Optimization of the computation of multidimensional integrals for estimating the meson lifetime

D. Goderidze, Yu. L. Kalinovsky, A.V.Friesen

The XXV International Scientific Conference of Young Scientists and Specialists (AYSS-2021)
Almaty, Republic of Kazakhstan
October 15, 2021

Task

- Motivation: calculation of meson lifetime in hot and dense nuclear matter

- Task: calculation of 5-dimensional integrals with a compicated integrand by the Monte-Carlo method

- Goal: optimize the calculation of integrals, parallelization of calculations

Meson width (Gamma)

$$\Gamma = \frac{1}{2p_0}\left(\Sigma^{>}(p_0) - \Sigma^{<}(p_0)\right)$$

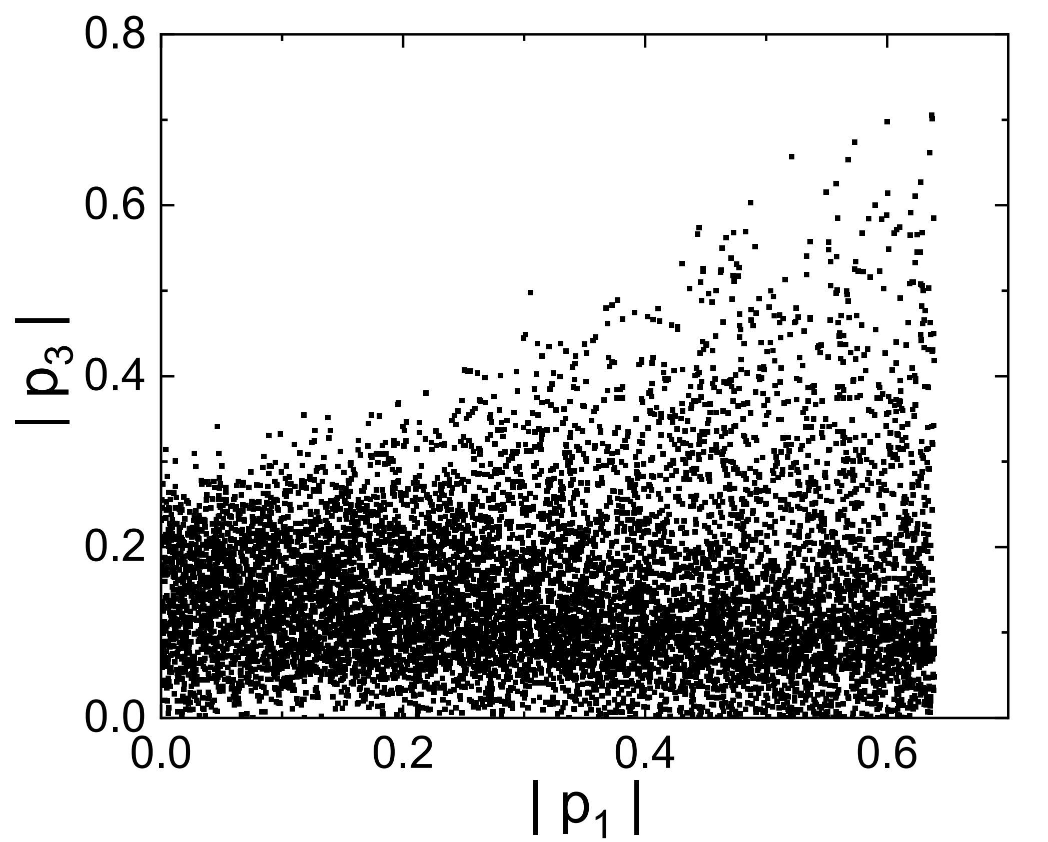
$$\Sigma^{>(<)}(p) = \frac{1}{(2\pi)^3}\int d\Omega \int \frac{\vec{p}_1}{2E_1}\frac{1}{8\pi}\int_{-1}^{1} d(cos\alpha)\frac{|\vec{p}_3|^2}{|\vec{p}_3|^2(\sqrt{s_2}+m_1)-|\vec{p}_1|E_3 cos\alpha}|A|^2 F^{>(<)}$$

$$d\Omega = \frac{1}{(2\pi)^3}\int ds_1\, A(s_1) ds_3 A(s_3) ds_4 A(s_4),$$ where $$A(s_i) = 2\frac{M_i\Gamma_i}{(s_i - M_i^2)^2 + M_i\Gamma_i^2}$$

$F^{>} = n_1\,(n_3+1)\,(n_4+1)$, $F^{<} = (1+n_1)\, n_3\, n_4$

$$|\vec{p_3}| = \frac{|\vec{p}_1|cos\alpha\, a \pm \sqrt{|\vec{p}_1|^2 cos^2\alpha\, a^2 + \left((\sqrt{s_2}+E_1)^2 - |\vec{p}_1|^2 cos^2\alpha\right)(a^2 - 4s_3(\sqrt{s_2}+E_1)^2)}}{2\left((\sqrt{s_2}+E_1)^2\right) - |\vec{p}_1|^2 cos^2\alpha}$$

D. Blaschke, M. K. Volkov, V. L. Yudichev: ar-Xiv:nucl-th/0303034v1



$$|\overrightarrow{p_3}| = \frac{|\vec{p}_1| cos\alpha\ a \pm \sqrt{|\vec{p}_1|^2 cos^2\alpha\ a^2 + \left((\sqrt{s_2}+E_1)^2 - |\vec{p}_1|^2 cos^2\alpha\right)(a^2 - 4s_3(\sqrt{s_2}+E_1)^2)}}{2\left((\sqrt{s_2}+E_1)^2\right) - |\vec{p}_1|^2 cos^2\alpha}$$

