag{2}$$

should be equal to 5.04 MeV $\equiv \mathrm{E}_\pi$.

The dynamical Casimir effect. A meson field in a rectangular potential well, $\varphi(\vec{r}, t) = e^{-iEt}\varphi_E(\vec{r})$, is described by the Klein - Gordon - Fock (KGF) steady-state equation,

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\varphi_E(r)}{dr} + (E^2 - m^2 + U_0))\varphi_E(r) = 0,$$

which has a solution $\varphi_E(r) = A\sin pr/r$ inside the well, and $\varphi_E(r) = Be^{-qr}/r$, $q = \sqrt{m^2 - E^2}$ outside it. The requirement of continuity of the logarithmic derivative at the edge of the well, $r = a$, leads to a transcendental equation

$$p\ \mathrm{ctg}(pa) = \sqrt{m^2 - E^2}, \tag{3}$$

which is suitable for an estimation of relevant physical values in the interaction region. Spatial dimensions, corresponding to a given value of momentum transfer, is [9]

$$a = \langle r^2 \rangle^{1/2} \approx \sqrt{6}/|\vec{q}| = 0.68\ \mathrm{Fm}, \qquad |\vec{q}|^2 = -t.$$

Solving eq. (3) with this value of a, one obtains $p \approx$ 0.53 GeV, and using (2), one finds $\sqrt{U_0} \approx 0.55$ GeV.

Touching dynamics of the bound pion production, we suggest that it is induced by a change of a position of walls forming the potential well, in close analogy with emission of electromagnetic waves due to a motion of resonators walls. This movement is capable to give energy to the virtual pions surrounding nucleons and turn them into real particles, the bound pions. Such a mechanism is known as the dynamical Casimir effect, firstly described in [10]. It is closely connected with the Hawking radiation phenomenon and the Fulling-Unruh effect [11]. The appeal of this model is it predicts the meson field with the vacuum quantum numbers, since the mesons are produced from the vacuum state due to the strong interaction, conserving all of them. Because of this, the pion field may be present at the ground state of deuteron, as it follows from the experimental data[3], without breaking the deuteron quantum numbers. As far as the vacuum state has positive parity and the intrinsic parity of pion is negative, only even number of pions may be created in the process. Similarly, isospin conservation leads to a conclusion that pions may be produced in pairs with $I = 0$, i.e. in the following vector of state:

$$\Psi_{2\pi} = \frac{1}{\sqrt{3}}(\pi_a^+\pi_b^- + \pi_a^-\pi_b^+ - \pi_a^0\pi_b^0).$$

A picture of the pion production may be depicted as follows. At some instant t_1 a potential well capable to hold a bound pion energy level of a value ε is formed. Then, rather quickly, the energy level $\mathrm{E}_\pi > \varepsilon$ is developed due to a shrinkage of the potential well in the nucleon collision process. After that at moment t_2, when nucleons is moving away, the energy level returns to the value ε, and afterwards it changes again to the Yukawa vacuum, corresponding $E = 0$ and $q = m$. From mathematical viewpoint, creation of bound pions in this framework is totally equivalent to the parametric excitation of the quantum oscillator, which appears after the quantization of the field.

Pion Bose-Einstein condensate. The time dependent KGF equation,

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} + m^2 - U_0\right]\psi(r,t) = 0, \qquad (4)$$

with the evolving boundary conditions gives the wave function inside the well,

$$\varphi(r,t) = \chi(t)\sin pr/r,$$

where $\chi(t)$ describes an increasing amplitude of the field, which manifests itself in the pion production. It obeys the equation

$$\frac{\partial^2\chi(t)}{\partial t^2} + (p^2 + m^2 - U_0)\chi(t) = 0, \qquad (5)$$

which has the same form as one for a classical oscillator with the varying frequency $\omega(t) = E(t)$. Therefore, it is possible to introduce the oscillator Hamiltonian

$$H = \frac{1}{2}\left(\pi_\omega^2 + \omega^2(t)\chi_\omega^2\right) = \omega(t)\left(a_\omega^+(t)a_\omega(t) + \frac{1}{2}\right), \qquad (6)$$

and draw eq. (5) in the Hamiltonian formalism framework:

$$\frac{\partial H}{\partial \pi_\omega} = \dot{\chi}_\omega, \qquad -\frac{\partial H}{\partial \chi_\omega} = \dot{\pi}_\omega,$$

where

$$\chi_\omega = \frac{a_\omega + a_\omega^+}{\sqrt{2\omega}}, \qquad \pi_\omega = \frac{a_\omega - a_\omega^+}{\sqrt{2\omega}}.$$

