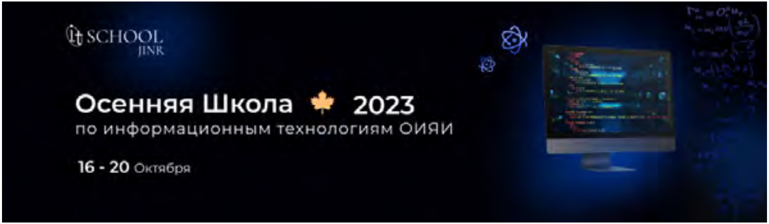

Кварконии в билокальной модели при конечной температуре и плотности

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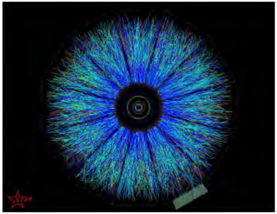
Introduction

Why is finite T/μ physics ($\approx$ QCD) interesting?

High energy physics applications

- Heavy ion experiments $\Rightarrow$ Need quantitive understanding of non-Abelian plasmas at
  - High T and small/moderate μ
  - Moderately large couplings
  - In and (especially) out of equilibrium
- Early universe thermodynamics
  - Signatures of phase transitions
  - EW baryogenesis
- Neutron star interiors
  - EoS at high μ and $T \simeq 0$
  - Transport in nucl. matter

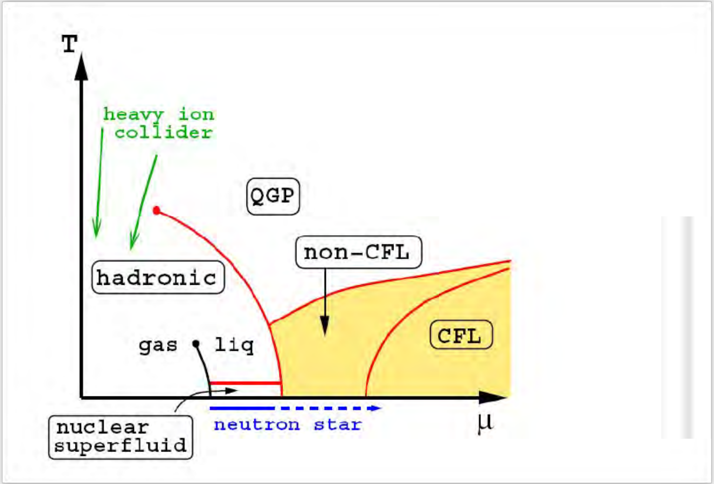


Challenges in thermal QCD

To understand heavy ion experiments / early universe thermodynamics want to know (among other things):

- Structure of QCD phase diagram: Phase structure, location of transition lines, critical points,...
- Properties of phase transitions, in particular the deconfinement transition
- Equation of state and other equilibrium quantities

Introduction



Covariant bound states in effective QCD

A systematic field-theoretic approach to the description of hadronic matter on the basis of an effective-action functional in the quark sector of QCD can be formulated within the path-integral approach. The quark-action functional can be represented in the form

$$S_{\text{eff}}[q,\bar{q}] = \int \mathrm{d}x_1\, \mathrm{d}x_2\, \bar{q}_A(x_1)\left[G_0^{-1}(x_1,x_2)\right]_{AB} q_B(x_2)$$

$$-\frac{1}{2}\int \mathrm{d}x_1\, \mathrm{d}x_2\, \mathrm{d}y_1\, \mathrm{d}y_2\, \left[\bar{q}_A(x_1) q_B(y_1)\right]$$

$$\times\left[\mathscr{K}^{\eta}(x_1,y_1;x_2,y_2)\right]_{AB;CD}\left[\bar{q}_C(x_2) q_D(y_2)\right].$$

Here, the first term is the free-quark action, with

$$\left[G_0^{-1}(x_1,x_2)\right]_{AB} = (i\not{\partial} - \hat{m}_0)\delta_{AB}\delta(x_1 - x_2),$$

and $\hat{\mathrm{m}}_0 = \mathrm{diag}(\mathrm{m}_0^{\mathrm{a}})$, $\mathrm{a} = 1,\ldots,\mathrm{N}$, is the current quark mass matrix. The indices A, B, C, D are a compact notation for Dirac as well as flavour and colour indices.

