Моделирование физических процессов в плотной и горячей ядерной среде

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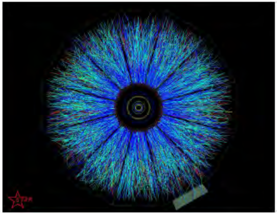
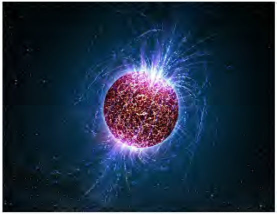
Introduction

Why is finite T/μ physics ($\approx$ QCD) interesting?

High energy physics applications

- Heavy ion experiments $\Rightarrow$ Need quantitive understanding of non-Abelian plasmas at
  - High T and small/moderate μ
  - Moderately large couplings
  - In and (especially) out of equilibrium
- Early universe thermodynamics
  - Signatures of phase transitions
  - EW baryogenesis
- Neutron star interiors
  - EoS at high μ and $T \simeq 0$
  - Transport in nucl. matter





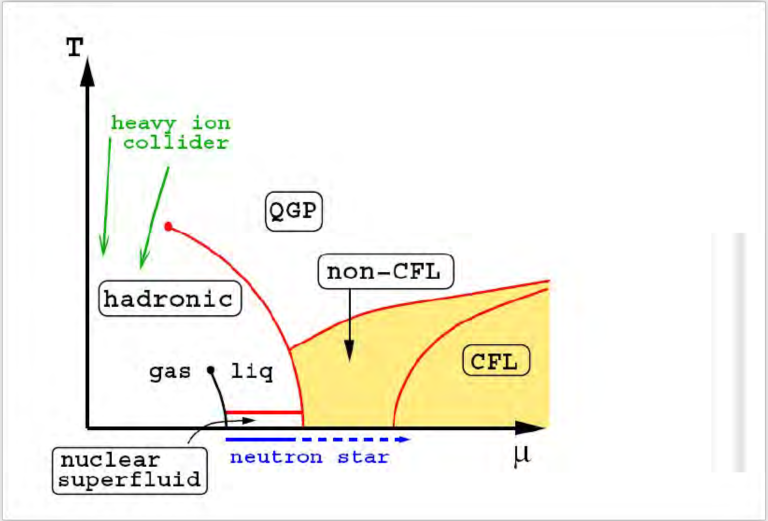
Introduction

Challenges in thermal QCD

To understand heavy ion experiments / early universe thermodynamics want to know (among other things):

- Structure of QCD phase diagram: Phase structure, location of transition lines, critical points,...
- Properties of phase transitions, in particular the deconfinement transition
- Equation of state and other equilibrium quantities

Introduction



The model

The NJL model [1]

We will use a two flavor model, with a degenerate mass matrix for quarks.

The associated Lagrangian reads:

$$\mathrm{L_{NJL}} = \bar{\mathrm{q}}(\mathrm{i}\gamma^{\mu}\partial_{\mu} - \hat{\mathrm{m}})\mathrm{q} + \mathrm{G_1}\left[(\bar{\mathrm{q}}\mathrm{q})^2 + (\bar{\mathrm{q}}\mathrm{i}\gamma_5\vec{\tau}\mathrm{q})^2\right]$$

$\bar{\mathrm{q}} = (\bar{\mathrm{u}}, \bar{\mathrm{d}})$, $\hat{\mathrm{m}} = \mathrm{diag}(\mathrm{m_u}, \mathrm{m_d})$, with $\mathrm{m_u} = \mathrm{m_d} \equiv \mathrm{m_0}$; τ^{a} $(\mathrm{a} = 1, 2, 3)$ - $\mathrm{SU_f}(2)$ Pauli matrices acting in flavor space.

This Lagrangian is invariant under a global color symmetry $\mathrm{SU}(\mathrm{N_c} = 3)$. It also satisfies the chiral $\mathrm{SU_L}(2) \times \mathrm{SU_R}(2)$ symmetry if $\hat{\mathrm{m}} = 0$ while $\hat{\mathrm{m}} \neq 0$ implies an explicit (but small) chiral symmetry breaking from $\mathrm{SU_L}(2) \times \mathrm{SU_R}(2)$ to $\mathrm{SU_f}(2)$.

[1] M.K. Volkov, Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961). S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992). T. Hatsuda and T. Kunihiro, Phys. Rep. 247, 221 (1994).