The quantization may be performed by analogy with the similar procedure for a quantum field in the box via replacing functions $a_\omega(t)$ and $a_\omega^+(t)$ by the corresponding operators. The only non-essential difference is that now the field does not vanish at the boundary, but terminates in an exponentially decaying tail outside the potential well. Fields of this type are met in solid-state physics [12]. Thus, the quantized field in the Heisenberg picture is written as

$$\hat{\varphi}(r,t) = \hat{\chi}_\omega(t)\sin pr/r = \left(\frac{\hat{a}_\omega^+(t) + \hat{a}_\omega(t)}{\sqrt{2\omega_1}}\right)\sin pr/r,$$

for any t in the range of the pion production, $t_1 \le t \le t_2$. Here $\omega_1 = \omega(t_1) = \varepsilon$. The time evolution of the field may be expressed in an equivalent form, using Bogoliubov's canonical transformation (BCT):

$$\begin{pmatrix} \hat{a}(\Delta t) \\ \hat{a}^+(\Delta t) \end{pmatrix} = \overbrace{\begin{pmatrix} u(\Delta t) & v(\Delta t) \\ u^*(\Delta t) & v^*(\Delta t) \end{pmatrix}}^{S(\Delta t)} \begin{pmatrix} \hat{a}_S \\ \hat{a}_S^+ \end{pmatrix}, \qquad (7)$$

where $\hat{a}_S$, $\hat{a}_S^+$ are the annihilation and production operators in the Schrödinger representation, $u(\Delta t)$ and $v(\Delta t)$ are usual (non-operator) functions. It is obvious that matrices $S(\Delta t)$ generate a group under multiplication,

$$S(\Delta t) \equiv S(\Delta t_1 + ... + \Delta t_n) = S(\Delta t_n)...S(\Delta t_1).$$

The commutation relation requirement $[\hat{a}(t), \hat{a}^+(t)] = 1$ leads to a constraint

$$|u(t)|^2 - |v(t)|^2 = 1, \qquad (8)$$

which means that the group of dynamical symmetry is $SU(1,1)$.

Now we turn to the Schrödinger picture and define the group action in the space of state vectors, rather than in a space of the parameters describing evolution of operators. Lie algebra of $SU(1,1)$ is defined by the commutation relations

$$\left[\hat{K}_1, \hat{K}_2\right] = -i\hat{K}_0, \quad \left[\hat{K}_2, \hat{K}_0\right] = i\hat{K}_1,$$

$$\left[\hat{K}_0, \hat{K}_1\right] = i\hat{K}_2,$$

or, after introducing

$$\hat{K}_\pm = \pm i(\hat{K}_1 \pm i\hat{K}_2),$$

by

$$\left[\hat{K}_0, \hat{K}_\pm\right] = \pm\hat{K}_\pm, \qquad \left[\hat{K}_-, \hat{K}_+\right] = 2\hat{K}_0.$$

One can express elements of the $SU(1,1)$ group through its generators:

$$\hat{S}(dt) = e^{(\beta\hat{K}_+ - \beta^*\hat{K}_- - i\gamma\hat{K}_0)dt}.$$

But in the case of the Hamiltonian evolution

$$\hat{S}(dt) = e^{-i\hat{H}dt},$$

so that it is possible to rewrite Hamiltonian (6) in the form

$$\hat{H} = i(\beta\hat{K}_+ - \beta^*\hat{K}_- - i\gamma\hat{K}_0).$$

Corresponding expressions for $\hat{K}_+$, $\hat{K}_-$ and $\hat{K}_0$ are

$$\hat{K}_+ = \frac{(\hat{a}^+)^2}{2}, \quad \hat{K}_- = \frac{\hat{a}^2}{2}, \quad \hat{K}_0 = \frac{\hat{a}\hat{a}^+ + \hat{a}^+\hat{a}}{4}$$

