Разработка схем метода конечных элементов для исследования коллективных моделей атомных ядер

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Физ. постановка задач

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Содержание

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Осенняя Школа по информационным технологиям ОИЯИ

Finite Element Method

Stages:

- BVP $\rightarrow$ minimization of quadratic functional problem
- Finite Element Mesh
- Construction of shape functions
  - Interpolation Polynomials
    - Lagrange Interpolation Polynomials
    - Hermite Interpolation Polynomials
  - ...
- Construction of piecewise polynomial functions by joining the shape functions
- Calculations of the integrals
  - Gaussian quadratures
  - ...
- Solving of Algebraic (Eigenvalue) Problem
  - Continuous Analog of Newton Method
  - ...

Problem statement

Self-adjoint system of N second-order ODEs for unknowns $\mathbf{\Phi}(z) \equiv \{\mathbf{\Phi}^{(i)}(z)\}_{i=1}^{N_o}$, $\mathbf{\Phi}^{(i)}(z) = (\Phi_1^{(i)}(z), \ldots, \Phi_N^{(i)}(z))^T$ by z in the region $z \in \Omega_z = (z^{\min}, z^{\max})$

$$\left(-\frac{1}{f_B(z)} \mathbf{I} \frac{d}{dz} f_A(z) \frac{d}{dz} + \mathbf{V}(z) + \frac{f_A(z)}{f_B(z)} \mathbf{Q}(z) \frac{d}{dz} + \frac{1}{f_B(z)} \frac{d\, f_A(z) \mathbf{Q}(z)}{dz} - E\, \mathbf{I} \right) \mathbf{\Phi}(z) = 0.$$

$f_B(z) > 0$ $f_A(z) > 0$, $\mathbf{I}$ is unit matrix; $\mathbf{V}(z)$ and $\mathbf{Q}(z)$ are a symmetric and an antisymmetric $N \times N$ matrices, with real or complex-valued coefficients from the Sobolev space $\mathcal{H}_2^{s \geq 1}(\Omega)$.
All coefficients are continuous (or piecewise continuous) functions that have derivatives up to the order of $\kappa^{\max} - 1 \geq 1$ in the domain $z \in \bar{\Omega}_z$.

The boundary conditions:

$$\text{(I)}: \qquad \mathbf{\Phi}(z^t) = 0,$$

$$\text{(II)}: \qquad \lim_{z \to z^t} f_A(z) \left(\mathbf{I} \frac{d}{dz} - \mathbf{Q}(z) \right) \mathbf{\Phi}(z) = 0,$$

$$\text{(III)}: \qquad \lim_{z \to z^t} \left(\mathbf{I} \frac{d}{dz} - \mathbf{Q}(z) \right) \mathbf{\Phi}(z) = \mathbf{G}(z^t) \mathbf{\Phi}(z^t).$$

Problem 1. For bound or metastable states

Case of the real potentials and real eigenvalues E: $E_1 \le E_2 \le \ldots \le E_{N_o}$

$$\langle \mathbf{\Phi}_m | \mathbf{\Phi}_{m'} \rangle = \int_{z^{\min}}^{z^{\max}} f_B(z) (\mathbf{\Phi}^{(m)}(z))^\dagger \mathbf{\Phi}^{(m')}(z) dz = \delta_{mm'}.$$

Case of the complex potentials and complex eigenvalues $E = \Re E + \imath \Im E$: $\Re E_1 \le \Re E_2 \le \ldots \le \Re E_{N_o}$,

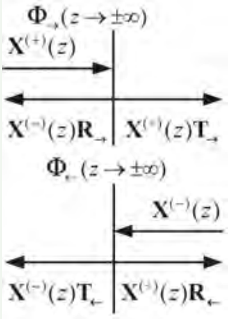
The eigenfunctions $\mathbf{\Phi}_m(z)$ obey the normalization and orthogonality conditions

$$(\mathbf{\Phi}_m | \mathbf{\Phi}_{m'}) = \int_{z^{\min}}^{z^{\max}} f_B(z) (\mathbf{\Phi}^{(m)}(z))^T \mathbf{\Phi}^{(m')}(z) dz = \delta_{mm'}.$$

J.G. Muga, J.P. Palao, B. Navarro, I.L. Egusquiza Complex absorbing potentials Physics Reports 395 (2004) 357–426
A.A. Gusev et al,Symbolic-numeric solution of boundary-value problems for the Schrodinger equation using the finite element method: scattering problem and resonance states, Lecture Notes in Computer Science 9301 (2015) 182–197.

Problem 2. The scattering problem

“incident wave + outgoing waves” asymptotic form



$$\boldsymbol{\Phi}_{\rightarrow}(z \rightarrow \pm\infty) = \begin{cases} \mathbf{X}^{(\rightarrow)}_{\min}(z) + \mathbf{X}^{(\leftarrow)}_{\min}(z)\mathbf{R}_{\rightarrow} + \mathbf{X}^{(c)}_{\min}(z)\mathbf{R}^{c}_{\rightarrow}, & z \rightarrow -\infty \\ \mathbf{X}^{(\rightarrow)}_{\max}(z)\mathbf{T}_{\rightarrow} + \mathbf{X}^{(c)}_{\max}(z)\mathbf{T}^{c}_{\rightarrow}, & z \rightarrow +\infty \end{cases}$$

$$\boldsymbol{\Phi}_{\leftarrow}(z \rightarrow \pm\infty) = \begin{cases} \mathbf{X}^{(\leftarrow)}_{\min}(z)\mathbf{T}_{\leftarrow} + \mathbf{X}^{(c)}_{\min}(z)\mathbf{T}^{c}_{\leftarrow}, & z \rightarrow -\infty \\ \mathbf{X}^{(\leftarrow)}_{\max}(z) + \mathbf{X}^{(\rightarrow)}_{\max}(z)\mathbf{R}_{\leftarrow} + \mathbf{X}^{(c)}_{\max}(z)\mathbf{R}^{c}_{\leftarrow}, & z \rightarrow +\infty \end{cases}$$

