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ULTRAVIOLET DIVERGENCES IN NON-RENORMALIZABLE SUPERSYMMETRIC THEORIES

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We present a pedagogical review of our current understanding of the ultraviolet structure of $\mathcal{N} = (1, 1)$ 6D supersymmetric Yang–Mills theory and of $\mathcal{N} = 8$ 4D supergravity. These theories are not renormalizable, they involve power ultraviolet divergences and, in all probability, an infinite set of higher-dimensional counterterms that contribute to on-mass-shell scattering amplitudes.

A specific feature of supersymmetric theories (especially of extended supersymmetric theories) is that these counterterms may not be invariant off-shell under the full set of supersymmetry transformations. The lowest-dimensional nontrivial counterterm is supersymmetric on-shell. Still higher counterterms may lose even the on-shell invariance. On the other hand, the full effective Lagrangian, generating the amplitudes and representing an infinite sum of counterterms, still enjoys the complete symmetry of original theory.

We also discuss simple supersymmetric quantum-mechanical models that exhibit the same behavior.

PACS: 04.65.+e; 12.60.Jv

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NEW DEVELOPMENTS IN NONRELATIVISTIC GRAVITY

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In this paper, we give a short overview of Newton–Cartan geometry and gravity and some recent results about its matter couplings.

PACS: 02.40.Ma; 04.25.Nx; 04.50.Kd

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INDUCED EFFECTIVE ACTION IN 6D HYPERMULTIPLY THEORY ON A VECTOR/TENSOR BACKGROUND

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We consider the six-dimensional hypermultiplet, vector and tensor multiplet models in (1,0) harmonic superspace and discuss the corresponding superfield actions. Manifestly a (1,0) supersymmetric procedure of computing the effective action is developed using superfield proper-time technique. The leading low-energy contributions to the effective action are calculated.

PACS: 11.25.Hf; 12.60.Jv

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HIGHER-DERIVATIVE COUPLINGS IN SUPERGRAVITY

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We review the construction of the $\mathcal{N} = 2$ supersymmetric completion of a scalar curvature squared term both in superspace and components in a completely gauge-independent form.

PACS: 04.65.+e

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**INVARIANT DIFFERENTIAL OPERATORS
FOR NONCOMPACT LIE GROUPS:
SUMMARY OF $su(4, 4)$ MULTIPLETS**

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The present paper is a part of the project of systematic construction of invariant differential operators of noncompact semisimple Lie algebras. Here we give a summary of all multiplets containing physically relevant representations including the minimal ones for the algebra $su(4, 4)$. Due to the recently established parabolic relations, the results are valid also for the algebras $sl(8, \mathbb{R})$ and $su^*(8)$.

PACS: 02.20.Sv

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W_4 TODA EXAMPLE AS HIDDEN LIOUVILLE CFT

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We construct correlators in the W_4 Toda 2d conformal field theory for a particular class of representations and demonstrate a relation to a W_2 (Virasoro) theory with different central charge. The relevance of the classical limits of the constructed 3-point functions and braiding matrices to problems in 4d conformal theories is discussed.

PACS: 11.25.Hf

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NUMBER THEORY MEETS HIGH ENERGY PHYSICS

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Feynman amplitudes in perturbative quantum field theory are being expressed in terms of an algebra of functions, extending the familiar logarithms, and associated numbers — *periods*. The study of these functions (including hyperlogarithms) and numbers (like the multiple zeta values), that dates back to Leibniz and Euler, has attracted anew the interest of algebraic geometers and number theorists during the last decades. The two originally independent developments are recently coming together in an unlikely collaboration between particle physics and what were regarded as the most abstruse branches of mathematics.

PACS: 02.10.De; 11.15.Bt

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A FRESH VIEW OF COSMOLOGICAL MODELS DESCRIBING VERY EARLY UNIVERSE: GENERAL SOLUTION OF THE DYNAMICAL EQUATIONS

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The dynamics of any spherical cosmology with a scalar field (“scalon”) coupling to gravity is described by the nonlinear second-order differential equations for two metric functions and the scalaron depending on the “time” parameter. The equations depend on the scalaron potential and on arbitrary gauge function that describes time parameterizations. This dynamical system can be integrated for flat, isotropic models with very special potentials. But, somewhat unexpectedly, replacing the independent variable t by one of the metric functions allows us to completely integrate the general spherical theory in any gauge and with arbitrary potentials. In this approach, inflationary solutions can be easily identified, explicitly derived, and compared to the standard approximate expressions. This approach is also applicable to intrinsically anisotropic models with a massive vector field (“vecton”) as well as to some noninflationary models.