for $\pi^0\pi^0$ and

$$\hat{K}_+ = \hat{a}_+^+\hat{a}_-^+, \quad \hat{K}_- = \hat{a}_+\hat{a}_-,$$

$$\hat{K}_0 = \frac{1}{2}(\hat{a}_+^+\hat{a}_+ + \hat{a}_-^+\hat{a}_- + 1)$$

for $\pi^+\pi^-$. In fact, the operators $\hat{K}_0$ do not lead to a change of a particle number and it is possible to omit them, at least for particle number distribution calculations. Thus, the evolution operator may be defined as an element of the $SU(1,1)$ group of a kind $\hat{S}(t) = \exp(\xi\hat{K}_+ - \xi^*\hat{K}_-)$. Therefore, the state of system at moment t is estimated as

$$|\psi_t\rangle = \exp(\xi\hat{K}_+ - \xi^*\hat{K}_-)\,|0\rangle\,. \tag{9}$$

It is possible to notice a similarity of this state to the Glauber coherent state [13]

$$|\psi_G\rangle = e^{\alpha a^+ - \alpha^* a}\,|0\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\,|n\rangle\,,$$

which leads to the Poisson distribution for the probability to find n particles in the $|\psi_G\rangle$ state,

$$w_n = |\langle n \mid \psi_G\rangle|^2 = e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!},\ \langle n\rangle = |\alpha|^2\,.$$

Similarly, the state $|\psi_t\rangle$ reads[14]

$$|\psi_t\rangle = (1-|\eta|^2)^k\sum_{m=0}^{\infty}\left(\frac{\Gamma(m+2k)}{m!\Gamma(2k)}\right)^{1/2}\eta^m\,|k, k+m\rangle\,.$$

Here k describes a representations of $SU(1,1)$, $k = 1/4$ for $\pi^0\pi^0$ and $k = \frac{1}{2}$ for $\pi^+\pi^-$, m is a number of pion pairs created, $\eta = \sqrt{\rho}e^{i\varphi}$. A value of ρ may be expressed through the coefficients $u(t_2)$ and $v(t_2)$ of BCT at the end of the pion production, $\rho = |v|^2 / |u|^2$, and $e^{i\varphi}$ is a phase factor, unessential here. The probability to find $n = 2m$ particles in the state is equal to

$$w_n = |\langle n \mid \psi_t\rangle|^2 = \sqrt{1-\rho}\frac{n!}{2^n\left[(n/2)!\right]^2}\rho^{n/2}, \tag{10}$$

for $\pi^0\pi^0$ system. For $\pi^+\pi^-$, it is

$$w_n = |\langle n \mid \psi_t\rangle|^2 = (1-\rho)\rho^{n/2}. \tag{11}$$

Calculation of ρ. The model under consideration allows to find an exact solution. To arrive at it, one should only calculate a value of ρ. This can be done in the framework of a certain scattering problem for a quantum mechanical particle[15, 16], if we accept the usual scattering matrix formalism assumption: $t_1 \to -\infty$ and $t_2 \to +\infty$.

In order to make sure of that, let us come back to the Bogoliubov transformation (7). One can see that the coefficients $u(t)$ and $v(t)$ should satisfy eq. (5), because the field should satisfy eq. (4), taken in the operator form. Boundary conditions for the appropriate solutions of (5) follow from requirements

$$\hat{a}(t) = \exp(i\omega_1 t)\hat{a}_S \qquad \text{for } t \to -\infty,$$

$$\hat{a}(t) = C_1\exp(i\omega_1 t)\hat{a}_S + C_2\exp(i\omega_1 t)\hat{a}_S^+$$
$$\text{for } t \to +\infty.$$