$\boldsymbol{\Phi}_{\rightarrow}(z)$, $\boldsymbol{\Phi}_{\leftarrow}(z)$ are the matrix solutions by dimension $N \times N_o^L$, $N \times N_o^R$

N_o^L, N_o^R are the numbers of open channels,

$\mathbf{X}^{(\rightarrow)}_{\min}(z)$, $\mathbf{X}^{(\leftarrow)}_{\min}(z)$ are open channel asymptotic solutions at $z \rightarrow -\infty$, dim. $N \times N_o^L$,

$\mathbf{X}^{(\rightarrow)}_{\max}(z)$, $\mathbf{X}^{(\leftarrow)}_{\max}(z)$ are open channel asymptotic solutions at $z \rightarrow +\infty$, dim. $N \times N_o^R$,

$\mathbf{X}^{(c)}_{\min}(z)$, $\mathbf{X}^{(c)}_{\max}(z)$ are closed channel solutions, dim. $N \times (N - N_o^L)$, $N \times (N - N_o^R)$,

$\mathbf{R}_{\rightarrow}$, $\mathbf{R}_{\leftarrow}$ are the reflection amplitude square matrices of dimension $N_o^L \times N_o^L$, $N_o^R \times N_o^R$,

$\mathbf{T}_{\rightarrow}$, $\mathbf{T}_{\leftarrow}$ are the transmission amplitude rectangular mat. of dim. $N_o^R \times N_o^L$, $N_o^L \times N_o^R$,

$\mathbf{R}^{c}_{\rightarrow}$, $\mathbf{T}^{c}_{\rightarrow}$, $\mathbf{T}^{c}_{\leftarrow}$, $\mathbf{R}^{c}_{\leftarrow}$ are auxiliary matrices.

Problem 2. The scattering problem

Wronskian conditions

$$\mathbf{Wr}(\mathbf{Q}(z);\mathbf{X}^{(\mp)}(z),\mathbf{X}^{(\pm)}(z)) = \pm 2\imath \mathbf{I}_{oo},\ \ \mathbf{Wr}(\mathbf{Q}(z);\mathbf{X}^{(\pm)}(z),\mathbf{X}^{(\pm)}(z)) = \mathbf{0}$$

$$\mathbf{Wr}(\mathbf{Q}(z);\mathbf{a}(z),\mathbf{b}(z)) = \mathbf{a}^T(z)\left(\frac{d\mathbf{b}(z)}{dz} - \mathbf{Q}(z)\mathbf{b}(z)\right) - \left(\frac{d\mathbf{a}(z)}{dz} - \mathbf{Q}(z)\mathbf{a}(z)\right)^T \mathbf{b}(z).$$

For real-valued potentials

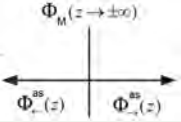
$$\mathbf{T}_{\rightarrow}^{\dagger}\mathbf{T}_{\rightarrow} + \mathbf{R}_{\rightarrow}^{\dagger}\mathbf{R}_{\rightarrow} = \mathbf{I}_{oo}, \quad \mathbf{T}_{\leftarrow}^{\dagger}\mathbf{T}_{\leftarrow} + \mathbf{R}_{\leftarrow}^{\dagger}\mathbf{R}_{\leftarrow} = \mathbf{I}_{oo},$$
$$\mathbf{T}_{\rightarrow}^{\dagger}\mathbf{R}_{\leftarrow} + \mathbf{R}_{\rightarrow}^{\dagger}\mathbf{T}_{\leftarrow} = \mathbf{0}, \quad \mathbf{R}_{\leftarrow}^{\dagger}\mathbf{T}_{\rightarrow} + \mathbf{T}_{\leftarrow}^{\dagger}\mathbf{R}_{\rightarrow} = \mathbf{0},$$
$$\mathbf{T}_{\rightarrow}^{T} = \mathbf{T}_{\leftarrow}, \quad \mathbf{R}_{\rightarrow}^{T} = \mathbf{R}_{\rightarrow}, \quad \mathbf{R}_{\leftarrow}^{T} = \mathbf{R}_{\leftarrow}.$$

For real-valued potentials the scattering matrix is symmetric and unitary, for complex potentials it is only symmetric

$$\mathbf{S} = \begin{pmatrix} \mathbf{R}_{\rightarrow} & \mathbf{T}_{\leftarrow} \\ \mathbf{T}_{\rightarrow} & \mathbf{R}_{\leftarrow} \end{pmatrix}, \qquad \mathbf{S}^{\dagger}\mathbf{S} = \mathbf{S}\mathbf{S}^{\dagger} = 1.$$

Problem 3. The metastable state pr. with complex e.v. $E = \Re E + \imath \Im E$:

Asymptotic form



$$\boldsymbol{\Phi}_{\rightarrow}(z \rightarrow \pm\infty) = \begin{cases} \mathbf{X}_{\min}^{(\leftarrow)}(z)\mathbf{O}_{\leftarrow} + \mathbf{X}_{\min}^{(c)}(z)\mathbf{O}_{\leftarrow}^{c}, & z \rightarrow -\infty \\ \mathbf{X}_{\max}^{(\rightarrow)}(z)\mathbf{O}_{\rightarrow} + \mathbf{X}_{\max}^{(c)}(z)\mathbf{O}_{\rightarrow}^{c}, & z \rightarrow +\infty \end{cases}$$

Robin (Siegert) BC

(III) :

$$\lim_{z \rightarrow z^t} \left(\mathbf{I} \frac{d}{dz} - \mathbf{Q}(z) \right) \boldsymbol{\Phi}(z) = \mathbf{G}(z^t)\boldsymbol{\Phi}(z^t), \quad t = \text{min and/or max}$$

$$\mathbf{G}(z^t) = \left(\lim_{z \rightarrow z^t} \left(\mathbf{I} \frac{d}{dz} - \mathbf{Q}(z) \right) \left(\mathbf{X}_t^{(\leftrightarrows)}(z), \mathbf{X}_t^{(c)}(z) \right) \right) \left(\mathbf{X}_t^{(\leftrightarrows)}(z), \mathbf{X}_t^{(c)}(z) \right)^{-1}$$

Orthonormalization conditions

$$(\boldsymbol{\Phi}_m | \boldsymbol{\Phi}_{m'}) = \int f_B(z) (\boldsymbol{\Phi}^{(m)}(z))^T \boldsymbol{\Phi}^{(m')}(z) dz = \delta_{mm'}.$$

J.G. Muga, J.P. Palao, B. Navarro, I.L. Egusquiza Complex absorbing potentials
Physics Reports 395 (2004) 357–426

Tests:

- BP for 1 ODE
  - Morse potential
  - Pöschl-Teller potential
  - Scarf complex potential
- BP for N ODE
  - system of piecewise constant potentials
- BP for multidimensional PDE
  - Helmholtz eq. for some domains (square, equilateral triangle, ...)
  - Coulomb potential
  - Harmonic oscillator

Test example (ODE System with Piecewise Constant Potentials)

$$\left(-\mathbf{I}\frac{d^2}{dz^2}+\mathbf{V}(z)-E\,\mathbf{I}\right)\mathbf{\Phi}(z)=0,\quad \mathbf{V}(z)=\{\mathbf{V}_1, z\le z_1, ..., \mathbf{V}_{k-1}, z\le z_{k-1}, \mathbf{V}_k, z>z_{k-1}\},$$

Matching the Fundamental Solutions

$$\left(-\mathbf{I}\frac{d^2}{dz^2}+\mathbf{V}_m-E\,\mathbf{I}\right)\mathbf{\Phi}_m(z)=0,\quad z\in(z_{m-1},z_m],\quad m=1,...,k,$$

$$\Rightarrow\quad \mathbf{\Phi}_m(z)=\sum_{i=1}^{N}\left(A_i^{(m)}\exp(-\sqrt{\lambda_i^{(m)}-E}z)\mathbf{\Psi}_i^{(m)}+B_i^{(m)}\exp(\sqrt{\lambda_i^{(m)}-E}z)\mathbf{\Psi}_i^{(m)}\right),$$

Here $\lambda_i^{(m)}$ and $\mathbf{\Psi}_i^{(m)}$ are the solutions of the algebraic eigenvalue problems

$$\mathbf{V}^{L,R}\mathbf{\Psi}_i^{(m)}=\lambda_i^{(m)}\mathbf{\Psi}_i^{(m)},\quad (\mathbf{\Psi}_i^{(m)})^T\mathbf{\Psi}_j^{(m)}=\delta_{ij}.$$

$$\lim_{z\to z_{m-1}}\mathbf{\Phi}_{m-1}(z)-\mathbf{\Phi}_m(z)=0,\quad \lim_{z\to z_{m-1}}\frac{\mathbf{\Phi}_{m-1}(z)}{dz}-\frac{\mathbf{\Phi}_m(z)}{dz}=0,\quad m=2,...,k$$

$\Rightarrow 2N(k-1)$ linear eqs. with $2N(k-1)$ unknowns.

Problem 2. The scattering problem. Example of asymptotic solutions

ODE in asymptotic regions $z \to \pm\infty$

$$\left(-\mathbf{I}\frac{d^2}{dz^2} + \mathbf{V}^{L,R} - E\,\mathbf{I}\right)\mathbf{\Phi}(z) = 0, \quad \text{where } \mathbf{V}^{L,R} \text{ are constant matrices.}$$

Asymptotic solutions

The open channel asymptotic solutions: $i_o = 1, ..., N_o^{L,R}$:

$$\mathbf{X}_{i_o}^{(\rightleftarrows)}(z \to \pm\infty) \to \frac{\exp\left(\pm\imath\sqrt{E - \lambda_{i_o}^{L,R}}z\right)}{\sqrt[4]{E - \lambda_{i_o}^{L,R}}}\mathbf{\Psi}_{i_o}^{L,R}, \quad \lambda_{i_o}^{L,R} < E.$$

The closed channels asymptotic solutions $i_c = N_o^{L,R} + 1, \ldots, N$:

$$\mathbf{X}_{i_o}^{(c)}(z \to \pm\infty) \to \exp\left(-\sqrt{\lambda_{i_c}^{L,R} - E}|z|\right)\mathbf{\Psi}_{i_c}^{L,R}, \quad \lambda_{i_c}^{L,R} \geq E.$$