The model

The parameters are usually fixed to reproduce the mass and decay constant of the pion as well as the chiral condensate.
The Hartree quark mass (or constituent quark mass) is m = 325 MeV and the pion decay constant and mass are obtained within a Hartree + RPA calculation.

Λ [GeV]	G_1 [GeV^{-2}]	m_0 [MeV]
0.651	5.04	5.5

m MeV	f_π MeV	m_π MeV
251	92.4	139.5

Pure gauge sector

The deconfinement phase transition in a pure $\mathrm{SU(N_c)}$ gauge theory. It can be conveniently described through the introduction of an effective potential for the complex Polyakov loop field.

Since we want to study the $\mathrm{SU(N_c)}$ phase structure, first of all an appropriate order parameter has to be defined.

The Polyakov line

$$\mathrm{L}\left(\vec{\mathrm{x}}\right) \equiv \mathcal{P}\exp\left[\mathrm{i}\int_0^\beta \mathrm{d}\tau\, \mathrm{A}_4\left(\vec{\mathrm{x}},\tau\right)\right]$$

$\mathrm{A_4 = iA^0}$ is the temporal component of the Euclidean gauge field $(\vec{\mathrm{A}}, \mathrm{A_4})$, in which the strong coupling constant $\mathrm{g_S}$ has been absorbed. $\mathcal{P}$ denotes path ordering and the usual notation $\beta = 1/\mathrm{T}$ has been introduced with the Boltzmann constant set to one ($\mathrm{k_B} = 1$).

Pure gauge sector

When the theory is regularized on the lattice, the Polyakov loop,

$$\mathrm{l}(\vec{\mathrm{x}}) = \frac{1}{\mathrm{N_c}} \mathrm{Tr} \mathrm{L}(\vec{\mathrm{x}}),$$

is a color singlet under $\mathrm{SU(N_c)}$, but transforms non-trivially, like a field of charge one, under $\mathrm{Z_{N_c}}$. Its thermal expectation value is then chosen as an order parameter for the deconfinement phase transition.
In fact, in the usual physical interpretation, $< \mathrm{l}(\vec{\mathrm{x}}) >$ is related to the change of free energy occurring when a heavy color source in the fundamental representation is added to the system.

$$< \mathrm{l}(\vec{\mathrm{x}}) >= \mathrm{e}^{-\beta \Delta \mathrm{F_Q}(\vec{\mathrm{x}})}.$$

Pure gauge sector

In the Z_{N_c} symmetric phase, $\langle l(\vec{x})\rangle = 0$, implying that an infinite amount of free energy is required to add an isolated heavy quark to the system: in this phase color is confined.

Phase transitions are usually characterized by large correlation lengths, i.e. much larger than the average distance between the elementary degrees of freedom of the system.

In particular, in the usual Landau-Ginzburg approach, the order parameter is viewed as a field variable and for the latter an effective potential is built, respecting the symmetries of the original lagrangian. In the case of the SU(3) gauge theory, the Polyakov line $L(\vec{x})$ gets replaced by its gauge covariant average over a finite region of space, denoted as $\langle\langle L(\vec{x})\rangle\rangle$.

The Polyakov loop field:

$$\Phi(\vec{x}) \equiv \langle\langle l(\vec{x})\rangle\rangle = \frac{1}{N_c}\,\mathrm{Tr}_c\,\langle\langle L(\vec{x})\rangle\rangle$$

Pure gauge sector

Effective potential for the (complex) Φ field, which is conveniently chosen to reproduce, at the mean field level, results obtained in lattice calculations.
In this approximation one simply sets the Polyakov loop field $\Phi(\vec{x})$ equal to its expectation value Φ =const., which minimizes the potential

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{b_2\left(T\right)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2 \ ,$$

where

$$b_2\left(T\right) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 .$$

A precision fit of the coefficients a_i, b_i has been performed to reproduce some pure-gauge lattice data.

Pure gauge sector

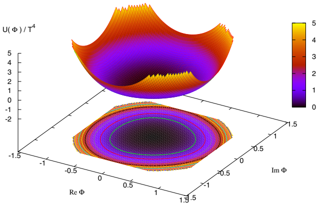
These parameters have been fixed to reproduce the lattice data for both the expectation value of the Polyakov loop and some thermodynamic quantities.
The parameter T_0 is the critical temperature for the deconfinement phase transition, fixed to 270 MeV according to pure gauge lattice findings. The range of applicability of our model is limited to temperatures $T \leq 2.5\ T_c$ and for these temperatures there is good agreement with lattice data for Φ.