D. Blaschke, M. K. Volkov, V. L. Yudichev: ar-Xiv:nucl-th/0303034v1

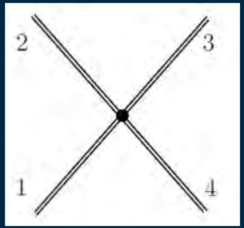
Amplitude

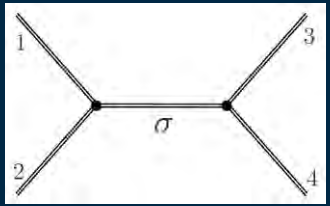
$$A_0 = 3A + B + C$$

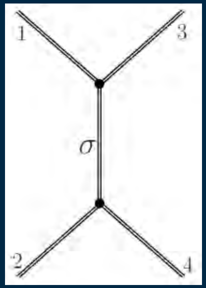
$$A_1 = B - C$$

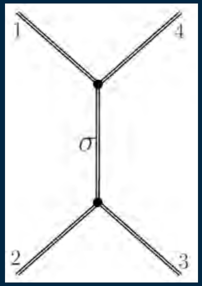
$$A_2 = B + C$$

$$\langle cp_c; dp_d | T | ap_a; bp_b \rangle = \delta_{ab}\delta_{cd}A(s,t,u) + \delta_{ac}\delta_{bd}B(s,t,u) + \delta_{ad}\delta_{bc}C(s,t,u)$$









$$\pi^{\pm}\pi^{\pm} \to \pi^{\pm}\pi^{\pm} = A_2$$

$$\pi^{\mp}\pi^{\pm} \to \pi^{\mp}\pi^{\pm} = \frac{1}{6}A_2 + \frac{1}{2}A_1 + \frac{1}{3}A_0$$

$$\pi^{\pm}\pi^{0} \to \pi^{\pm}\pi^{0} = \frac{1}{2}A_2 + \frac{1}{2}A_1$$

$$\pi^{\pm}\pi^{\mp} \to \pi^{0}\pi^{0} = \frac{1}{3}A_2 - \frac{1}{3}A_0$$

$$\pi^{0}\pi^{0} \to \pi^{0}\pi^{0} = \frac{2}{3}A_2 + \frac{1}{3}A_0$$

$$\mathrm{A} = (\pi^0\pi^0 \to \pi^0\pi^0) + (\pi^{\pm}\pi^{\mp} \to \pi^0\pi^0) + + 2(\pi^{\pm}\pi^0 \to \pi^{\pm}\pi^0)$$

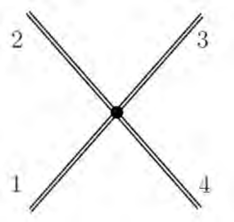
Amplitude

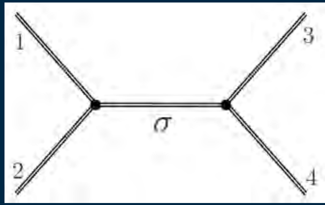
$$A_0 = 3A + B + C$$

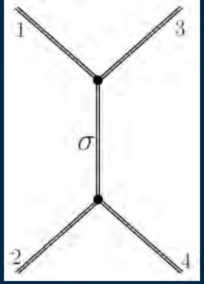
$$A_1 = B - C$$

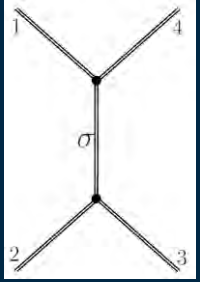
$$A_2 = B + C$$

$$\langle cp_c; dp_d|T|ap_a; bp_b\rangle = \delta_{ab}\delta_{cd}A(s,t,u) + \delta_{ac}\delta_{bd}B(s,t,u) + \delta_{ad}\delta_{bc}C(s,t,u)$$









$$\pi^{\pm}\pi^{\pm} \to \pi^{\pm}\pi^{\pm} = A_2$$

$$\pi^{\mp}\pi^{\pm} \to \pi^{\mp}\pi^{\pm} = \frac{1}{6}A_2 + \frac{1}{2}A_1 + \frac{1}{3}A_0$$

$$\pi^{\pm}\pi^{0} \to \pi^{\pm}\pi^{0} = \frac{1}{2}A_2 + \frac{1}{2}A_1$$

$$\pi^{\pm}\pi^{\mp} \to \pi^{0}\pi^{0} = \frac{1}{3}A_2 - \frac{1}{3}A_0$$

$$\pi^{0}\pi^{0} \to \pi^{0}\pi^{0} = \frac{2}{3}A_2 + \frac{1}{3}A_0$$

$$\mathbf{A} = (\pi^0\pi^0 \to \pi^0\pi^0) + (\pi^{\pm}\pi^{\mp} \to \pi^0\pi^0) + 2(\pi^{\pm}\pi^0 \to \pi^{\pm}\pi^0)$$