Here $\lambda_i^{L,R}$ and $\mathbf{\Psi}_{i_c}^{L,R}$ are the solutions of the algebraic eigenvalue problems

$$\mathbf{V}^{L,R}\mathbf{\Psi}_i^{L,R} = \lambda_i^{L,R}\mathbf{\Psi}_i^{L,R}, \quad (\mathbf{\Psi}_i^{L,R})^T\mathbf{\Psi}_j^{L,R} = \delta_{ij}.$$

Problem 3. The metastable state pr. with complex e.v. $E = \Re E + \imath \Im E$:

Example of asymptotic solutions

The open channel asymptotic solutions: $i_o = 1, ..., N_o^{L,R}$:

$$\mathbf{X}_{i_o}^{(\rightleftarrows)}(z\to\infty)\to\exp\left(+\imath\sqrt{E-\lambda_{i_o}^{L,R}}|z|\right)\mathbf{\Psi}_{i_o}^{L,R},\quad \lambda_{i_o}^{L,R}<\Re E,\quad i_o=1,...,N_o^{L,R},$$

The closed channels asymptotic solutions $i_c = N_o^{L,R} + 1, \ldots, N$:

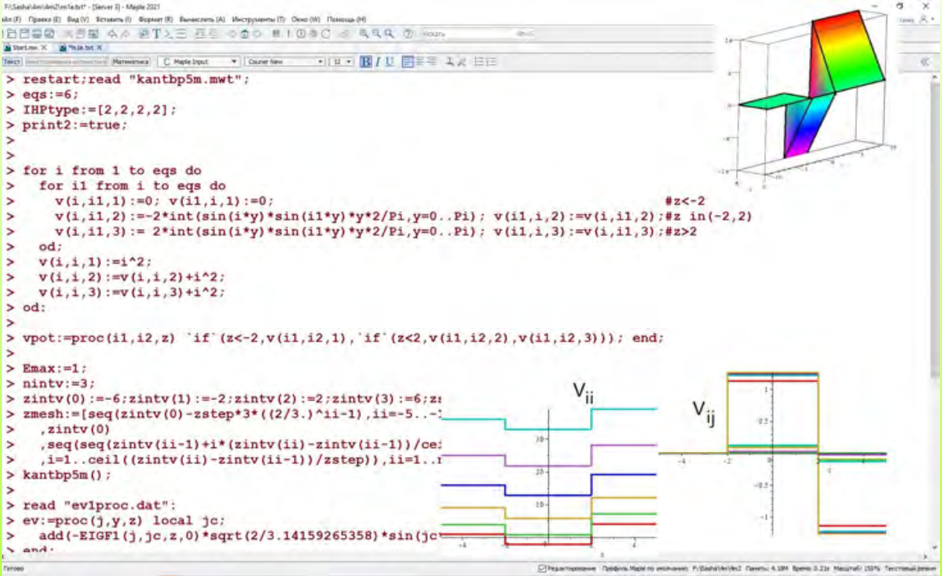
$$\mathbf{X}_{i_c}^{(c)}(z\to\infty)\to\exp\left(-\sqrt{\lambda_{i_c}^{L,R}-E}|z|\right)\mathbf{\Psi}_{i_c}^{L,R},\quad \lambda_{i_c}^{L,R}\geq\Re E,\quad i_c=N_o^{L,R}+1,\ldots,N.$$

Robin BC

$$\mathcal{R}(z^t) = \mathbf{\Psi}^{L,R}\mathbf{F}^{L,R}\left(\mathbf{\Psi}^{L,R}\right)^{-1},$$

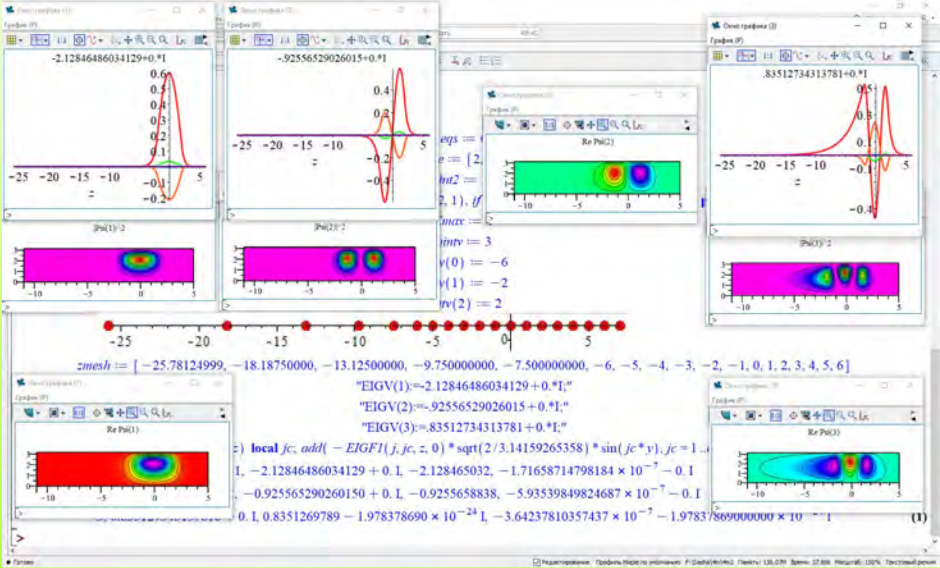
$$\mathbf{F}^{L,R}=\mathrm{diag}(..., \pm\sqrt{\lambda_{i_c}^{L,R}-E}, ..., \mp\imath\sqrt{E-\lambda_{i_o}^{L,R}}, ...)$$