PACS: 98.80.Cq

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TETRAHEXAHEDRIC CALOGERO MODEL

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We consider the spherical reduction of the rational Calogero model (of A_{n-1} type, without the center of mass) as a maximally superintegrable quantum system. It describes a particle on the $(n-2)$ -sphere in a very special potential. A detailed analysis is provided of the simplest nonseparable case, $n = 4$, whose potential blows up at the edges of a spherical tetrahedron, tessellating the two-sphere into 24 identical right isosceles spherical triangles in which the particle is trapped. We construct a complete set of independent conserved charges and Hamiltonian intertwiners and elucidate their algebra. The key structure is the ring of polynomials in Dunkl-deformed angular momenta, in particular, the subspaces invariant and anti-invariant under all Weyl reflections, respectively.

PACS: 03.65.-w; 02.30.Ik

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PHENOMENOLOGICAL LAGRANGIANS, GAUGE MODELS AND BRANES

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Phenomenological Lagrangians for physical systems with spontaneously broken symmetries are reformulated in terms of gauge field theory. Description of the Dirac p -branes in terms of the Yang–Mills–Cartan gauge multiplets interacting with gravity is proved to be equivalent to their description as a closed dynamical system with the symmetry $ISO(1, D - 1)$ spontaneously broken to $ISO(1, p) \times SO(D - p - 1)$.

PACS: 11.25.-w

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MASSIVE HIGHER SPIN SUPERMULTIPLETS IN 3D

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Here we give a brief review of the explicit construction for massive higher spin supermultiplets developed in collaboration with I. L. Buchbinder and T. V. Snegirev.

PACS: 12.60.Jv; 11.15.Wx; 04.20.Cz

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TRINIFICATION MODEL $SU(3)^3$ FROM ORBIFOLDS FOR FUZZY SPHERES

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In this review, we consider the $\mathcal{N} = 4$ supersymmetric $SU(3N)$ gauge theory defined on the Minkowski spacetime. Then we apply an orbifold projection leading to the $\mathcal{N} = 1$ supersymmetric $SU(N)^3$ model, with a truncated particle spectrum. Then, we present the dynamical generation of (twisted) fuzzy spheres as vacuum solutions of the projected field theory, breaking the $SU(N)^3$ spontaneously to a chiral effective theory with unbroken gauge group, the trinification group $SU(3)^3$.

PACS: 02.40.Gh; 04.50.-h; 12.60.Jv

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EISENHART LIFT IN PSEUDO-EUCLIDEAN SPACE AND HIGHER RANK KILLING TENSORS

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It is emphasized that the Eisenhart lift applied to integrable systems in pseudo-Euclidean space may result in Ricci-flat metrics of ultrahyperbolic signature which admit higher rank Killing tensors.

PACS: 04.50.-h

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INTEGRABILITY OF CALOGERO–COULOMB PROBLEMS

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In this short review, we describe the integrability properties of the Calogero-type perturbations of one- and two-center Coulomb problems and of the Stark–Coulomb problem. We present the explicit expressions of their constants of motion and show that these systems admit partial separation of variables.

PACS: 03.65.-w; 02.30.Ik

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BETHE VECTORS AND FORM FACTORS FOR TWO-COMPONENT BOSE GAS

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This short note presents works done in collaboration with S. Pakuliak (JINR, Dubna) and N. Slavnov (Steklov Mathematical Institute, Moscow). It is a summary of the articles arXiv:1412.6037, arXiv:1501.07566, arXiv:1502.01966, and arXiv:1503.00546. Here are given only references used for our calculations. A detailed list of references can be found in our articles mentioned here.