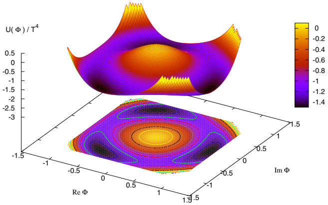
a_0	a_1	a_2	a_3	b_3	b_4
6.75	-1.95	2.625	-7.44	0.75	7.5

Pure gauge sector

$\mathrm{T} = 0.26$ GeV$< \mathrm{T}_0$
"Color confinement"
$\langle \Phi \rangle = 0 \longrightarrow$ No breaking of $\mathbb{Z}_3$

$\mathrm{T} = 1$ GeV $> \mathrm{T}_0$
"Color deconfinement"
$\langle \Phi \rangle \neq 0 \longrightarrow$ breaking of $\mathbb{Z}_3$





H. Hansen, W. M. Alberico, A. Beraudo, A. Molinari, M. Nardi, C. Ratti, Phys.Rev.D75:065004,2007

Coupling between quarks and the gauge sector: the PNJL model

In the presence of dynamical quarks the Z_3 symmetry is explicitly broken.
The PNJL model attempts to describe in a simple way the two characteristic phenomena of QCD, namely deconfinement and chiral symmetry breaking.
In order to describe the coupling of quarks to the chiral condensate: an NJL description of quarks (global $SU_c(3)$ symmetric point-like interaction), coupled in a minimal way to the Polyakov loop, via the following Lagrangian

$$L_{PNJL} = \bar{q}\left(i\gamma_\mu D^\mu - \hat{m}_0\right) q + G_1 \left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] - \mathcal{U}\left(\Phi[A], \bar{\Phi}[A]; T\right),$$

where the covariant derivative reads $D^\mu = \partial^\mu - iA^\mu$ and $A^\mu = \delta^\mu_0 A^0$ (Polyakov gauge), with $A^0 = -iA_4$. $A^\mu(x) = g_S \mathcal{A}^\mu_a(x)\frac{\lambda_a}{2}$, $\mathcal{A}^\mu_a$ is the gauge field ($SU_c(3)$) and λ_a are the Gell–Mann matrices.

$$L'_{PNJL} = L_{PNJL} + \mu\bar{q}\gamma^0 q$$

Field equations

Hartree approximation
One defines the vertices Γ_{M}, where $\mathrm{M} = \{\mathrm{S}, \mathrm{P}\}$, in the scalar ($\Gamma_{\mathrm{S}} \equiv \mathbb{I}$) and pseudoscalar ($\Gamma_{\mathrm{P}}^{\mathrm{a}} \equiv \mathrm{i}\gamma_5\tau^{\mathrm{a}}$) channel.

$$\mathrm{S}_0(\mathrm{p}) = \frac{1}{\not{\mathrm{p}} - \mathrm{m}_0 + \gamma^0(\mu - \mathrm{i}\mathrm{A}_4)},$$

$$\mathrm{S}(\mathrm{p}) = \frac{1}{\not{\mathrm{p}} - \mathrm{m} + \gamma^0(\mu - \mathrm{i}\mathrm{A}_4)},$$

The Hartree equation then reads:

$$\mathrm{m} - \mathrm{m}_0 = 2\mathrm{G}_1\mathrm{T}\ \mathrm{Tr} \sum_{\mathrm{n}=-\infty}^{+\infty} \int_{\Lambda} \frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3} \frac{-1}{\not{\mathrm{p}} - \mathrm{m} + \gamma^0(\mu - \mathrm{i}\mathrm{A}_4)}$$

Field equations

$p_0 = i\omega_n$ and $\omega_n = (2n+1)\pi T$ is the Matsubara frequency for a fermion; the trace is taken over color, Dirac and flavor indices.
The symbol $\int_\Lambda$ denotes the three dimensional momentum regularisation; we use an ultraviolet cut-off Λ for both the zero and the finite temperature contributions.

$$m - m_0 = 2G_1 N_f \sum_{i=1}^{N_c} \int_\Lambda \frac{d^3p}{(2\pi)^3} \frac{2m}{E_p} [1 - f(E_p - \mu + iA_4^{ii}) - f(E_p + \mu - iA_4^{ii})] \,.$$

