Solving the Optimization Problem for Designing a Pulse Cryogenic Cell

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Solving the optimization problem for the characteristics of the thermal source of a cryogenic cell – a multilayer cylindrical sandwich-type configuration (see Fig. 1) designed for a pulsed dosed injection of the working substance into the ionization chamber of the source of multiply charged ions – is considered. To solve the optimization problem, we have developed the MPI+OpenMP hybrid parallel calculation algorithm based on the brute force method (see Fig. 2) to search for the maximum of the integral of proportionality to the volume of gas evaporated from the cell surface. The algorithm leads to multiple solutions of the initial boundary value problem for the heat equation, which is solved numerically by the alternating direction implicit method (ADI) (see Fig. 2). A method of simple iterations with an adaptive time-step is implemented to solve nonlinear difference equations. The solution of the optimization problem for a specific cell configuration on the GOVORUN supercomputer has demonstrated a ten- to hundredfold acceleration of calculations (see Fig. 3).

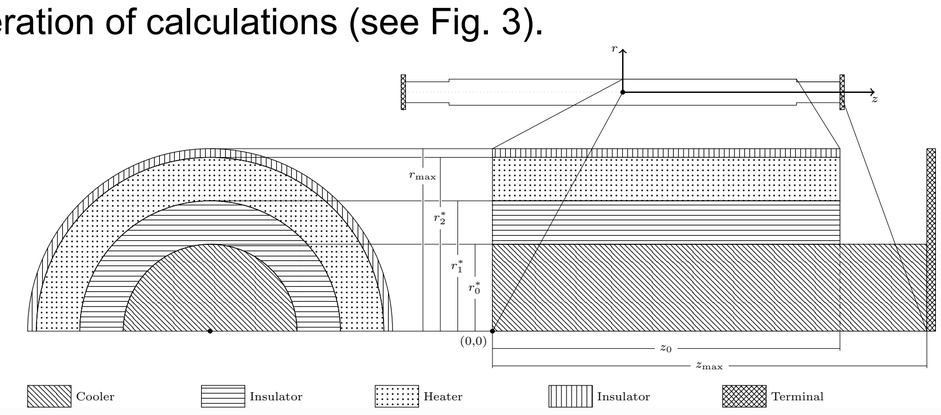

Fig. 1. A schematic view of a quarter of the cell slice through the axis and its cross-section. The bottom line is r=0. The cooler (the copper core rod) cools the cell by contact with the temperature terminal (liquid helium with 4.2 K). The heater (the conductive layer) heats the cell up by the way of passing the electrical current. The inner insulator is needed to prevent the electrical current outflow from the heater to the cooler. The heat process starts by passing the pulse electrical current through the conductive layer.