Here the annihilation operator for the outgoing field is taken in the most general form consistent with its $\exp(i\omega_1 t)$ time dependence and the ingoing annihilation operator corresponds to the field without pions. This implies

$$u(t) = \exp(i\omega_1 t), \qquad v(t) = 0 \qquad \text{for } t \to -\infty,$$

$$u(t) = C_1\exp(i\omega_1 t), \qquad v(t) = C_2\exp(i\omega_1 t)$$
$$\text{for } t \to +\infty.$$

Thus, the unknown parameter ρ may be written as

$$\rho(t_2) = \frac{|v(t_2)|^2}{|u(t_2)|^2} = \frac{|C_2|^2}{|C_1|^2}.$$

The requirement (8) means that $|C_1|^2$ and $|C_2|^2$ are not independent. This gives

$$|C_1|^2 = \frac{1}{1-\rho}, \qquad |C_2|^2 = \frac{\rho}{1-\rho}.$$

A variable

$$w(t) = (u(t) + v(t)^*)/C_1$$

also satisfies (5) together with boundary conditions

$$w(t) = e^{i\omega_1 t}/C_1 \qquad \text{for } t \to -\infty, \tag{12}$$

$$w(t) = e^{i\omega_1 t} + \frac{C_2^*}{C_1} e^{-i\omega_1 t} \qquad \text{for } t \to +\infty. \tag{13}$$

There is a close analogy between eq. (5) for $w(t)$, and its solutions (12), (13) and the Schrödinger equation

$$\frac{\partial^2 \psi(x)}{\partial x^2} + \left(\frac{k^2}{2m} - V(x)\right)\psi(x) = 0,$$

corresponding to the scattering problem of a particle by a potential $V(x)$ [17]. In this framework, the value of ρ corresponds to the reflection coefficient, $\rho = R$, of the scattering problem. To achieve the total mathematical equivalence of the both models, it is necessary to replace $2m$ by 1 in the Schrödinger equation, to transpose ingoing and outgoing states, and to map:

$$t \leftrightarrow x, \qquad \mathrm{E}^2(\mathrm{t}) - \mathrm{V}(\mathrm{t}) \leftrightarrow \mathrm{k}^2(\mathrm{x}) - \mathrm{V}(\mathrm{x}),$$

where a time-dependent potential $V(t)$ simulates the changing boundary conditions. In a simple case when

$$E(t) = \begin{cases} 5.04 \text{ MeV}, & \text{for } 0 < t < \tau, \\ \varepsilon, & \text{for } 0 > t, \text{ or } t > \tau, \end{cases}$$

one has the scattering by a rectangular potential well of a depth

$$V_0 = (5.04\text{MeV})^2 - \varepsilon^2.$$

Subject to this proviso, it is possible to find:

$$\rho = \frac{1}{1+\delta^2}, \qquad \delta = \frac{2\varepsilon E}{V_0 \sin E\tau},$$

where

$$E = 5.04 \text{ MeV}, \qquad \tau \sim 1/\Gamma,$$

Γ is the dibaryon width, ε is the only unknown parameter, which can be found in further experiments. The data accuracy in [4, 5] does not permit to estimate ε but it allows to conclude that ρ is very close to 1, see (11) for the registered value of $n = 80$. The distribution (10) rapidly decreases with n therefore only the bound $\pi^+\pi^-$ pairs contribute to the heavy dibaryon tail observed in [4, 5].

Conclusion. The dibaryons observed in [3, 4, 5] and obeying the equidistant spectrum regularity hardly can be interpreted in the frame of the 6-q bag model. It is very likely to assign them to the production of pion pairs strongly bound to compressed nucleon matter. The analysis of the data from [3] reveals the possibility of presence of the pion Bose-Einstein condensate (BEC) in the ground state of deuteron, see (9). According to this analysis, the condensed pion field in deuteron can change in hard nuclear collisions. The pion BEC condensate can also appear in the compressed proton-proton system subjected to a proper cooling, according to the experimental hints from [4, 5]. The theory predicts the characteristic mass distribution for dibaryons of this type, which may be considered as an experimentally feasible signature of the pion BEC condensate.

It is reasonable to ask whether the pion Bose condensate arises in compressed k-nucleon systems for $k > 2$. If this is true, it can impact essentially on collective flows at the final stage of high-energy nuclear collisions, especially on the sideflow [18].

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