Optimization of integral

1. General parts of integral

$$\Gamma = \frac{1}{2p_0}\left(\Sigma^{>}(p_0) - \Sigma^{<}(p_0)\right)$$

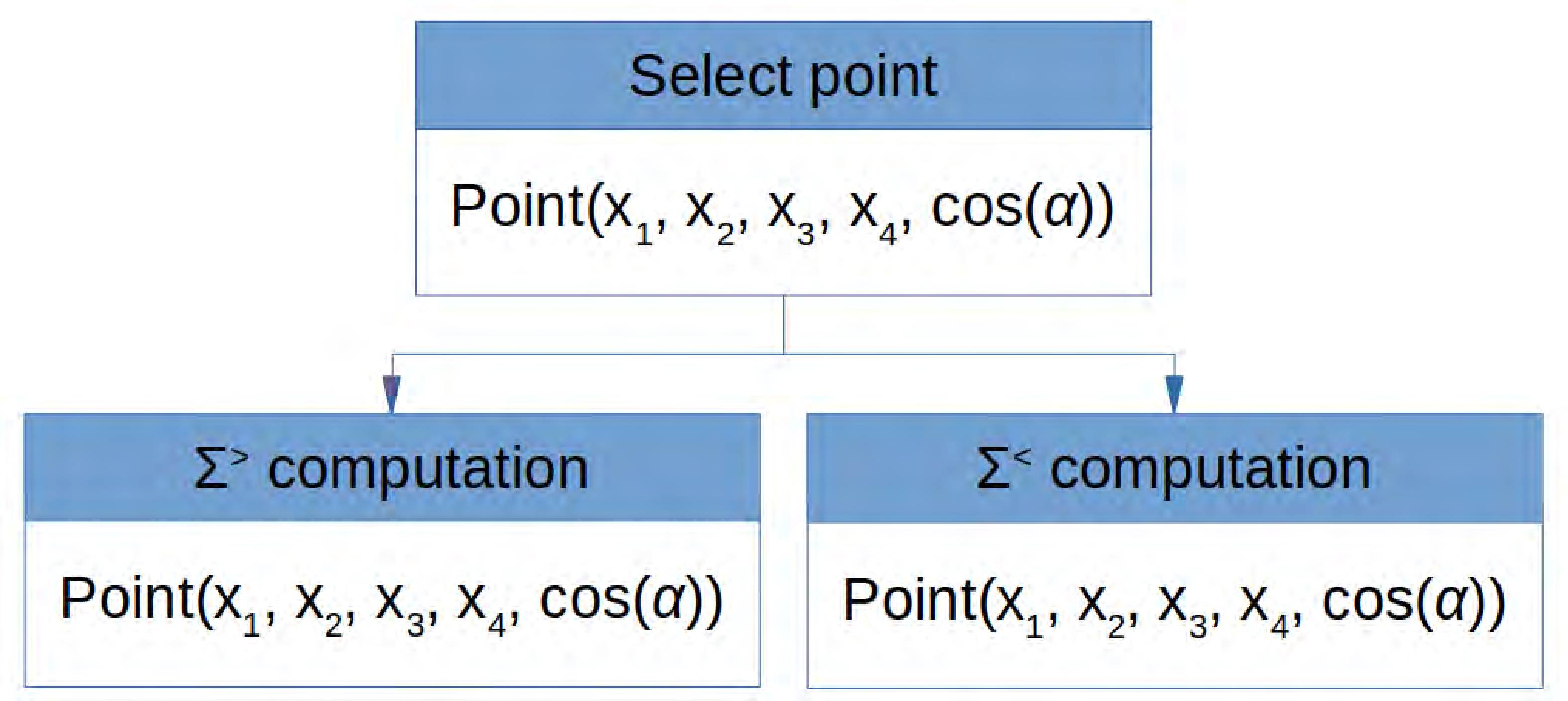
$$\Sigma^{>(<)}(p) = \frac{1}{(2\pi)^3}\int d\Omega \int \frac{\vec{p}_1}{2E_1}\frac{1}{8\pi}\int_{-1}^{1} d(\cos\alpha)\frac{|\vec{p}_3|^2}{|\vec{p}_3|^2(\sqrt{s_2}+m_1)-|\vec{p}_1|E_3\cos\alpha}|A|^2 F^{>(<)}$$

$$d\Omega = \frac{1}{(2\pi)^3}\int ds_1\, A(s_1)ds_3 A(s_3)ds_4 A(s_4),$$ where $$A(s) = 2\frac{M_i\Gamma_i}{(s_i-M_i^2)^2+M_i\Gamma_i^2}$$

$F^{>} = n_1\,(n_3+1)\,(n_4+1)$, $F^{<}=(1+n_1)\,n_3\,n_4$

$$|\vec{p}_3| = \frac{|\vec{p}_1|\cos\alpha\, a \pm \sqrt{|\vec{p}_1|^2\cos^2\alpha\, a^2 + \left((\sqrt{s_2}+E_1)^2 - |\vec{p}_1|^2\cos^2\alpha\right)(a^2-4s_3(\sqrt{s_2}+E_1)^2)}}{2\left((\sqrt{s_2}+E_1)^2\right)-|\vec{p}_1|^2\cos^2\alpha}$$

1. General parts of integral



2. $\int_{-1}^{1} d(cos\alpha)$

$$\Gamma = \frac{1}{2p_0}\left(\Sigma^{>}(p_0) - \Sigma^{<}(p_0)\right)$$

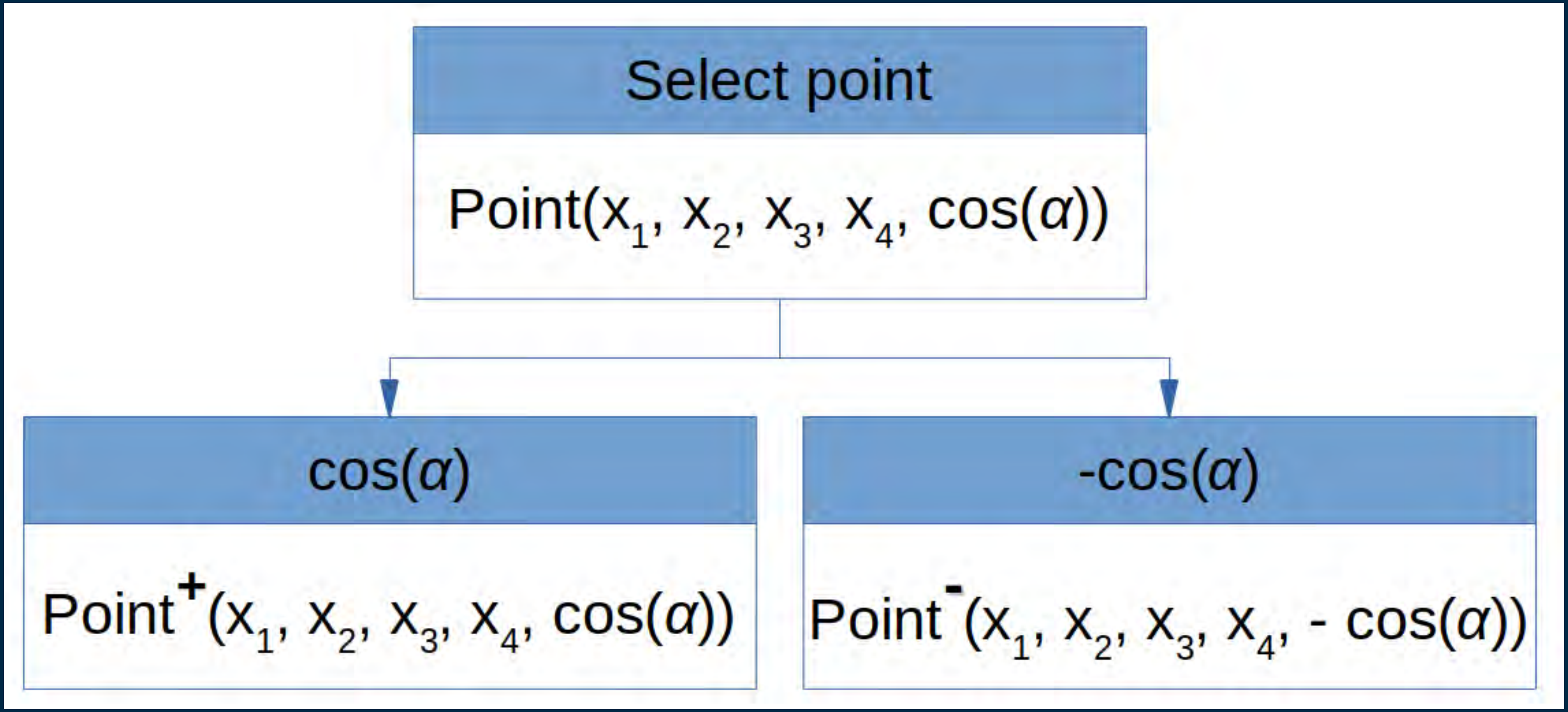
$$\Sigma^{>(<)}(p) = \frac{1}{(2\pi)^3}\int d\Omega \int \frac{\vec{p}_1}{2E_1}\frac{1}{8\pi}\boldsymbol{\int_{-1}^{1} d(cos\alpha)\frac{|\vec{p}_3|^2}{|\vec{p}_3|^2(\sqrt{s_2}+m_1)-|\vec{p}_1|E_3 cos\alpha}}|A|^2 F^{>(<)}$$

