A Strong Laser Impact on Spin Precession of a Charged Particle in the Semi-Relativistic Interaction Regime

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• **Motivation and main results** • With this note we aim to report on studies of classical and quantum nonlinear effects in dynamics of a charged particle's spin caused by the interaction with a high intensity laser modeled by the monochromatic plane wave.

The standard dipole approximation [1] in laser-matter interaction is well justifiable and consistent with the non-relativistic treatment of a charged particle dynamics if the laser intensity is low. With the growing intensity the retardation effects as well as the influence of the magnetic part of the Heaviside-Lorentz force become important and modification of the dipole approximation schemes becomes unavoidable.

Quantity characterizing the intensity of a coherent radiation interactions with a massive (m), charged particle (e) is given by the so-called laser field strength parameter:

$$\eta^2 := -2\,\frac{e^2}{m^2c^4}\,\langle\langle\, A_\mu A^\mu \,\rangle\rangle\,,$$

where $\langle\langle \cdots \rangle\rangle$ denotes time average and A_μ is a four vector describing the laser field [2, 3]. It is well established that for a vanishing field strength $\eta^2 \ll 1$, a charged particle's spin behaves adiabatically, linearly responding to the magnetic component of the electromagnetic field. When η^2 is increasing the dipole approximation breaks down, non-linear effects come in force and the adiabatic picture of the spin precession turns to be inadequate. Below, having in mind the necessity of the dipole approximation scheme modification, we briefly state a method allowing to describe the evolution of a spin-1/2 particle in a strong monochromatic plane wave for the semi-relativistic interaction regime. Particularly, the question what is the effect of complete treatment of the Heaviside-Lorentz force (non ignoring its magnetic part) on the solution to the Larmor precession is addressed. Our consideration includes both classical as well as the non-relativistic quantum mechanical analysis. The results stated below developed out of our studies of a charged particle's classical dynamics beyond the dipole approximation [4] and semiclassical analysis of the spin-1/2 evolution in a strong laser field [5, 6]. Our description of the spin dynamics is applicable in the semi-relativistic regime of light-matter interactions, when η^2 is not negligible any more, but still $\eta^2 < 1$ and expansions over η^2 are correct.

Employing the above methods, the following effects of a strong coherent radiation have been found:

1. A nonlinear dependence of the spin precession frequency on a laser intensity and polarization;

2. Appearance of the quantum phase of spin's wave function due to the nonvanishing laser intensity.

• **Background and calculations** • We follow the semiclassical attitude towards the spin dynamics; spin degrees of a charged particle are described in a standard quantum mechanical way using the Pauli equation with the leading relativistic corrections. The effective spin-radiation interaction has a conventional form with magnetic field evaluating along charged particle's classical trajectory found from Newton's equations beyond the dipole approximation [4].

The basic assumption on a charged particle state vector $|\Psi\rangle$ consists in charge & spin decomposition:[1]

$$|\Psi\rangle = \sum_{i=0,1}\sum_{\alpha=\pm} c_{\alpha,i}|\psi_\alpha\rangle \otimes |\chi_i\rangle\,. \tag{1}$$

Two states $|\psi_\pm\rangle$, are linearly independent WKB solutions to the Schrödinger equation for a charge interacting with a laser radiation in the radiation gauge, $A_0 = 0$, $\boldsymbol{\nabla}\cdot\mathbf{A} = 0$. The vector $|\chi_i\rangle$ describes the states of a non-relativistic spin-1/2 subject to the Pauli equation with an effective Hamiltonian derived in the semiclassical approximation from the Schrödinger equation. Our calculations lead to the

[1]Here to simplify the expressions, we keep only one nonzero coefficient, $c_{+,0}$. The unit normalization condition on the WKB wave function fixes this coefficient, $\pi c^2_{+,0} = 2m\,\omega_P$, where ω_P is fundamental frequency of a particle driven by a laser.

following Hamiltonian operator governing the evolution of a two component spin-1/2 wave function $|\chi(t)\rangle$:

$$H(t) := -\frac{\kappa\hbar}{2}\,\mathbf{\Omega}(t)\cdot\boldsymbol{\sigma}\,.$$