The piecewise constant potentials

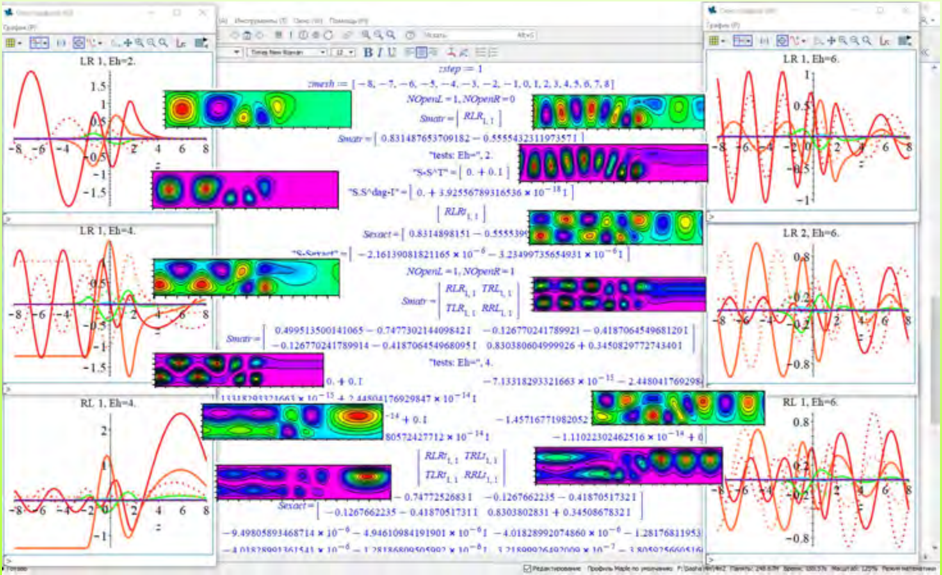


A. Gusev, S. Vinitsky, V. Gerdt, O. Chuluunbaatar, G. Chuluunbaatar, L. Le Hai, E. Zima, A Maple implementation of the finite element method for solving boundary problems of the systems of ordinary second order differential equations. Maple

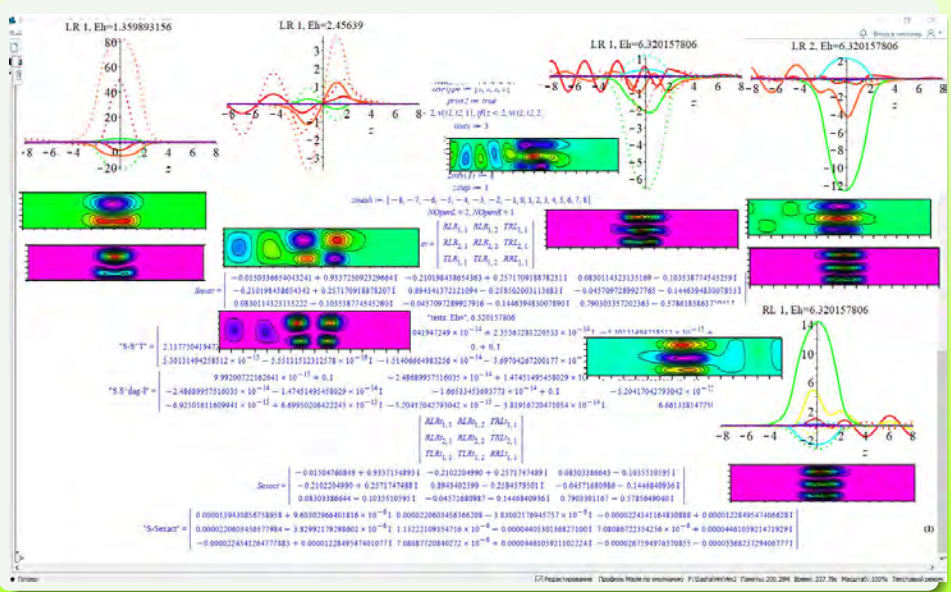
The piecewise constant potentials (eigenvalue problem)



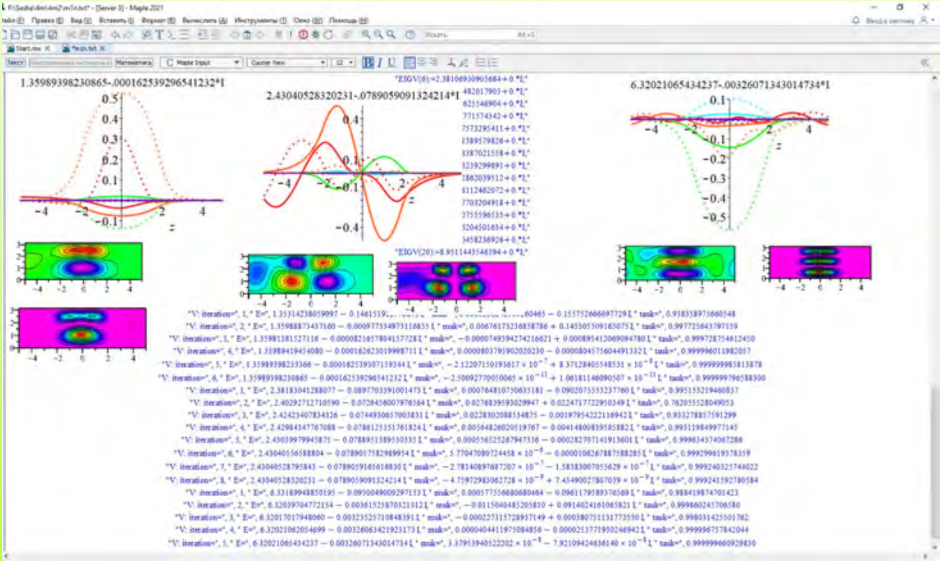
The piecewise constant potentials (multichannel scattering problem)