$$d\Omega = \frac{1}{(2\pi)^3}\int ds_1\, A(s_1) ds_3 A(s_3) ds_4 A(s_4),$$ where $A(s) = 2\frac{M_i\Gamma_i}{(s_i-M_i^2)^2+M_i\Gamma_i^2}$

$F^{>} = n_1\,(n_3+1)\,(n_4+1),$ $F^{<} = (1+n_1)\,n_3\,n_4$

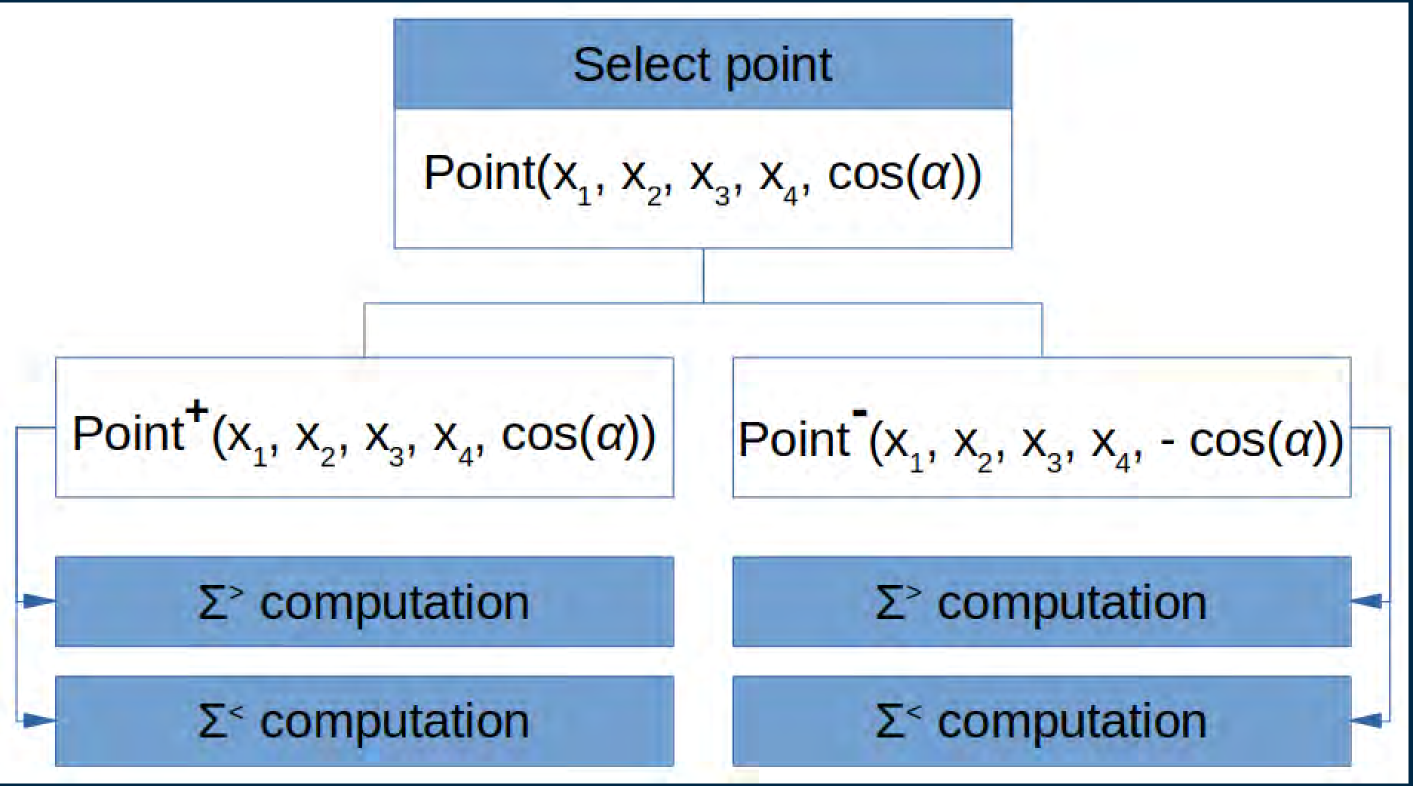
$$|\vec{p}_3| = \frac{|\vec{p}_1| cos\alpha\, a \pm \sqrt{|\vec{p}_1|^2 cos^2\alpha\, a^2 + \left((\sqrt{s_2}+E_1)^2 - |\vec{p}_1|^2 cos^2\alpha\right)(a^2 - 4s_3(\sqrt{s_2}+E_1)^2)}}{2\left((\sqrt{s_2}+E_1)^2\right) - |\vec{p}_1|^2 cos^2\alpha}$$

2. $\int_{-1}^{1} d(cos\alpha)$



Union

Both methods reduce the number of the generated points and the total number of the required calculations by 4 times, thus accelerating the computation process.