PACS: 03.65.-w; 02.30.Ik

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ADIABATIC LIMIT IN ABELIAN HIGGS MODEL WITH APPLICATION TO SEIBERG–WITTEN EQUATIONS

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In this paper, we deal with the $(2 + 1)$ -dimensional Higgs model governed by the Ginzburg–Landau Lagrangian. The static solutions of this model, called otherwise vortices, are described by Taubes’ theorem. This theorem gives, in particular, an explicit description of the moduli space of vortices (with respect to gauge transforms). However, much less is known about the moduli space of dynamical solutions. A description of slowly moving solutions may be given in terms of the adiabatic limit. In this limit, the dynamical Ginzburg–Landau equations reduce to the adiabatic equation coinciding with the Euler equation for geodesics on the moduli space of vortices with respect to the Riemannian metric (called T-metric) determined by the kinetic energy of the model.

A similar adiabatic limit procedure can be used to describe approximately solutions of the Seiberg–Witten equations on 4-dimensional symplectic manifolds. In this case, the geodesics of T-metric are replaced by the pseudoholomorphic curves, while the solutions of Seiberg–Witten equations reduce to the families of vortices defined in the normal planes to the limiting pseudoholomorphic curve. Such families should satisfy a nonlinear $\bar{\partial}$ -equation which can be considered as a complex analogue of the adiabatic equation. Respectively, the arising pseudoholomorphic curves may be considered as complex analogues of adiabatic geodesics in $(2 + 1)$ -dimensional case. In this sense, the Seiberg–Witten model may be treated as a $(2 + 2)$ -dimensional analogue of the $(2 + 1)$ -dimensional Abelian Higgs model.

PACS: 14.80.Da

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NONLINEAR SCHRÖDINGER EQUATION WITH COMPLEX SUPERSYMMETRIC POTENTIALS

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Using the concept of supersymmetry, we obtain exact analytical solutions of nonlinear Schrödinger equation with a number of complex supersymmetric potentials and power law nonlinearity. Linear stability of these solutions for self-focusing as well as defocusing nonlinearity has also been examined.

PACS: 12.60.Jv

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HOW QUANTUM MECHANICS PROBES SUPERSPACE

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We study the relation between the partition function of a nonrelativistic particle, that describes the equilibrium fluctuations implicitly, and the partition function of the same system, deduced from the Langevin equation, that describes the fluctuations explicitly, of a bath with additive white-noise properties. We show that both can be related to the partition function of the $\mathcal{N} = 1$ supersymmetric theory with one-dimensional bosonic worldvolume and that they can all describe the same physics, since the correlation functions of the observables satisfy the same identities for all systems. The supersymmetric theory provides the consistent closure for describing the fluctuations, even though supersymmetry may be broken, when their backreaction is taken into account. The trajectory of the classical particle becomes a component of a superfield, when fluctuations are taken into account. These statements can be tested by the identities the correlation functions satisfy, by using the lattice regularization of an action that describes commuting fields only.

PACS: 02.50Ey; 02.70Uu; 03.65Ca; 05.10Gg; 11.15Tk; 11.30Pb

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METAPLECTIC R -MATRICES

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We consider R -matrices, corresponding to the central (projective) extensions of orthogonal and symplectic groups, called *Metagonal* and *Metaplectic* groups, respectively.

PACS: 02.10.Yn

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STATIC AND NONSTATIC BLACK HOLES WITH THE LIOUVILLE MODE

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We present a new class of static and nonstatic quasi-spherical black hole solutions in four-dimensional Minkowski and anti-de Sitter spaces and briefly discuss its employing in the gauge/gravity duality.

PACS: 04.70.-s; 04.20.Jb; 11.25.Tq

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ESTIMATION OF CONFORMAL COSMOLOGICAL MODEL PARAMETERS WITH SDSS AND SNLS SUPERNOVA SAMPLES

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In spite of enormous progress of standard Λ CDM cosmology (SC), a number of alternative approaches have been suggested because there are great puzzles with an origin and essence of dark matter and dark energy which unavoidably arise in the framework of the standard approach. Alternative approaches have to pass a number of observational tests including one with distant type Ia supernovae (SNe Ia) data. A conformal cosmological (CC) approach can explain cosmological SNe Ia data without introducing Λ -term, however, introducing an exotic rigid equation of state is needed. Later on, these statements were confirmed with larger samples of observational data. In the paper, we check previous claims with joint SDSS-II and SNLS supernova samples.