By introducing the modified distribution functions f_Φ^+ and f_Φ^-

$$f_\Phi^+(E_p) = \frac{\left(\Phi + 2\bar{\Phi} e^{-(E_p-\mu)/T}\right) e^{-(E_p-\mu)/T} + e^{-3(E_p-\mu)/T}}{1 + 3\left(\Phi + \bar{\Phi} e^{-(E_p-\mu)/T}\right) e^{-(E_p-\mu)/T} + e^{-3(E_p-\mu)/T}}$$

$$f_\Phi^-(E_p) = \frac{\left(\bar{\Phi} + 2\Phi e^{-(E_p+\mu)/T}\right) e^{-(E_p+\mu)/T} + e^{-3(E_p+\mu)/T}}{1 + 3\left(\Phi + \bar{\Phi} e^{-(E_p+\mu)/T}\right) e^{-(E_p+\mu)/T} + e^{-3(E_p+\mu)/T}},$$

Field equations

The gap equation reads:

$$\mathrm{m} - \mathrm{m}_0 = 2\mathrm{G}_1 \mathrm{N}_\mathrm{f} \mathrm{N}_\mathrm{c} \int_\Lambda \frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3} \frac{2\mathrm{m}}{\mathrm{E}_\mathrm{p}} [1 - \mathrm{f}_\Phi^+(\mathrm{E}_\mathrm{p}) - \mathrm{f}_\Phi^-(\mathrm{E}_\mathrm{p})] \,.$$

Field equations

$$m - m_0 \quad = \quad 8iG_1 m N_c N_f \int_\Lambda \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2},$$

after adopting the following symbolic replacements:

$$\begin{aligned} p = (p_0, \vec{p}) \quad &\rightarrow \quad (i\omega_n + \mu - iA_4, \vec{p}) \\ i \int_\Lambda \frac{d^4p}{(2\pi)^4} \quad &\rightarrow \quad -T \frac{1}{N_c} Tr_c \sum_n \int_\Lambda \frac{d^3p}{(2\pi)^3}, \end{aligned}$$

Grand potential at finite temperature and density in Hartree approximation

The usual techniques Klevansky:1992, Schwarz,1999 can be used to obtain the PNJL grand potential from the Hartree propagator (Ratti,2005):

$$\begin{aligned}\Omega = \Omega(\Phi, \bar{\Phi}, \mathrm{m}; \mathrm{T}, \mu) \quad &= \quad \mathcal{U}\left(\Phi, \bar{\Phi}, \mathrm{T}\right) + \frac{(\mathrm{m}-\mathrm{m}_0)^2}{4\mathrm{G}_1} - 2\mathrm{N_c N_f} \int_\Lambda \frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3}\,\mathrm{E_p} \\ &-2\mathrm{N_f}\,\mathrm{T} \int_\Lambda \frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3} \left\{ \mathrm{Tr_c} \ln \left[1 + \mathrm{L}^\dagger \mathrm{e}^{-(\mathrm{E_p}-\mu)/\mathrm{T}} \right] \right. \\ &\left. + \mathrm{Tr_c} \ln \left[1 + \mathrm{L} \mathrm{e}^{-(\mathrm{E_p}+\mu)/\mathrm{T}} \right] \right\} .\end{aligned}$$

$\mathrm{E_p} = \sqrt{\vec{\mathrm{p}}^{\,2} + \mathrm{m}^2}$ is the Hartree single quasi-particle energy (which includes the constituent quark mass).

Grand potential at finite temperature and density in Hartree approximation

$z_\Phi^{+,-}$ and compute them for $N_c = 3$:

$$
\begin{aligned}
z_\Phi^+ &\equiv \mathrm{Tr}_c \ln \left[1 + L^\dagger e^{-(E_p-\mu)/T}\right] \\
&= \ln\left\{1 + 3\left(\bar{\Phi} + \Phi e^{-(E_p-\mu)/T}\right) e^{-(E_p-\mu)/T} + e^{-3(E_p-\mu)/T}\right\} \\
z_\Phi^- &\equiv \mathrm{Tr}_c \ln \left[1 + L e^{-(E_p+\mu)/T}\right] \\
&= \ln\left\{1 + 3\left(\Phi + \bar{\Phi} e^{-(E_p+\mu)/T}\right) e^{-(E_p+\mu)/T} + e^{-3(E_p+\mu)/T}\right\}
\end{aligned}
$$