Decreasing of computation time

Step of increasing the number of points

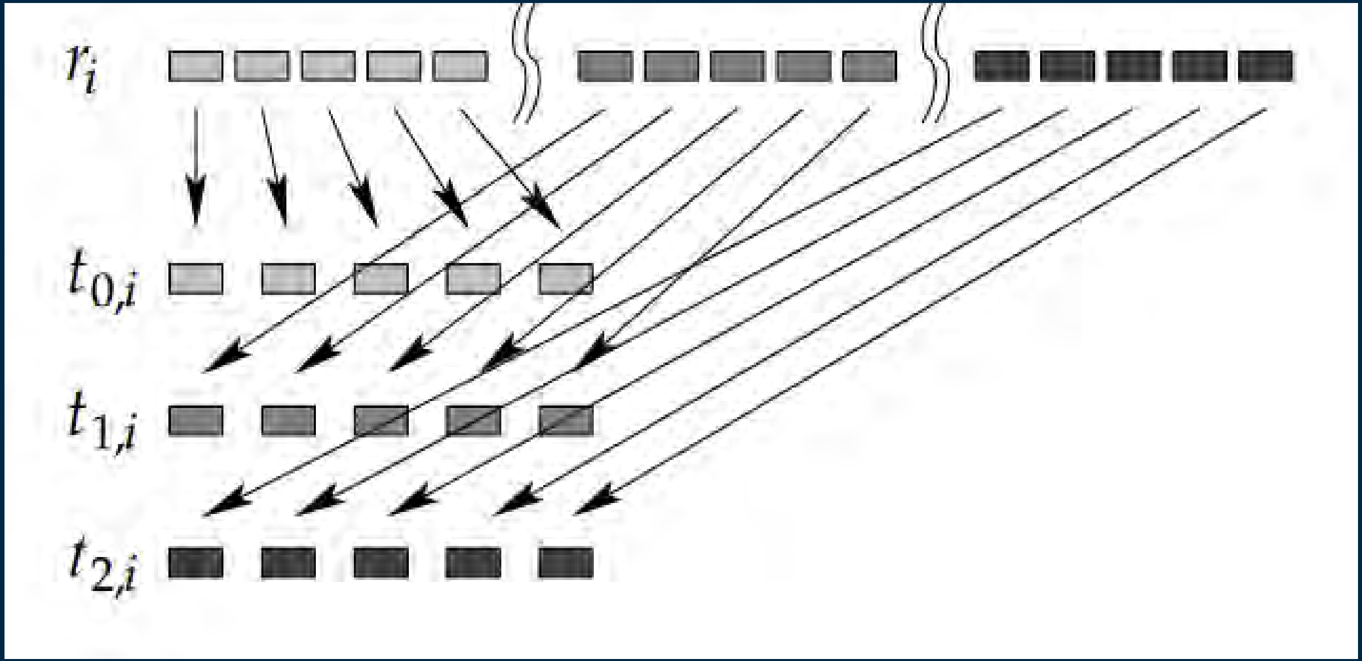
$\sum_N$ – sum of integral with calculating N points

$\sum_M$ – sum of integral with calculating M points

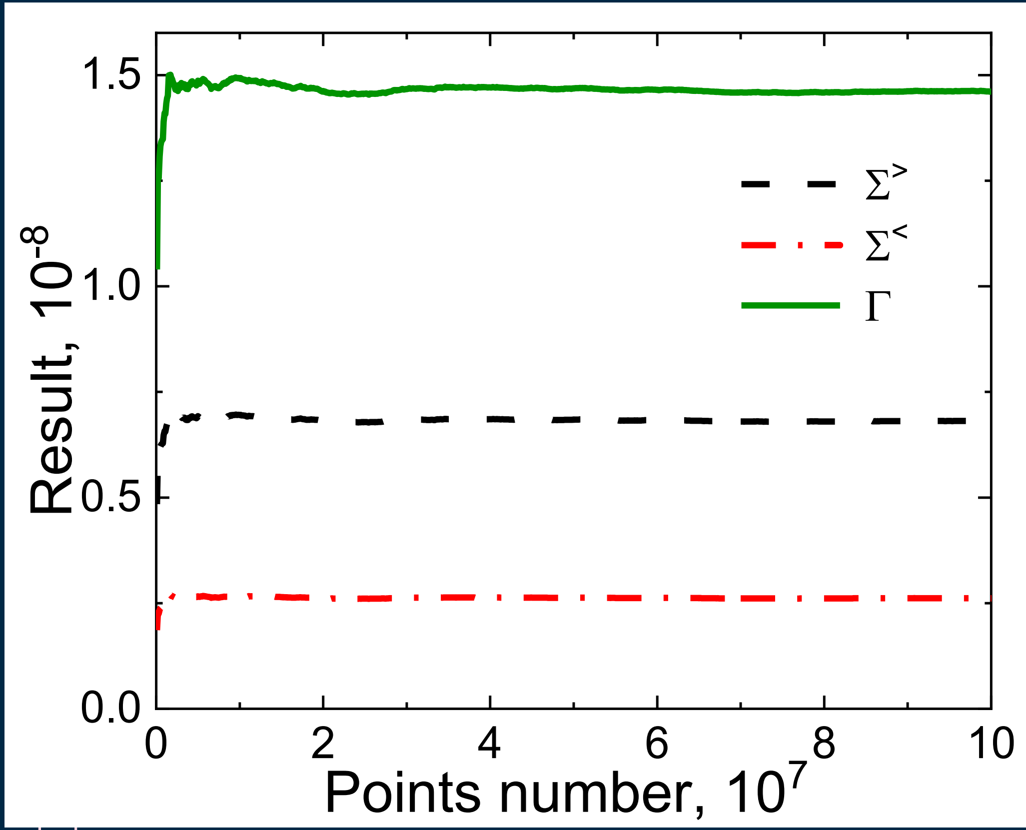
$$\int_a^b f(x)dx = \frac{(b-a)}{(N+M)}\left(\sum_{i=1}^{N} f(x_i) + \sum_{j=1}^{M} f(x_j)\right)$$