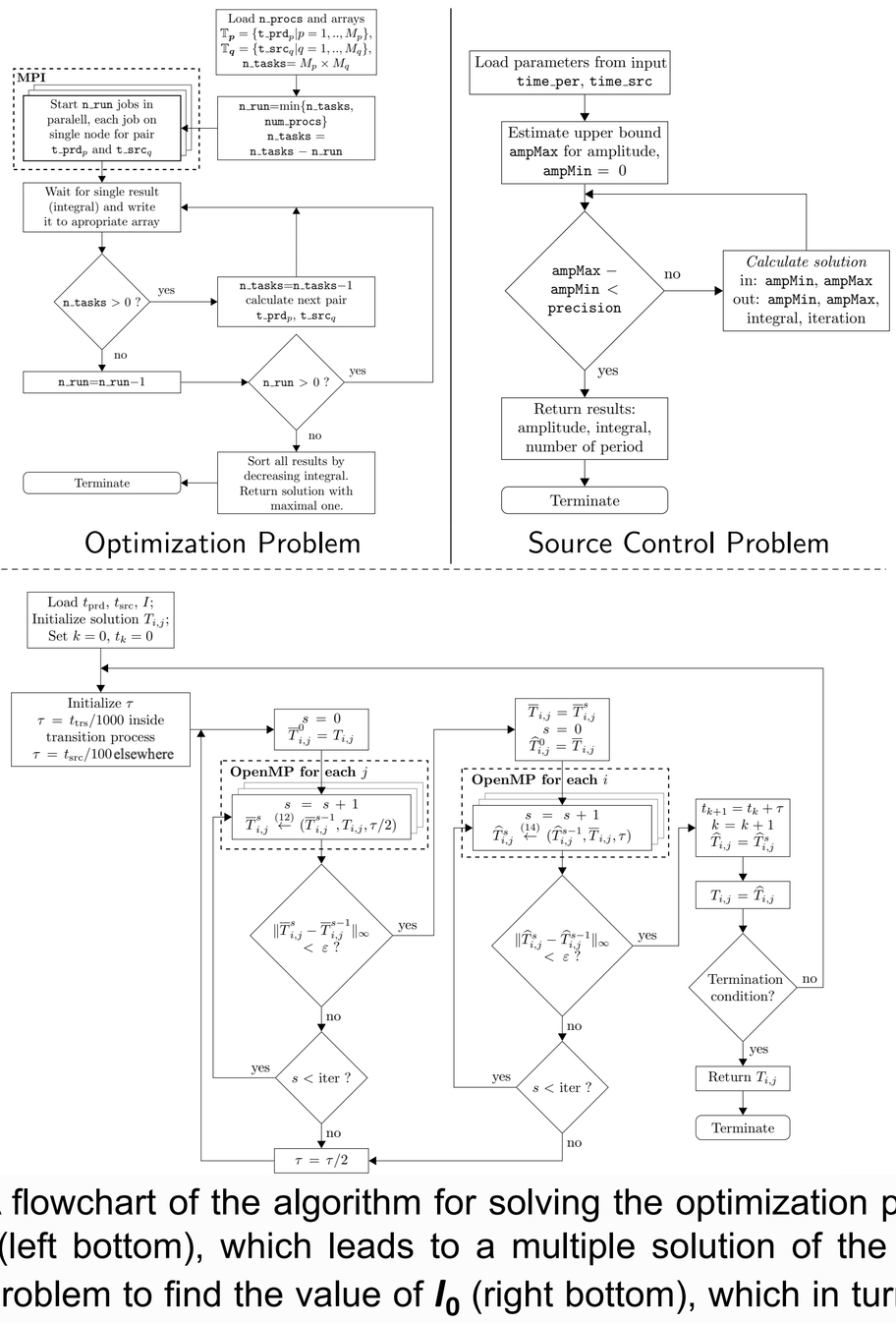

Fig. 2. A flowchart of the algorithm for solving the optimization problem on MPI (left bottom), which leads to a multiple solution of the source control problem to find the value of I_0 (right bottom), which in turn leads to a multiple solution of the direct problem on OpenMP (upper).

The optimization problem is formulated as following:

$$\{t_{\mathrm{prd}}, t_{\mathrm{src}}, I_0\} = \arg\max \left(\int_{n\,t_{\mathrm{prd}}}^{(n+1)\,t_{\mathrm{prd}}} \iint_S F(t, z; I_0, t_{\mathrm{prd}}, t_{\mathrm{src}}) \mathrm{d}s\,\mathrm{d}t \right),$$

here S is the surface area of the cylinder with the radius $r_{\max}$ and the length z_0, and the function $F(t, z)$ is expressed as above:

$$F(t, z) = \begin{cases} 0, & T(t, r = r_{\max}, z) < T^{\mathrm{c}}_{\max} \\ 0, & T(t, r = r_{\max}, z) > T_{\mathrm{limit}} \\ T(t, r = r_{\max}, z) - T^{\mathrm{c}}_{\max}, & \text{otherwise} \end{cases}$$