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GAUGE-INVARIANT LAGRANGIANS FOR MIXED-ANTISYMMETRIC HIGHER SPIN FIELDS

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Lagrangian descriptions of integer HS representations of the Poincare group subject to the Young tableaux $Y[\hat{s}_1, \hat{s}_2]$ with two columns are constructed within a metric-like formulation in a d -dimensional flat space-time based on the BRST approach. A Lorentz-invariant resolution of the BRST complex within BRST formulations produces a gauge-invariant Lagrangian in terms of the initial tensor field $\Phi_{[\mu]_{\hat{s}_1}, [\mu]_{\hat{s}_2}}$ subject to $Y[\hat{s}_1, \hat{s}_2]$ with an additional tower of gauge parameters realizing the $(\hat{s}_1 - 1)$ -th stage reducible theory with a specific dependence on the value $(\hat{s}_1 - \hat{s}_2) = 0, 1, \dots, \hat{s}_1$. Minimal BRST–BV action is suggested, providing objects appropriate to construct interacting models with mixed-antisymmetric fields in a general framework.

PACS: 02.20.-a; 11.15.Wx; 04.20.Gz

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FRAME-LIKE GAUGE-INVARIANT DESCRIPTION OF MASSIVE FERMIONIC HIGHER SPINS IN 3D

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We give the frame-like gauge-invariant Lagrangian description for massive fermionic arbitrary spin fields in three-dimensional AdS space. The Lagrangian, complete set of gauge transformations and gauge-invariant curvatures are obtained.

PACS: 04.20.Gz; 11.15.Wx

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SPINOR DESCRIPTION OF THE CURVATURES OF $D = 5$ GAUGE FIELDS

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Spinor description of the curvatures of $D = 5$ Yang–Mills, Rarita–Schwinger, and gravitational fields is considered. Restrictions imposed on the curvature spinors by the dynamical equations and Bianchi identities are studied. In the absence of sources, symmetric curvature spinors with $2s$ indices obey first-order equations that in the linearized limit reduce to Dirac-type equations for massless free fields. These equations allow for a higher-spin generalization similarly to $4d$ case. Their solution in the form of the integral over Lorentz-harmonic variables parameterizing coset manifold $SO(1, 4)/(SO(1, 1) \times ISO(3))$ isomorph to the three-sphere is considered.

PACS: 02.40.Re; 04.60.Rt; 11.15.Wx

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GENERAL SOLUTIONS OF INTEGRABLE COSMOLOGICAL MODELS WITH NONMINIMAL COUPLING

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We study the integrable model with minimally and nonminimally coupled scalar fields and the correspondence of their general solutions. Using the model with a minimally coupled scalar field and the constant potential as an example we demonstrate the difference between the general solutions of the corresponding models in the Jordan and Einstein frames.

PACS: 98.80Jk; 98.80Cq; 04.20-q; 04.20Jb

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RENORMALIZATION-GROUP IMPROVED INFLATIONARY SCENARIOS

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The possibility to construct an inflationary scenario for renormalization-group improved potentials corresponding to the Higgs sector of quantum field models is investigated. Taking into account quantum corrections to the renormalization-group potential, which sums all leading logs of perturbation theory, is essential for a successful realization of the inflationary scenario, with very reasonable values of parameters. The scalar electrodynamics inflationary scenarios thus obtained are seen to be in good agreement with the most recent observational data.

PACS: 98.80Cq; 04.50.Kd; 98.80.-k

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REVISITING THE ROLE OF OCTONIONS IN HADRONIC PHYSICS

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Octonions and their split versions are shown to be applicable to the solutions of a large number of problems in hadronic physics, from the foundations of exceptional groups that are used in grand unified theories, to heterotic strings, the non-Desarguesian geometric property of space-time symmetries, twistors, harmonic superspace, conformal field theories, etc. Upon a brief review of these investigations we proceed to show how they are used in the unification of ancient and modern geometries, which in turn open new avenues for, and goes far beyond in providing, geometric foundations for the existence of internal symmetries such as color and flavor.

PACS: 13.75.-n; 13.85.-t

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DARBOUX–CRUM TRANSFORMATIONS, JORDAN STATES AND PT -SYMMETRY

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We describe how Darboux–Crum transformations with Jordan states provide a useful method for the design of complex optical systems. We focus on the construction of PT -symmetric models with features like invisible periodic defects and vanishing reflection coefficient. Some illustrative examples are discussed.