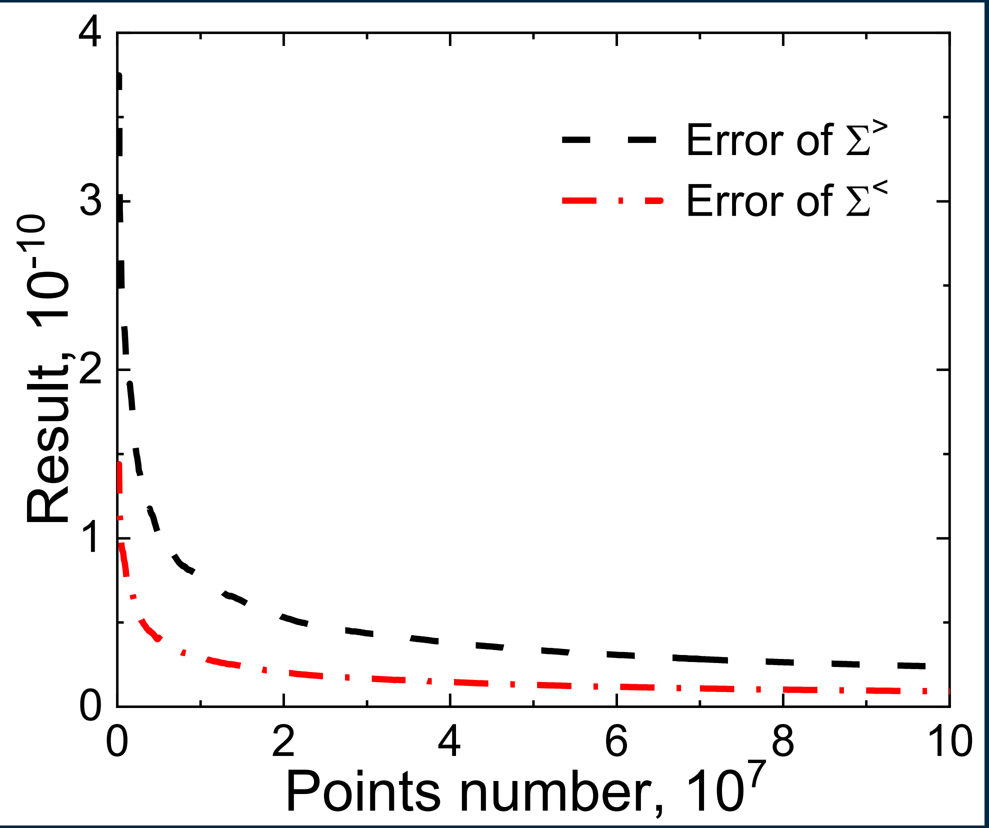
Tina’s Random Number Generator

Dividing row into blocks for threads

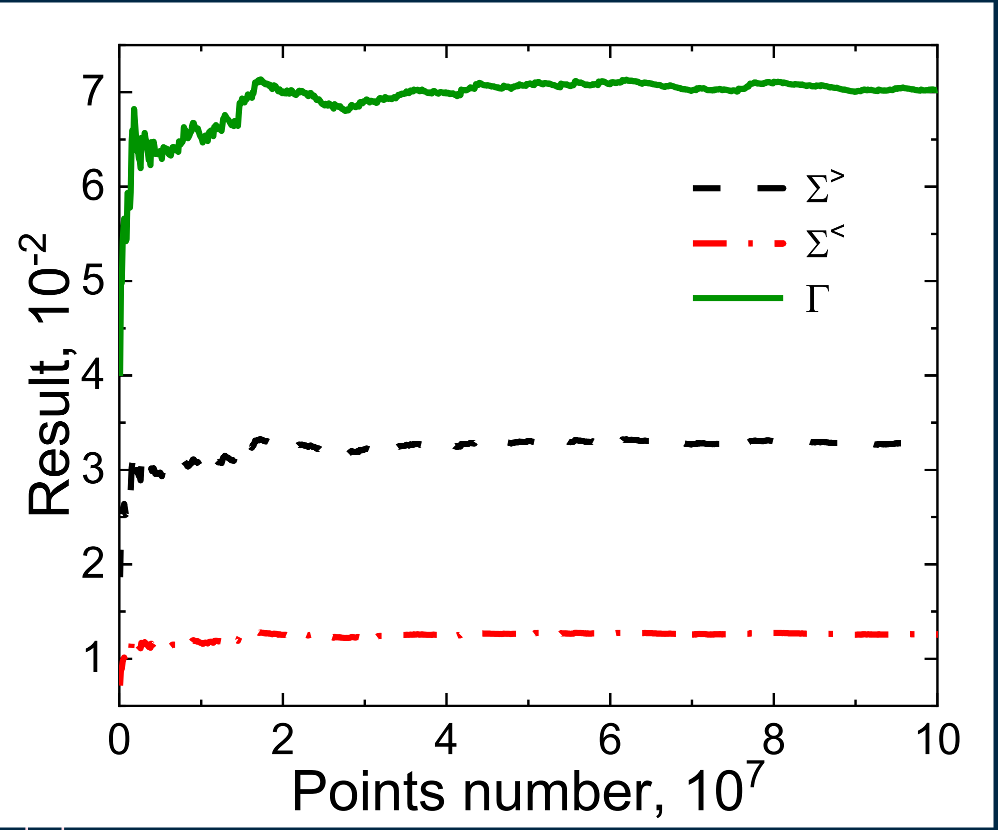


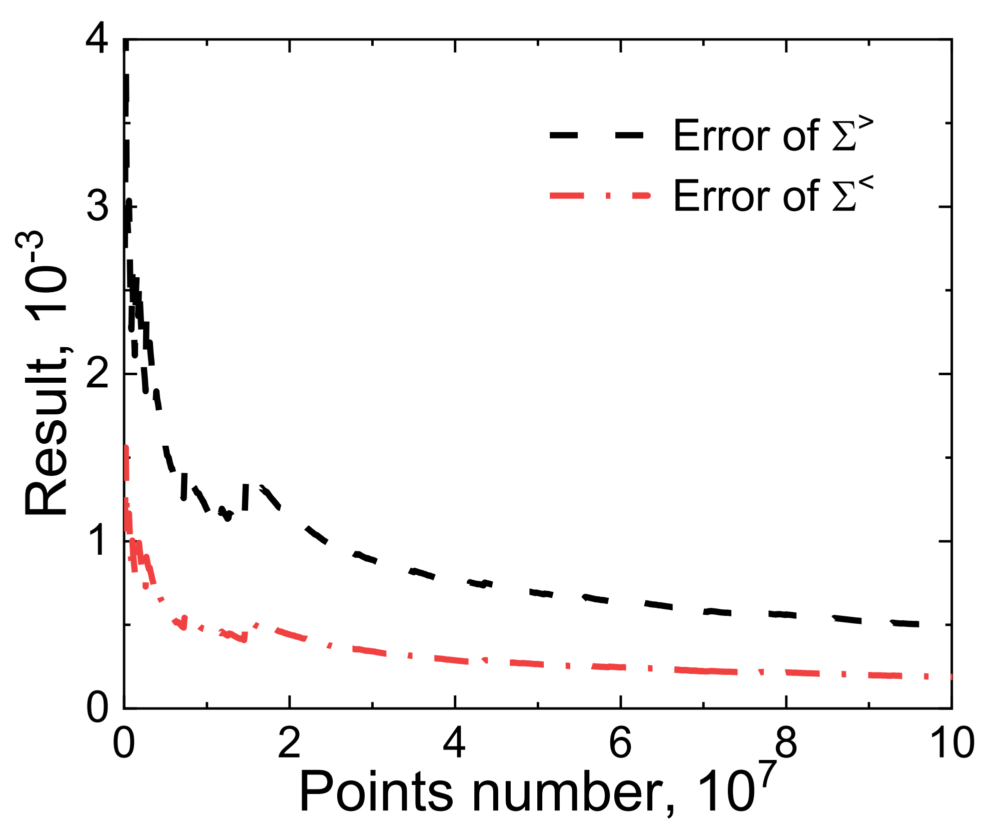
Comparing error upper bound with constant temperature without amplitude. T = 0.15 GeV