Grand potential at finite temperature and density in Hartree approximation

The solutions of the mean field equations are obtained by minimizing the grand potential with respect to m, Φ and $\bar{\Phi}$, namely (again below $N_c = 3$)

$$\begin{aligned}\frac{\partial\Omega}{\partial\Phi} &= 0\\ &= \frac{T^4}{2}(-b_2(T)\bar{\Phi} - b_3\Phi^2 + b_4\Phi\bar{\Phi}^2)\end{aligned}$$

$$-6N_fT\int_\Lambda \frac{d^3p}{(2\pi)^3}\left\{\frac{e^{-2(E_p-\mu)/T}}{1+3\left(\bar{\Phi}+\Phi e^{-(E_p-\mu)/T}\right)e^{-(E_p-\mu)/T}+e^{-3(E_p-\mu)/T}}\right.$$
$$\left.+\frac{e^{-(E_p+\mu)/T}}{1+3\left(\Phi+\bar{\Phi}e^{-(E_p+\mu)/T}\right)e^{-(E_p+\mu)/T}+e^{-3(E_p+\mu)/T}}\right\},$$

Grand potential at finite temperature and density in Hartree approximation

$$
\begin{aligned}
\frac{\partial\Omega}{\partial\bar{\Phi}} &= 0 \\
&= \frac{T^4}{2}\left(-b_2(T)\Phi - b_3\bar{\Phi}^2 + b_4\bar{\Phi}\Phi^2\right)
\end{aligned}
$$

$$
-6N_f T\int_\Lambda \frac{d^3p}{(2\pi)^3}\left\{\frac{e^{-(E_p-\mu)/T}}{1+3\left(\bar{\Phi}+\Phi e^{-(E_p-\mu)/T}\right)e^{-(E_p-\mu)/T}+e^{-3(E_p-\mu)/T}} + \frac{e^{-2(E_p+\mu)/T}}{1+3\left(\Phi+\bar{\Phi}e^{-(E_p+\mu)/T}\right)e^{-(E_p+\mu)/T}+e^{-3(E_p+\mu)/T}}\right\}
$$

and

$$
\frac{\partial\Omega}{\partial m} = 0
$$

Mesonic correlators

$$\mathrm{J_P}^{\mathrm{a}}(\mathrm{x}) \quad = \quad \bar{\mathrm{q}}(\mathrm{x})\mathrm{i}\gamma_5\tau^{\mathrm{a}}\mathrm{q}(\mathrm{x}) \qquad \text{(pion)}$$

and the scalar iso-scalar current:

$$\mathrm{J_S}(\mathrm{x}) \quad = \quad \bar{\mathrm{q}}(\mathrm{x})\mathrm{q}(\mathrm{x}) - < \bar{\mathrm{q}}(\mathrm{x})\mathrm{q}(\mathrm{x}) > \qquad \text{(sigma).}$$

In terms of the above currents, the following mesonic correlation functions and their Fourier transforms are defined:

$$\mathrm{C_{ab}^{PP}}(\mathrm{q}^2) \equiv \mathrm{i}\int \mathrm{d}^4\mathrm{x}\mathrm{e}^{\mathrm{iq.x}} < \mathrm{T}\left(\mathrm{J_P^a}(\mathrm{x})\mathrm{J_P^{b+}}(0)\right) > = \mathrm{C^{PP}}(\mathrm{q}^2)\delta_{\mathrm{ab}}$$

and

$$\mathrm{C^{SS}}(\mathrm{q}^2) \equiv \mathrm{i}\int \mathrm{d}^4\mathrm{x}\mathrm{e}^{\mathrm{iq.x}} < \mathrm{T}\left(\mathrm{J_S}(\mathrm{x})\mathrm{J_S^+}(0)\right) >.$$

Mesonic correlators

$\mathrm{T} = \mu = 0$

$$\mathrm{C}^{\mathrm{MM}}(\mathrm{q}^2) \quad = \quad \Pi^{\mathrm{MM}}(\mathrm{q}^2) + \sum_{\mathrm{M}'} \Pi^{\mathrm{MM}'}(2\mathrm{G}_1)\mathrm{C}^{\mathrm{M}'\mathrm{M}}$$

where the

$$\Pi^{\mathrm{MM}'} \quad \equiv \quad \int_\Lambda \frac{\mathrm{d}^4\mathrm{p}}{(2\pi)^4} \mathrm{Tr}\left(\Gamma_{\mathrm{M}} \mathrm{S}(\mathrm{p}+\mathrm{q}) \Gamma_{\mathrm{M}'} \mathrm{S}(\mathrm{q})\right)$$