Covariant bound states in effective QCD

The channel decomposition of the four-point interaction kernel $\mathcal{K}^\eta$

$$[\mathscr{K}^\eta(x_1, y_1; x_2, y_2)]_{AB;CD} = (\Lambda^{\mathrm{H}}_{AD} \cdot \Lambda^{\mathrm{H}}_{CB}) \mathscr{K}^\eta(x, y \,|\, X, Y),$$

where

$$\Lambda^{\mathrm{H}}_{AD} = (\xi^k \cdot \rho^b \cdot \zeta^j)_{AD}$$

$$\xi^k = \left(1,\, i\gamma_5,\, \frac{i}{\sqrt{2}}\gamma_\mu,\, \frac{i}{\sqrt{2}}\gamma_\mu\gamma_5\right),$$

$$\rho^b = \left(\sqrt{\tfrac{3}{2}}\,1_{\mathrm{f}},\, \frac{i}{\sqrt{2}}\tau^a\right), \quad a = 1, 2, 3 \quad \text{for } \mathrm{SU}(2)_{\mathrm{f}},$$

$$\zeta^j = \left(\tfrac{4}{3}1_{\mathrm{c}},\, \frac{i}{\sqrt{3}}\lambda^i\right), \quad i = 1, 2, \ldots, 8 \quad \text{for } \mathrm{SU}(3)_{\mathrm{c}}.$$

The orbital part of the interaction,

$$\mathscr{K}^{\eta}(x, y \,|\, X, Y) = W(x^{\perp}, y^{\perp})\delta^4(X - Y)\delta(x \cdot \eta)\delta(y \cdot \eta),$$

has a relativistic covariant form, with $\mathrm{x} = \mathrm{x}_1 - \mathrm{x}_2$, $\mathrm{X} = (\mathrm{x}_1 + \mathrm{x}_2)/2$ being the four-vectors of the relative and the center-of-mass coordinates of the incoming quark-antiquark pair;
$\mathrm{y} = \mathrm{y}_1 - \mathrm{y}_2$, $\mathrm{Y} = (\mathrm{y}_1 + \mathrm{y}_2)/2$ are those of the outgoing one, respectively. The four-dimensional δ-function guarantees the condition of the center-of-mass conservation, which is a consequence of the homogeneity of the space-time continuum.

The instantaneous interaction kernel is further assumed to neglect retardation effects in the s-channel so that it depends via $W(x^\perp, y^\perp)$ only on the transverse components of the four-distances x_μ and y_μ, with respect to the (conserved) four-vector P_μ, of the center-of-mass momentum:

$$x_\mu^\perp = x_\mu - x_\mu^\parallel, \quad x_\mu^\parallel = \eta_\mu (x \cdot \eta), \quad \eta_\mu = \frac{P_\mu}{\sqrt{P^2}}.$$

$y^\parallel$ and $y^\perp$ are introduced accordingly.
The neglect of retardation effects in the s-channel is motivated by the analogy with quantum electrodynamics where the assumption of the dominance of instantaneous interactions in the formation of bound states can be justified. The projection on the subspace of equal-time processes is represented by $\delta(x \cdot \eta)$.
The potential of the effective quark interaction can be constructed in a gauge-invariant way within the reduced phase-space- quantization scheme.