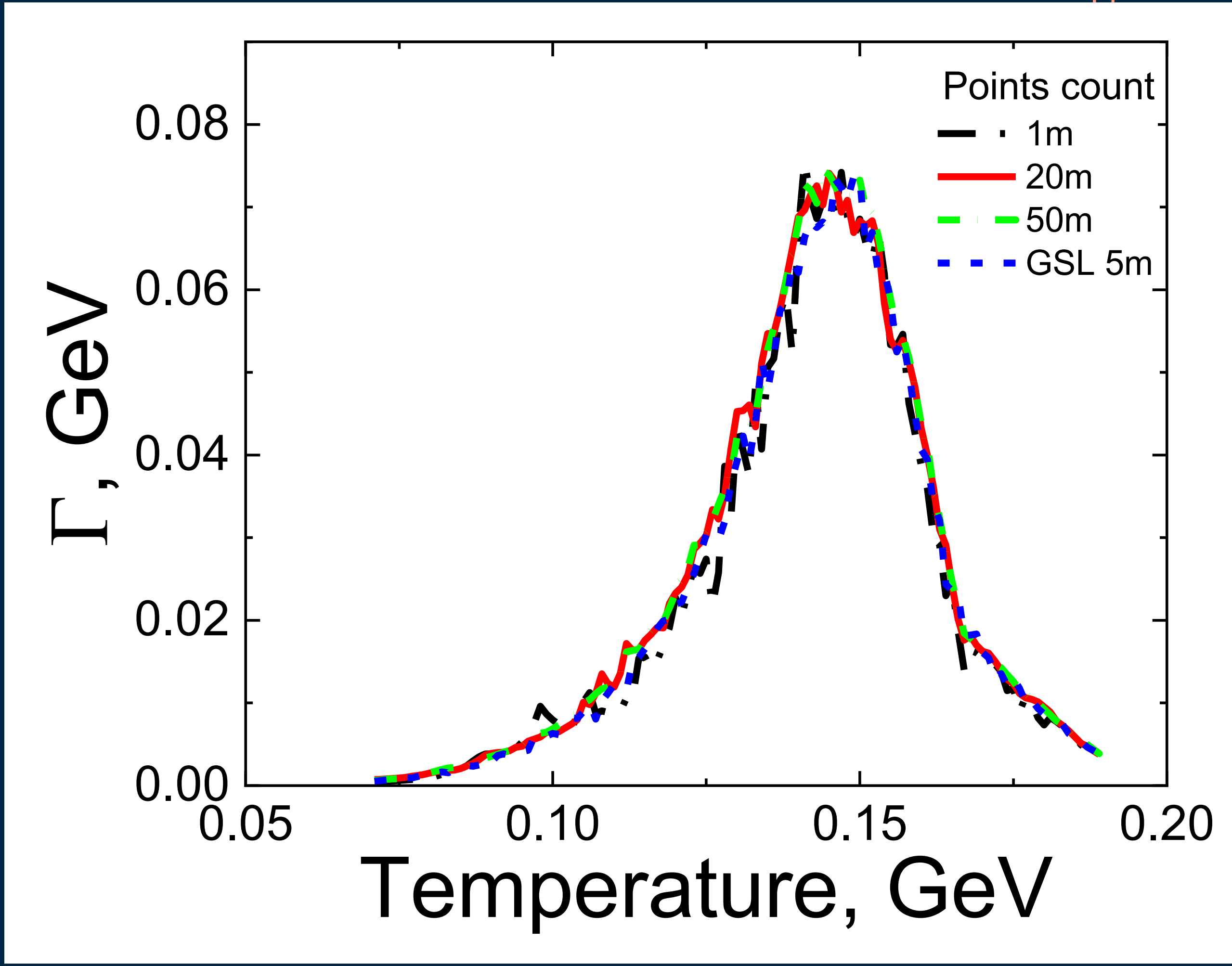


Comparing upper error bound with constant temperature with adding amplitude. T = 0.15 GeV