The piecewise constant potentials (resonance scattering states)



The piecewise constant potentials (metastable state problem)



Sub-barrier reactions of the fusion of heavy ions

The coupled-channels Schrödinger equation ^{64}Ni+^{100}Mo ($N = 27$)

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{l(l+1)\hbar^2}{2\mu r^2}+V_N^{(0)}(r)+\frac{Z_P Z_T e^2}{r}+\epsilon_n-E\right]\ \psi_{nn_o}(r)+\sum_{n'=1}^{N}V_{nn'}(r)\psi_{n'n_o}(r)=0,$$

$V_{nn'}(r)$ are mat. elem. of Coulomb and the Nuclear (Woods-Saxon, $V_N^{(0)}(r)$) potentials.

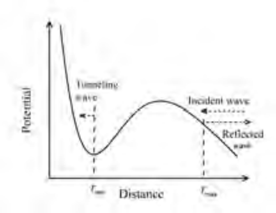
P.W. Wen, O. Chuluunbaatar, A.A. Gusev, R.G. Nazmitdinov, A.K. Nasirov, S.I. Vinitsky, C.J. Lin, and H.M. Jia, Near-barrier heavy-ion fusion: Role of boundary conditions in coupling of channels, Phys. Rev. C 101, pp. 014618 (2020).
P. W. Wen , C. J. Lin, R. G. Nazmitdinov , S. I. Vinitsky, O. Chuluunbaatar, A. A. Gusev, A. K. Nasirov, H. M. Jia, and A. Góźdź Potential roots of the deep subbarrier heavy-ion fusion hindrance phenomenon within the sudden approximation approach, Physical Review C 103, 054601 (2021)

Sub-barrier reactions of the fusion of heavy ions

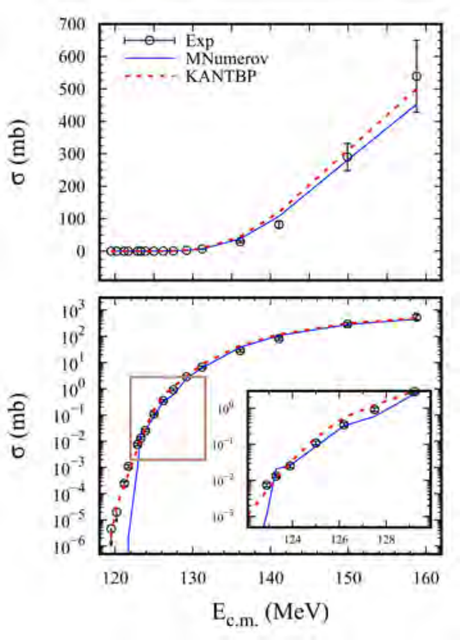
$$\psi^{as}_{nn_o}(r) = \sum_{m=1}^{M_o} A_{nm}\frac{\exp(-iK_m r)}{\sqrt{K_m}}\hat{T}_{mn_o} + \sum_{m=M_o+1}^{N} A_{nm}\frac{\exp(|K_m|r)}{\sqrt{|K_m|}}\hat{T}^c_{mn_o}, \quad r = r_{\min},$$

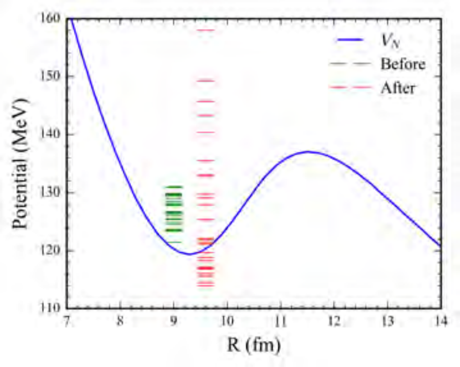
$$\psi^{as}_{nn_o}(r) = \begin{cases} \hat{H}^-_l(k_n r)\delta_{n,n_o} + \hat{H}^+_l(k_n r)\hat{R}_{nn_o}, & r = r_{\max}, \\ 2|k_n|^{1/2} r\exp(-|k_n|r)U(1+\eta_n, 2, 2|k_n|r), & r = r_{\max}. \end{cases}$$

$\hat{H}^{\pm}_l(k_n r)$ are Coulomb functions, $U(1+\eta_n, 2, 2|k_n|r)$ is Whittaker function



^{64}Ni+^{100}Mo: Deep sub-barrier fusion





MNumerov, are the results obtained by means of CCFULL [K. Hagino, N. Rowley, and A.T. Kruppa, A FORTRAN77 program for coupled-channels calculations with all order couplings for heavy-ion fusion reactions; Comput. Phys. Comm 123 (1999) 143 - 152, also CCFULL Home Page]