The heat conduction equation describing the considered heat evolution is the following

$$\rho(r) c_V(T, r) \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \lambda(T, r) \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(\lambda(T, r) \frac{\partial T}{\partial z} \right) + X(T, t, r)$$

$$X(T, t, r) = \chi(T, r) \frac{I^2}{S_C} \sum_{n=0}^{\infty} \left[\theta\left(t - n t_{\mathrm{per}}\right) - \theta\left(t - n t_{\mathrm{per}} - t_{\mathrm{src}}\right) \right],$$

here $n \in \mathbb{N}_0$, $\chi(T, r)$ – temperature depended resistivity, $I(r)$ – electrical current amplitude, S_C – cross-section of the conductive layer, $t_{\mathrm{per}} > t_{\mathrm{src}}$ – period of one operation and time of heating correspondingly, and $\theta(t)$ is the Heaviside step function

$$T(r, z, t = 0) = T_0,$$

where $T_0 \equiv 4.2\,\mathrm{K}$ is the temperature of liquid helium.

$$\begin{cases} \dfrac{\partial T}{\partial \mathbf{n}} = 0 & \forall (r, z) \in \delta\mathbf{\Omega} \setminus \{(r, z) : z = z_{\max}\}, \\ T = T_0 & \forall (r, z) \in \{(r, z) : z = z_{\max}\}, \end{cases}$$

where $\delta\mathbf{\Omega}$ is the boundary of $\mathbf{\Omega}$, $\mathbf{n}$ is the normal vector of the boundary $\delta\mathbf{\Omega}$.

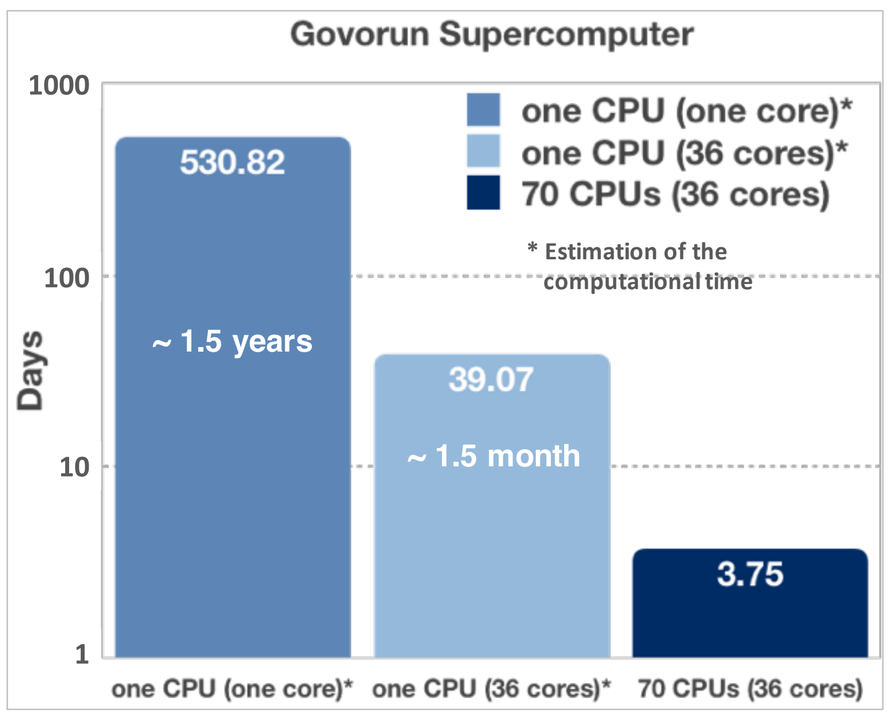

Fig. 3. The calculation time and its estimation. The right column corresponds to the time of solving the optimization problem; it is the time obtained for the master MPI process from the beginning of the program run to the result output. The middle column is the estimate of the calculation time of one Intel Xeon Gold 6154 (SkyLake) processor with 18 cores and multithreading enabled (36 threads), this estimate is obtained by summing the calculation time of slave MPI processes. The left column is the estimate obtained as a product of the average acceleration with the use of 36 OpenMP-threads by slave MPI processes and the estimate of the calculation time of one processor (middle column).

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