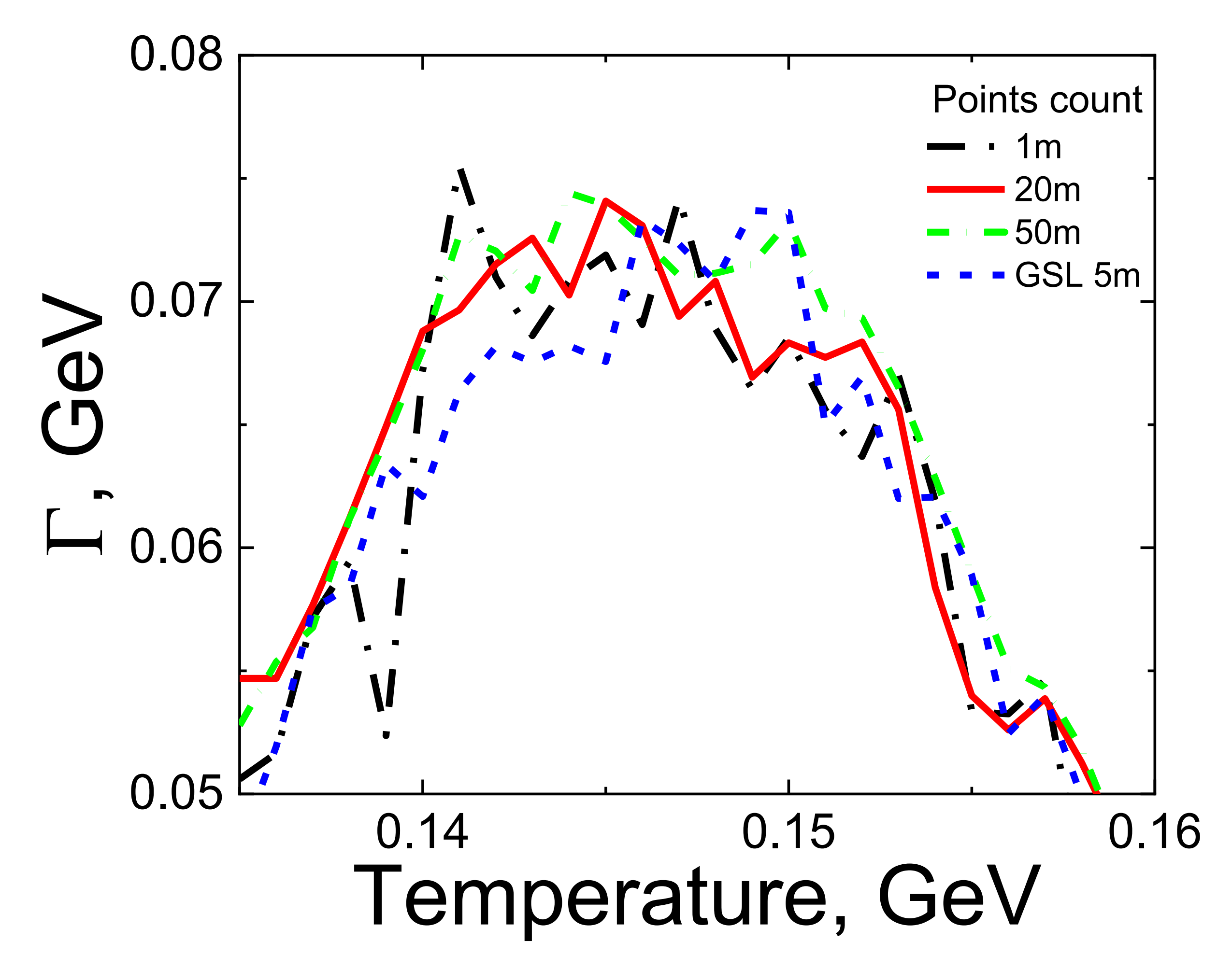




Mesons width



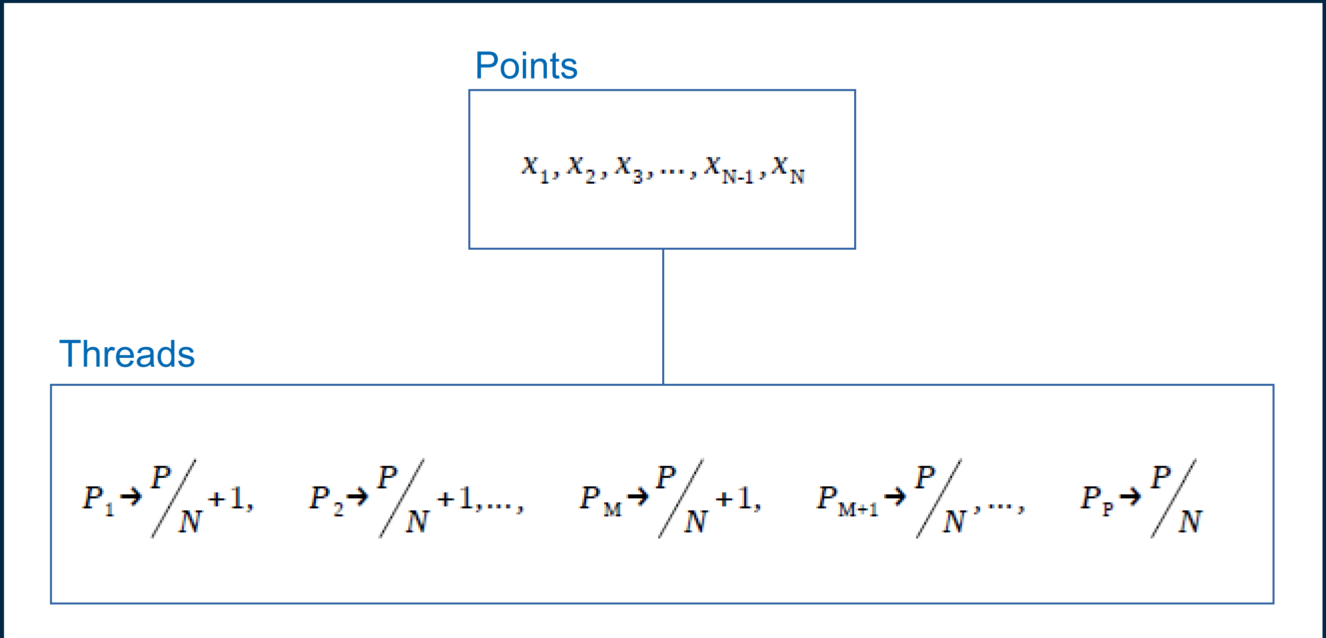
Mesons width



Realization

- Single
- OpenMP
- NVIDIA CUDA
- CUDA + OpenMP

Parallelization



Computation time

	None	OpenMP			CUDA			CUDA+OpenMP		
Threads	-	4	32	64	8/64	16/128	32/256	8/64+4	16/128+16	32/256+32
Time, sec	42	8.95	2.09	1.89	3.21	0.83	0.51	2.01	0.79	0.50

The calculations performed on the HybriLIT heterogeneous platform of the Meshcheryakov Laboratory of Information Technologies, JINR, Dubna.

CPU: Intel Xeon Phi
GPU: Tesla K40

Summary

- Our task consisted of solving 5-dimension integral
- We studied, how to solve this task with Monte-Carlo integration method
- For computation it was necessary to generate many points. We realized, that it would took a long time to get answer
- Optimized integral and used parallel technologies to accelerate calculations
- Used HybriLIT heterogeneous platform to reduce calculation time

Thank you for attention!