5DBVP for the five-dimensional quarupole Hamiltonian(5DQH)

The Schrödinger equation with respect to eigenfunction $\Psi_{nIM} \equiv \Psi_{nIM}(\beta, \gamma, \vartheta_i)$ and the corresponding eigenvalues of energy E_{nI} has the form

$$\frac{2}{\hbar^2}(\hat{H} - E_{nI})\Psi_{nIM} = \left(\hat{T}_{\rm vib} + \hat{T}_{\rm rot} + \frac{2}{\hbar^2}(V - E_{nI})\right)\Psi_{nIM} = 0. \quad (1)$$

orthogonality and normalization conditions

$$\int_{\Omega_5} \Psi_{nIM}\Psi_{n'I'M'} g_0(\beta,\gamma) d\beta d\gamma \sin\vartheta_2 d\vartheta_1 d\vartheta_2 d\vartheta_3 = \delta_{nn'}\delta_{II'}\delta_{MM'}. \quad (2)$$

The eigenfunction Ψ_{nIM} in the representation of the angular momentum I and its projections K and M on the third axes of the intrinsic and laboratory frames

$$\Psi_{nIM}(\beta,\gamma,\vartheta_i) = \sum_{K\geq 0, even}^{I} \mathcal{D}^{I*}_{MK}(\vartheta_i)\Phi_{nIK}(\beta,\gamma), \quad (3)$$

where $\mathcal{D}^{I*}_{MK}(\vartheta_i)$ are the normalized D-functions with the space parity $\hat{\pi} = \pm 1$

$$\mathcal{D}^{I*}_{MK}(\vartheta_i) = \sqrt{\frac{2I+1}{8\pi^2}} \frac{(D^{I*}_{MK}(\vartheta_i) + \hat{\pi}(-1)^I D^{I*}_{M-K}(\vartheta_i))}{\sqrt{2(1+\delta_{K0})}}. \quad (4)$$

2DBVP for five-dimensional quarupole Hamiltonian(5DQH)

The unknown set of $I_{\max}$ internal components $\Phi_{nIK} \equiv \Phi_{nIK}(\beta,\gamma)$, where $K = 0, 2, \ldots, I$ for even I, or $K = 2, 4, \ldots, (I-1)$ for odd I, compose the vector eigenfunction Φ_{nI} corresponding to the eigenvalue E_n^I (in MeV) of the BVP for a system of $I/2+1$ or $(I-1)/2$ equations for even or odd I, respectively:

$$\left[\hat{T}_{\rm vib} + T^I_{KK} + \frac{2}{\hbar^2}\left(V - E_{nI}\right)\right]\Phi_{nIK} + T^I_{KK+2}\Phi_{nIK+2} + T^I_{KK-2}\Phi_{nIK-2} = 0,$$

$$\hat{T}_{\rm vib}(x_1,x_2) = -\frac{1}{g_0(x_1,x_2)}\sum_{i,j=1}^{2}\frac{\partial}{\partial x_i}g_{ij}(x_1,x_2)\frac{\partial}{\partial x_j},$$

$$T^I_{KK} = (I(I+1)-K^2)\left(\frac{1}{2J_1}+\frac{1}{2J_2}\right)+\frac{K^2}{J_3},\ \ T^I_{KK\pm2} = \left(\frac{1}{4J_1}-\frac{1}{4J_2}\right)C^I_{KK\pm2},$$

$$C^I_{KK+2} = C^I_{K+2K} = (1+\delta_{K0})^{1/2}[(I-K)(I+K+1)(I-K-1)(I+K+2)]^{1/2},$$

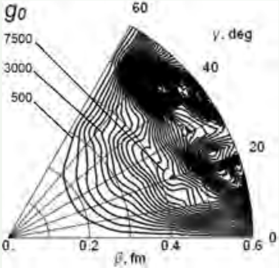
$$J_k(x_1,x_2) = J_k(\beta,\gamma) = 4B_k(\beta,\gamma)\beta^2\sin^2(\gamma - 2\pi k/3). \tag{5}$$

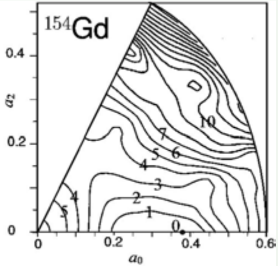
The components Φ_{nIK} are subject to Neumann or Dirichlet boundary conditions at the boundary $\partial\Omega_2$ of the domain Ω_2 and the orhtogonality and normalization conditions

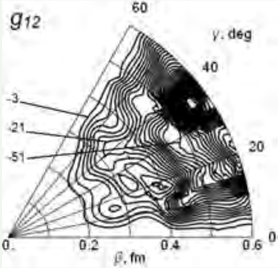
$$\int_0^{\beta_{\max}}\int_0^{\pi/3} g_0(\beta,\gamma)d\beta d\gamma \sum_{K\geq0,even}^{I_{\max}} \Phi_{nIK}(\beta,\gamma)\Phi_{n'IK}(\beta,\gamma) = \delta_{nn'}. \tag{6}$$

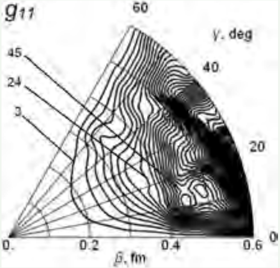
Benchmark calculations of ^{154}Gd in the RMF model