Covariant bound states in effective QCD

For the application of the general nonlocal theory, there are two important classes of potentials which are contained as special cases in the general form of $\mathrm{W}(\mathrm{x}^{\perp}, \mathrm{y}^{\perp})$

bilocal potentials:

$$W(x^{\perp}, y^{\perp}) = \delta^3(x^{\perp} - y^{\perp}) V(x^{\perp}),$$

separable potentials:

$$W(x^{\perp}, y^{\perp}) = V_0 g(x^{\perp}) g(y^{\perp}).$$

Covariant bound states in effective QCD

Within the standard functional-integral approach to bilocal field theory the action can be transformed by introducing bilocal bosonic fields $\mathcal{M}_{\rm AB}(x_1, x_2)$.
After integration over the quark fields, the effective bosonized action takes the form

$$S_{\rm eff}[\mathscr{M}] = -N_c\left[\frac{1}{2}\mathscr{M}(\mathscr{K}^\eta)^{-1}\mathscr{M} + i\ \mathrm{Tr}\ \ln(-G_0^{-1} + \mathscr{M})\right].$$

Due to the particular choice of the instantaneous interaction kernel, the fields $\mathcal{M}_{\rm AB}(x_1, x_2) = \Lambda^{\rm H}_{\rm AB}\mathcal{M}_{\rm AB}(x, X)$ can be introduced in such a way that they satisfy the Markov-Yukawa condition for instantaneous bilocal meson fields:

$$x_\mu \frac{\partial}{\partial X_\mu}\mathscr{M}^{\rm H}(x|X) = 0.$$

These fields are irreducible representations of the Poincare group with definite mass, $P^2 = M_H^2$, spin and orbital parts which can be expressed as

$$\mathscr{M}^{H}(x|X) = \int \frac{d\boldsymbol{P}}{(2\pi)^{3/2}\sqrt{2\omega_H(\boldsymbol{P})}} \Big[e^{-i(P\cdot X)} \Gamma_H(x^\perp|P) a_H^+(\boldsymbol{P}) + e^{i(P\cdot X)} \bar{\Gamma}_H(x^\perp|-P) a_H^-(\boldsymbol{P}) \Big] \delta(x\cdot\eta^H),$$

where $a_H^+(P)$, $a_H^-(P)$ are creation and annihilation operators of a bound state. The index H denotes the set of hadron quantum numbers and P_0 is fixed as $P_0 = \omega_H = \sqrt{P^2 + M_H^2}$ and $\Gamma_H(p^\perp|P)$, $\Gamma_H(p^\perp|-P)$ are the quark-meson-vertex amplitudes which are functions of the transverse component of the four-distance $x_\mu^\perp$ with respect to the four-vector P of the center-of-mass momentum.

$$\Sigma(x-y) = m^0\delta^{(4)}(x-y) + i\mathscr{K}(x,y)G_\Sigma(x-y),$$

$$\Gamma = i\mathscr{K}(x,y)\int d^4z_1\, d^4z_2\, G_\Sigma(x-z_1)\Gamma(z_1,z_2)G_\Sigma(z_2-y),$$

In momentum space we obtain with

$$\underline{\Sigma}(k) = \int d^4x\, \Sigma(x)e^{ikx},$$

$$\underline{\Gamma}(q|\mathscr{P}) = \int d^4x\, d^4y \exp\left[i\frac{x+y}{2}\mathscr{P}\right]\exp[i(x-y)q]\,\Gamma(x,y)$$

$$\underline{\Sigma}(k) = m^0 + i\int \frac{d^4q}{(2\pi)^4}\underline{V}(k^\perp - q^\perp)\not\eta\, \underline{G}_\Sigma(q)\not\eta,$$

$$\underline{\Gamma}(k,\mathscr{P}) = i\int \frac{d^4q}{(2\pi)^4}\underline{V}(k^\perp - q^\perp)\not\eta\left[\underline{G}_\Sigma\left(q+\frac{\mathscr{P}}{2}\right)\Gamma(q|\mathscr{P})\underline{G}_\Sigma\left(q-\frac{\mathscr{P}}{2}\right)\right]\not\eta,$$

The quantities $\underline{\Sigma}$ and $\underline{\Gamma}$ depend only on the transversal momentum

$$\underline{\Sigma}(k) = \underline{\Sigma}(k^{\perp}), \qquad \underline{\Gamma}(k|\mathscr{P}) = \underline{\Gamma}(k^{\perp}|\mathscr{P}),$$

because of the instantaneous form of the potential $\underline{V}(k^{\perp})$ in any frame.