Mesonic correlators

Polarization functions:

$$\begin{aligned}\Pi^{PP}_{ab}(q^2) &= \int_\Lambda \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left(i\gamma_5\tau^a S(p+q) i\gamma_5\tau^b S(q)\right) = \Pi^{PP}(q^2)\delta_{ab} \\ \Pi^{SS}(q^2) &= \int_\Lambda \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left(S(p+q)S(q)\right).\end{aligned}$$

Thus, for example, for the pion channel:

$$\begin{aligned}\Pi^{PP}(q^2) &= -4iN_cN_f \int_\Lambda \frac{d^4p}{(2\pi)^4} \frac{m^2 - p^2 + q^2/4}{[(p+q/2)^2 - m^2][(p-q/2)^2 - m^2]} \\ &= 4iN_cN_f I_1 - 2iN_cN_f q^2 I_2(q^2)\end{aligned}$$

Mesonic correlators

The loop integrals:

$$\begin{aligned} I_1 &= \int_\Lambda \frac{d^4p}{(2\pi)^4}\frac{1}{p^2-m^2} \\ I_2(q^2) &= \int_\Lambda \frac{d^4p}{(2\pi)^4}\frac{1}{[(p+q)^2-m^2]\,[p^2-m^2]}. \end{aligned}$$

By defining[2]:

$$F_P^2(q^2) = -4iN_c m^2 I_2(q^2)$$

and owing to the fact that the Hartree equation implies

[2] $F_P^2(q^2)=0$ is the pion decay constant f_π^2 in the chiral limit

Mesonic correlators

$$I_1 = \frac{m - m_0}{8iG_1 m N_c N_f},$$

one shows that

$$\Pi^{PP}(q^2) = \frac{m - m_0}{2G_1 m} + F_P^2(q^2)\frac{q^2}{m^2}$$

$$\Pi^{SS}(q^2) = \frac{m - m_0}{2G_1 m} + F_P^2(q^2)\frac{q^2 - 4m^2}{m^2}.$$

Mesonic correlators

The explicit solutions:

- Scalar iso-scalar sector

$$\begin{aligned} C^{SS}(q^2) &= \Pi^{SS}(q^2) + \Pi^{SS}(q^2)(2G_1)C^{SS}(q^2) \\ \Rightarrow C^{SS} &= \frac{\Pi^{SS}(q^2)}{1 - 2G_1\Pi^{SS}(q^2)}. \end{aligned}$$

- Pseudo-scalar iso-vector sector

$$\begin{aligned} C^{PP}(q^2) &= \Pi^{PP}(q^2) + \Pi^{PP}(q^2)(2G_1)C^{PP}(q^2) \\ \Rightarrow C^{PP} &= \frac{\Pi^{PP}(q^2)}{1 - 2G_1\Pi^{PP}(q^2)}. \end{aligned}$$

Mesonic correlators

$T \neq 0$ and $\mu \neq 0$

In order to study the problem at finite temperature and baryon density in the imaginary time formalism ($t = -i\tau$ with $\tau \in [0, \beta]$), the τ-ordered product of the operators replaces the usual time-ordering and all the expectation values are taken over the grand-canonical ensemble.

One can decompose all the integrands, for example in I_2, as a sum of partial fractions of the form

$$\frac{1}{i\omega_n - E + \mu}.$$

The sum over Matsubara frequencies is then computed by using:

$$\frac{1}{\beta}\sum_n \frac{1}{i\omega_n - E + \mu} = f(E - \mu)$$

where the Fermi – Dirac distribution function is given by:

$$f(E) = \frac{1}{1 + e^{\beta E}}.$$

Mesonic correlators

The integrals I_1 and I_2 at finite temperature and density are then expressed as

$$
\begin{aligned}
I_1 &= -i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1-f(E_p-\mu)-f(E_p+\mu)}{2E_p} \\
I_2(\omega,\vec{q}) &= i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p 2E_{p+q}}\frac{f(E_p+\mu)+f(E_p-\mu)-f(E_{p+q}+\mu)-f(E_{p+q}-\mu)}{\omega-E_{p+q}+E_p} \\
&+i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1-f(E_p-\mu)-f(E_{p+q}+\mu)}{2E_p 2E_{p+q}}\left(\frac{1}{\omega+E_{p+q}+E_p}-\frac{1}{\omega-E_{p+q}-E_p}\right)
\end{aligned}
$$

(these expression are implicitly taken at $\omega \to \omega + i\eta$ to obtain retarded correlation functions).