PACS: 42.70.-a; 12.60.Jv; 11.10.-z

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CONSTANTS OF MOTION IN DEFORMED OSCILLATOR AND COULOMB SYSTEMS

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In this paper, we propose a unified description of the constants of motion for superintegrable deformations of the oscillator and Coulomb systems on N -dimensional Euclidean space, sphere and hyperboloid.

PACS: 02.30.Ik

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ON THE KONTSEVICH \star -PRODUCT ASSOCIATIVITY MECHANISM

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The deformation quantization by Kontsevich is a way to construct an associative noncommutative star-product $\star = \times + \hbar \{, \}_\mathcal{P} + \bar{o}(\hbar)$ in the algebra of formal power series in \hbar on a given finite-dimensional affine Poisson manifold: here \times is the usual multiplication; $\{, \}_\mathcal{P} \neq 0$ is the Poisson bracket; and \hbar is the deformation parameter. The product \star is assembled at all powers $\hbar^{k \geq 0}$ via summation over a certain set of weighted graphs with $k + 2$ vertices; for each $k > 0$, every such a graph connects the two co-multiples of \star using k copies of $\{, \}_\mathcal{P}$. Cattaneo and Felder interpreted these topological portraits as genuine Feynman diagrams in the Ikeda–Izawa model for quantum gravity.

By expanding the star-product up to $\bar{o}(\hbar^3)$, i.e., with respect to graphs with at most five vertices but possibly containing loops, we illustrate the mechanism Assoc = \diamond (Poisson) that converts the Jacobi identity for the bracket $\{, \}_\mathcal{P}$ into the associativity of \star .

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TWO-LOOP EFFECTIVE KÄHLER POTENTIAL IN THREE-DIMENSIONAL $\mathcal{N} = 2$ SQED

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We consider three-dimensional Abelian $\mathcal{N} = 2$ supersymmetric Chern–Simons-matter model with two chiral superfields and study local superspace contributions to low-energy effective action in the matter superfields sector of the theory. Using supergraph technique, we compute the effective Kähler potential in the explicit form up to the two-loop approximation.

PACS: 11.30.Pb; 12.60.Jv

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COMPARATIVE ANALYSIS OF FINITE FIELD-DEPENDENT BRST TRANSFORMATIONS

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We review our recent study, introducing the concept of finite field-dependent BRST and BRST–anti-BRST transformations for gauge theories and investigating their properties. The algorithm of exact calculation for the Jacobian of a respective change of variables in the path integral is presented. Applications to the Yang–Mills theory, in view of infrared (Gribov) peculiarities, are discussed.

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GRAVITATIONAL LENS MODELS FOR COSMOLOGICAL BLACK HOLES

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If such objects as cosmological black holes really exist, they can be studied with a standard technique like strong and weak gravitational lensing. Cosmological voids can be explained as the result of the collapse of large perturbations into black hole with masses of the order of $10^{14}M_{\odot}$ and the expansion of the Universe. The resulting image of the Universe is that it is more homogeneous than expected from present observations. In this paper, we discuss some lensing properties related to the cosmological black holes (CBHs), namely, we consider differences in gravitational lensing for point-like mass and extended mass distributions. We consider the singular isothermal sphere model as a toy (illustrative) model for an extended distribution of Dark Matter and a slightly more complicated isothermal sphere with a core.

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RENORMDYNAMICS, DISCRETE DYNAMICS, AND QUANPUTERS

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In the Standard Model (SM) of particle physics, the values of the coupling constants and masses of particles evolve according to the renormdynamic equations of motion. In SM, minimal supersymmetric extension of the SM, standard pion–nucleon field theory and other models show how to define the values of coupling constants and masses.

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THE EARLY UNIVERSE HISTORY FROM CONTRACTION-DEFORMATION OF THE STANDARD MODEL

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The elementary particles evolution in the early Universe from Planck time up to several milliseconds is presented. The developed theory is based on the high-temperature (high-energy) limit of the Standard Model, which is generated by the contractions of its gauge groups. At the infinite temperature all particles lose masses. Only massless neutral Z bosons, massless u quarks, neutrinos, and photons survive in this limit. The weak interactions become long-range and are mediated by neutral currents, quarks have only one color degree of freedom.

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