$$\underline{\Sigma}_a(q) = \not{q}^{\perp} + E_a(q^{\perp})S_a^{-2}(q^{\perp})$$

for the self-energy with

$$S_a^{-2}(q^{\perp}) = \exp\{-\hat{\not{q}}^{\perp} 2v_a(q^{\perp})\}, \qquad \hat{q}_{\mu}^{\perp} = q_{\mu}^{\perp}/|q^{\perp}|,$$

where S_a is the Foldy-Wouthuysen type transformation matrix with the parameter v_a.

Then, one has

$$G_{\Sigma_a} = [q_0 \not\eta - E_a(q^\perp) S_a^{-2}(q^\perp)]^{-1}$$
$$= \left[\frac{\Lambda^{(\eta)}_{(+)a}(q^\perp)}{q_0 - E_a(q^\perp) + i\varepsilon} + \frac{\Lambda^{(\eta)}_{(-)a}(q^\perp)}{q_0 + E_a(q^\perp) + i\varepsilon} \right] \not\eta,$$

where

$$\Lambda^{(\eta)}_{(\pm)a}(q^\perp) = S_a(q^\perp) \Lambda^{(\eta)}_{(\pm)}(0) S_a^{-1}(q^\perp), \qquad \Lambda^{(\eta)}_{(\pm)}(0) = (1 \pm \not\eta)/2$$

are the operators separating the states with positive $(+E_a)$ and negative $(-E_a)$ energies.

As a result, we obtain the following equations for the single-particle energy E and the angle v,

$$E_a(k^\perp) \cos 2v(k^\perp) = m_a^0 + \frac{1}{2} \int \frac{d^3 q^\perp}{(2\pi)^3} \underline{V}(k^\perp - q^\perp) \cos 2v(q^\perp),$$

$$E_a(k^\perp) \sin 2v(k^\perp) = |k^\perp| + \frac{1}{2} \int \frac{d^3 q^\perp}{(2\pi)^3} \underline{V}(k^\perp - q^\perp) |k^\perp \cdot q^\perp| \sin 2v(q^\perp).$$

Covariant bound states in effective QCD

The vertex function is given by

$$\Gamma_{ab}(k^{\perp}|\mathscr{P}) = \int \frac{d^3q^{\perp}}{(2\pi)^3} \underline{V}(k^{\perp} - q^{\perp}) \not\eta \psi_{ab}(q^{\perp}) \not\eta ,$$

where the bound-state wave function ψ_{ab} is given by

$$\psi_{ab}(q^{\perp}) = \not\eta \left[\frac{\bar{\Lambda}_{(+)a}(q^{\perp})\Gamma_{ab}(q^{\perp}|\mathscr{P})\Lambda_{(-)b}(q^{\perp})}{E_T - \sqrt{\mathscr{P}^2} + i\varepsilon} + \frac{\bar{\Lambda}_{(-)a}(q^{\perp})\Gamma_{ab}(q^{\perp}|\mathscr{P})\Lambda_{(+)b}(q^{\perp})}{E_T + \sqrt{\mathscr{P}^2} - i\varepsilon} \right] \not\eta .$$

$E_T = E_a + E_b$ means the sum of one-particle energies of the two particles (a) and (b)

$$\bar{\Lambda}_{(\pm)}(q^{\perp}) = S^{-1}(q^{\perp})\Lambda_{(\pm)}(0)S(q^{\perp}) = \Lambda_{(\pm)}(-q^{\perp})$$

has been introduced.