Mesonic correlators

Then all the zero temperature results can be continued to finite temperature and density by a redefinition of I_1 and I_2.
At $\vec{q} = \vec{0}$, the integral I_2 reduces to:

$$I_2\left(\omega, \vec{0}\right) = -i \int_\Lambda \frac{d^3p}{(2\pi)^3} \frac{1 - f(E_p + \mu) - f(E_p - \mu)}{E_p\left(\omega^2 - 4E_p^2\right)}$$

so that we obtain:

$$\Pi^{PP}\left(\omega, \vec{0}\right) = -8N_cN_f \int_\Lambda \frac{d^3p}{(2\pi)^3} \frac{E_p}{\omega^2 - 4E_p^2} \left(1 - f(E_p + \mu) - f(E_p - \mu)\right)$$

$$\Pi^{SS}\left(\omega, \vec{0}\right) = -8N_cN_f \int_\Lambda \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} \frac{E_p^2 - m^2}{\omega^2 - 4E_p^2} \left(1 - f(E_p + \mu) - f(E_p - \mu)\right).$$

Mesonic correlators

It then follows:

$$\Im\mathrm{m}\,(-\mathrm{i}\mathrm{I}_2(\omega,0)) = \frac{1}{16\pi}\left(1-\mathrm{f}\left(\frac{\omega}{2}-\mu\right)-\mathrm{f}\left(\frac{\omega}{2}+\mu\right)\right)\sqrt{\frac{\omega^2-4\mathrm{m}^2}{\omega^2}}$$
$$\times\Theta(\omega^2-4\mathrm{m}^2)\Theta(4(\Lambda^2+\mathrm{m}^2)-\omega^2)$$

(and of course, the real part is given by the Cauchy principal value of the integral).

Mesonic correlators

Hence:

$$\Im\mathrm{m}\,\Pi^{PP}(\omega,0) = 2N_fN_c\omega^2\Im\mathrm{m}\left(-iI_2(\omega)\right)$$

$$= \frac{N_cN_f\omega^2}{8\pi}\sqrt{\frac{\omega^2-4m^2}{\omega^2}}\,N(\omega,\mu)\,\Theta(\omega^2-4m^2)\Theta(4(\Lambda^2+m^2)-\omega^2)$$

$$\Im\mathrm{m}\,\Pi^{SS}(\omega,0) =$$

$$= \frac{N_cN_f(\omega^2-4m^2)}{8\pi}\sqrt{\frac{\omega^2-4m^2}{\omega^2}}\,N(\omega,\mu)\,\Theta(\omega^2-4m^2)\Theta(4(\Lambda^2+m^2)-\omega^2$$

with

$$N(\omega,\mu) = \left(1 - f\left(\frac{\omega}{2}-\mu\right) - f\left(\frac{\omega}{2}+\mu\right)\right).$$

Mesonic correlators

PNJL. $T \neq 0$ and $\mu \neq 0$
Again, all the summation over Matsubara frequencies can be reduced to the sum of fractions.

$$F(E_p - \mu + iA_4) \equiv \frac{1}{\beta} \sum_n \frac{1}{i\omega_n - E_p + \mu - iA_4}$$

one shows that:

$$\mathrm{Tr}_c F(E_p - \mu + iA_4) \\ = f(E_p - \mu + i(A_4)_{11}) + f(E_p - \mu + i(A_4)_{22}) + f(E_p - \mu + i(A_4)_{33})$$

where $(A_4)_{ii}$ are the elements of the diagonalized A_4 matrix.