Here $\kappa = e/mc$ and $\boldsymbol{\sigma}$ are the Pauli matrices. The Larmor vector $\mathbf{\Omega}(t) = (\Omega_1, \Omega_2, \Omega_3)$ depends on time by means of the Jacobian elliptic functions, whose argument is $u = \omega' t$ and the modulus reads $\mu^2 = (1-2\varepsilon^2)\eta^2\gamma_z$:

$$\Omega_1 = \eta\omega'\varepsilon'\mathrm{sn}(u,\mu)\mathrm{dn}(u,\mu)\,, \tag{2}$$

$$\Omega_2 = \eta\omega'\varepsilon\,\mathrm{cn}(u,\mu)\mathrm{dn}(u,\mu)\,, \tag{3}$$

$$\Omega_3 = -\eta^2\varepsilon\varepsilon'\omega\,, \tag{4}$$

where ε denotes a laser polarization, $\varepsilon' = \sqrt{1-\varepsilon^2}$ and the $\omega'\gamma_z = \omega$ stands for the non-relativistically Doppler shifted laser frequency ω with a contraction factor fixed by particle's z-component velocity, $\gamma_z^{-1} = 1 - v_z(0)/c$. Our analysis of the spin evolution equation for two extreme polarizations, *linear* and *circular*, reveals the following pattern.

Linear polarization

In this case the spin-1/2 wave function exposes nontrivial periodicity:

$$|\chi(t + 4\mathrm{K}(\mu)/\omega')\rangle = |\chi(t)\rangle\,, \tag{5}$$

where $\mathrm{K}(\mu)$ is the quarter period of the Jacobian function. Apart from the pure kinematical non-relativistic Doppler shift the particle's spin precession frequency depends on the laser intensity through the period $\mathbb{K}$. Furthermore, a spin precesses at frequency that depends nonlinearly on the laser intensity. For the semi-relativistic intensities $\eta \ll 1$, the period of the particle oscillation can be represented in the form of the expansion

$$T_P = \frac{2\pi}{\omega'}\left[1 + \left(\frac{1}{2}\right)^2 1 - 2\,\varepsilon^2\gamma_z^2\,\eta^2 + \dots\right].$$

The presence of $\mathbb{K}$ in (5) exposes a fundamental peculiarity of the particle dynamics which is beyond the dipole approximation.

Circular polarization

For a particle interaction with a circularly polarised laser there is no modification of the precision period caused by a laser intensity, but another, pure quantum mechanical effect occurs. The integration of the spin precession equation gives

$$|\chi(t)\rangle = U(0,t)\,|\chi(0)\rangle\,, \qquad U(0,0) = I\,, \tag{6}$$

where the evolution operator $U(0,t)$ in the Eulerian form reads

$$U(0,t) = e^{-i\omega' t\,\frac{\sigma_3}{2}}\cdot e^{-i\beta\,\frac{\sigma_1}{2}}\, e^{-i\eta\kappa\omega'\sqrt{1+\Delta^2}t\frac{\sigma_3}{2}}\, e^{i\beta\,\frac{\sigma_1}{2}}\,,$$

where $\tan\beta := \Delta\,,\ \Delta := -1/\kappa\eta + \eta^2\gamma_z$. From this solution we conclude that the precession period is not modified but the state changes under the cyclic evolution highly nontrivially. The spin evolution operator $U(0,t)$ undergoes the following change under the cyclic evolution

$$U\left(t + 2\pi/\omega'\right) = e^{i\pi}U\left(t\right)M\,,$$

with the so-called *monodromy* matrix

$$M(\omega') = e^{-i\beta\,\frac{\sigma_1}{2}}\, e^{i\pi\eta\kappa\sqrt{1+\Delta^2}\sigma_3}\, e^{i\beta\,\frac{\sigma_1}{2}}\,. \tag{7}$$

The monodromy matrix M depends explicitly on the laser intensity. If the special initial conditions on the state vector are chosen the monodromy matrix (7) diagonalises

$$M_D = e^{i\pi\eta\kappa\sqrt{1+\Delta^2}\,\sigma_3}.$$

Note, that for the vanishing laser field strength, the induced quantum phase reduces to its non-relativistic value.

• **Conclusion** • Our studies point on new effects associated with a spin-1/2 charged particle motion in laser fields with intensities corresponding to the semi-relativistic interaction regime. These effects are identified beyond the dipole approximation and could be attributed to the fact that the magnetic forces associated with the laser field have been retained and alter the spin evolution appreciably.

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