Covariant bound states in effective QCD

$$(E_T(k^\perp) \mp \sqrt{\mathscr{P}^2})\Lambda^{(\eta)}_{(\pm)a}(k^\perp)\psi_{ab}(k^\perp)\Lambda^{(\eta)}_{(\mp)b}(-k^\perp)$$
$$= \Lambda^{(\eta)}_{(\mp)a}(k^\perp)\left[\int \frac{d^3q^\perp}{(2\pi)^3} \underline{V}(k^\perp - q^\perp)\psi_{ab}(q^\perp)\right]\Lambda^{(\eta)}_{(\mp)b}(-k^\perp).$$

$$E_a(\mathbf{k}) = \sqrt{(m_a^0)^2 + \mathbf{k}^2} \simeq m_a^0 + \frac{1}{2}\frac{\mathbf{k}^2}{m_a^0},$$

$$\tan 2v = \frac{k}{m^0} \to 0, \qquad S(\mathbf{k}) \simeq 1, \qquad \Lambda_{(\pm)} \simeq \frac{1 \pm \gamma_0}{2}.$$

$$\Lambda_{(+)}\psi\Lambda_{(-)} \simeq \psi_{\rm Sch}, \qquad \Lambda_{(-)}\psi\Lambda_{(+)} \simeq 0,$$

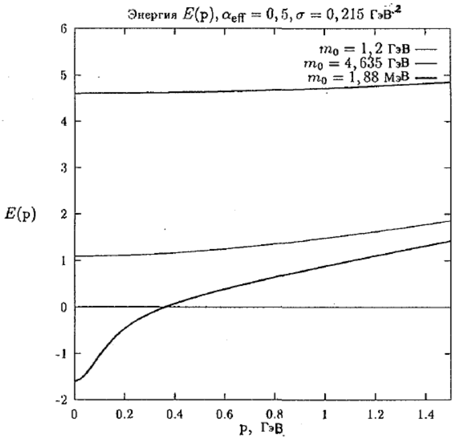
and finally the Schrödinger equation results in

$$\left[\frac{1}{2\mu}\mathbf{k}^{-2} + (m_a^0 + m_b^0 - M_A)\right]\psi_{\rm Sch}(\mathbf{k}) = \int \frac{d\mathbf{q}}{(2\pi)^3}\underline{V}(\mathbf{k}-\mathbf{q})\psi_{\rm Sch}(\mathbf{q}),$$

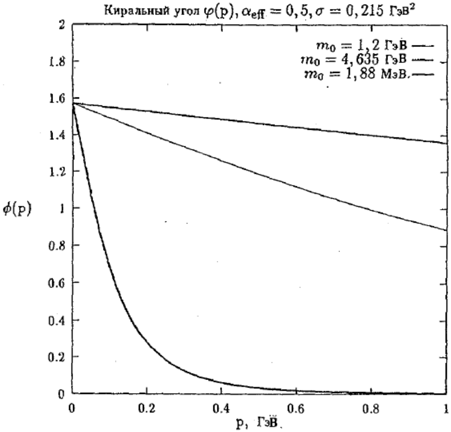
where $\mu = m_a \cdot m_b/(m_a + m_b)$. For an arbitrary total momentum $\mathscr{P}_\mu$,

$$\left[-\frac{1}{2\mu}(k_v^\perp)^{-2} + (m_a^0 + m_b^0 - \sqrt{\mathscr{P}^2})\right]\psi_{\rm Sch}(k^\perp) = \int \frac{d^3q^\perp}{(2\pi)^3}\underline{V}(k^\perp - q^\perp)\psi_{\rm Sch}(q^\perp),$$

Covariant bound states in effective QCD



Covariant bound states in effective QCD



Задачи

1. Моделирование и численный анализ поведения псевдоскалярных мезонов в плотной и горячей ядерной материи в зависимости от вида потенциала взаимодействия.
2. Моделирование и численный анализ поведения векторных мезонов в плотной и горячей ядерной материи в зависимости от вида потенциала взаимодействия.
3. Свойства J/ψ и Υ в в плотной и горячей ядерной материи.
4. Моделирование и численный анализ релятивистских уравнений для глюонных систем.
5. Моделирование и численный анализ релятивистских уравнении для многокварковых систем.
6. Моделирование распадов тяжелых кварконием в нелокальной эффективной теории.

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