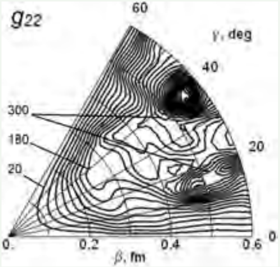
Spectrum E, isolines of $V(\beta,\gamma)$ counted from the minimum of $V(\beta{=}0.3875,\gamma{=}0)$ $=-1270.6$MeV, $g_0(\beta,\gamma)$ and $g_{ij}(\beta,\gamma)$ of ^{154}Gd calculated in PC-F1 of RMF model



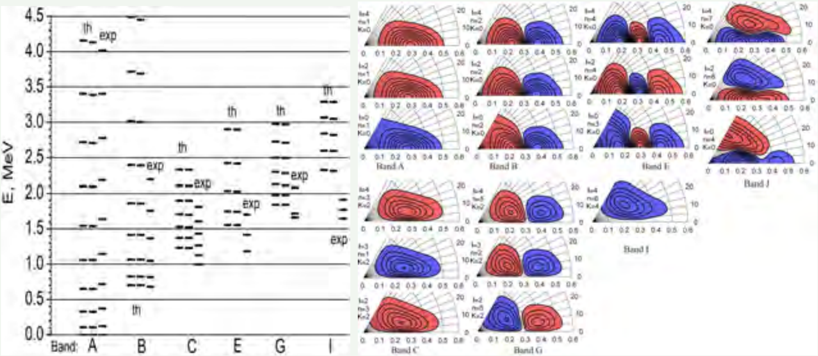








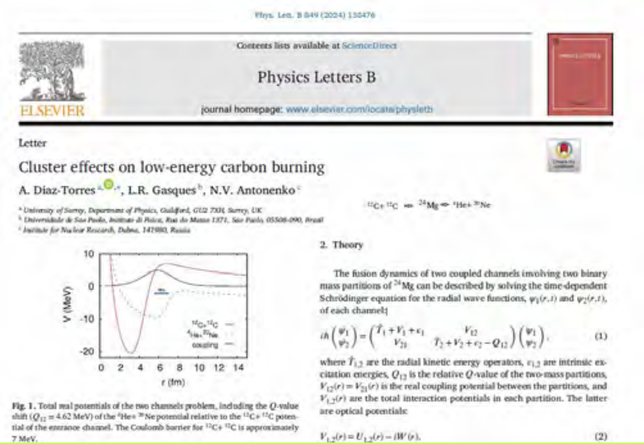
Energy spectrum of ^{154}Gd



Energy spectrum of ^{154}Gd. For each state of the bands A, B, E, C, G, and I, three short bars correspond to the diagonal approximation (left), nondiagonal one (middle), and experiment (right)[http://www.nndc.bnl.gov/ensdf/].

Band(A) is the $K^{\pi} = 0+$ ground state band;
Band(B): the first excited $K^{\pi} = 0^{+}$ (β-vibrational) band;
Band(E), Band(J), Band(K): the second, third and forth excited $K^{\pi} = 0+$ bands;
Band(C): the $K^{\pi} = 2^{+}$ (γ-vibrational) band;
Band(G): the second excited $K^{\pi} = 2^{+}$ ($\beta\gamma$-vibrational) band;
Band(I): the $K^{\pi} = 4^{+}$ band.

Возможная постановка задачи для дипломных работ



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Letter

Cluster effects on low-energy carbon burning

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$^{12}C + ^{12}C \Leftrightarrow ^{24}Mg \Leftrightarrow ^{4}He + ^{20}Ne$

Fig. 1. Total real potentials of the two channels problem, including the Q-value shift ($Q_{12} = 4.62$ MeV) of the $^4He + ^{20}Ne$ potential relative to the $^{12}C + ^{12}C$ potential of the entrance channel. The Coulomb barrier for $^{12}C + ^{12}C$ is approximately 7 MeV.

2. Theory

The fusion dynamics of two coupled channels involving two binary mass partitions of ^{24}Mg can be described by solving the time-dependent Schrödinger equation for the radial wave functions, $\psi_1(r,t)$ and $\psi_2(r,t)$, of each channel:

$$i\hbar \begin{pmatrix} \dot\psi_1 \\ \dot\psi_2 \end{pmatrix} = \begin{pmatrix} \hat T_1 + V_1 + \epsilon_1 & V_{12} \\ V_{21} & \hat T_2 + V_2 + \epsilon_2 - Q_{12} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad (1)$$

where $\hat T_{1,2}$ are the radial kinetic energy operators, $\epsilon_{1,2}$ are intrinsic excitation energies, Q_{12} is the relative Q-value of the two-mass partitions, $V_{12}(r) = V_{21}(r)$ is the real coupling potential between the partitions, and $V_{1,2}(r)$ are the total interaction potentials in each partition. The latter are optical potentials:

$$V_{1,2}(r) = U_{1,2}(r) - iW(r), \quad (2)$$

Краевая задача для системы ОДУ

$$\left(-\frac{1}{f_B(z)}\frac{d}{dz}\mathbf{f_A}(\mathbf{z})\frac{d}{dz} + \mathbf{V}(z) + ?\,\frac{f_A(z)}{f_B(z)}\mathbf{Q}(z)\frac{d}{dz} + ?\,\frac{1}{f_B(z)}\frac{d\, f_A(z)\mathbf{Q}(z)}{dz} - E\,\mathbf{I}\right)\boldsymbol{\Phi}(z) = 0.$$