Mesonic correlators

Let us write the Fermi–Dirac distribution function according to:

$$f(E_p-\mu) \equiv -\frac{1}{\beta}\frac{\partial z^+}{\partial E_p},$$

where

$$z^+ \equiv \ln\left(1+e^{-(E_p-\mu)/T}\right)$$

can be viewed as a density of partition function. We then obtain

$$\begin{aligned}
\mathrm{Tr_c}F(E_p-\mu+iA_4) &= -\frac{1}{\beta}\sum_i \frac{\partial \ln\left(1+e^{-(E_p-\mu)/T}e^{-i\beta(A_4)_{ii}}\right)}{\partial E_p} \\
&= -\frac{1}{\beta}\,\mathrm{Tr_c}\frac{\partial \ln\left(1+e^{-(E_p-\mu)/T}e^{-i\beta A_4}\right)}{\partial E_p} \\
&= -\frac{1}{\beta}\,\mathrm{Tr_c}\frac{\partial \ln\left(1+L^+e^{-(E_p-\mu)/T}\right)}{\partial E_p} = -\frac{1}{\beta}\frac{\partial z_\Phi^+}{\partial E_p}
\end{aligned}$$

where $z_\Phi^+ = \ln\left(1+L^+e^{-(E_p-\mu)/T}\right)$ is the corresponding density of partition function in PNJL.

Mesonic correlators

Hence

$$\mathrm{Tr_c F}(\mathrm{E_p} - \mu + \mathrm{i}A_4) = 3\frac{\left(\bar{\Phi} + 2\Phi e^{-(E_p-\mu)/T}\right) e^{-(E_p-\mu)/T} + e^{-3(E_p-\mu)/T}}{1 + 3\left(\bar{\Phi} + \Phi e^{-(E_p-\mu)/T}\right) e^{-(E_p-\mu)/T} + e^{-3(E_p-\mu)/T}}$$

We can do the same for the $\mathrm{F}(\mathrm{E_p} + \mu - \mathrm{i}A_4)$ case.
Hence, we can define:

$$f_\Phi^+(E_p) \equiv \frac{1}{N_c}\mathrm{Tr_c F}(E_p - \mu + \mathrm{i}A_4) = -\frac{1}{\beta N_c}\frac{\partial z_\Phi^+}{\partial E_p}$$

and

$$f_\Phi^-(E_p) \equiv \frac{1}{N_c}\mathrm{Tr_c F}(E_p + \mu - \mathrm{i}A_4) = -\frac{1}{\beta N_c}\frac{\partial z_\Phi^-}{\partial E_p} ,$$

where z_Φ^+ and z_Φ^- are the densities of the partition function in PNJL.

Mesonic correlators

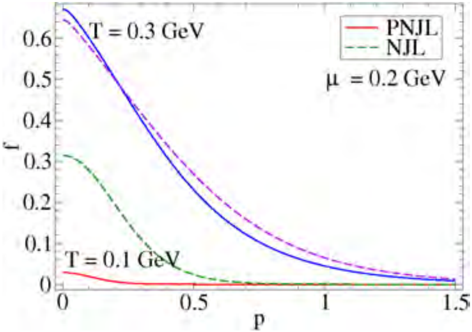
The only changes in going from NJL to PNJL can then be summarized in the following prescriptions:

$$\mathrm{f}(\mathrm{E_p}-\mu) \quad \Longrightarrow \quad \mathrm{f}_\Phi^+(\mathrm{E_p}) = \frac{\left(\bar{\Phi} + 2\Phi \mathrm{e}^{-(\mathrm{E_p}-\mu)/\mathrm{T}}\right)\mathrm{e}^{-(\mathrm{E_p}-\mu)/\mathrm{T}} + \mathrm{e}^{-3(\mathrm{E_p}-\mu)/\mathrm{T}}}{1 + 3\left(\bar{\Phi} + \Phi \mathrm{e}^{-(\mathrm{E_p}-\mu)/\mathrm{T}}\right)\mathrm{e}^{-(\mathrm{E_p}-\mu)/\mathrm{T}} + \mathrm{e}^{-3(\mathrm{E_p}-\mu)/\mathrm{T}}}$$

$$\mathrm{f}(\mathrm{E_p}+\mu) \quad \Longrightarrow \quad \mathrm{f}_\Phi^-(\mathrm{E_p}) = \frac{\left(\Phi + 2\bar{\Phi} \mathrm{e}^{-(\mathrm{E_p}+\mu)/\mathrm{T}}\right)\mathrm{e}^{-(\mathrm{E_p}+\mu)/\mathrm{T}} + \mathrm{e}^{-3(\mathrm{E_p}+\mu)/\mathrm{T}}}{1 + 3\left(\Phi + \bar{\Phi} \mathrm{e}^{-(\mathrm{E_p}+\mu)/\mathrm{T}}\right)\mathrm{e}^{-(\mathrm{E_p}+\mu)/\mathrm{T}} + \mathrm{e}^{-3(\mathrm{E_p}+\mu)/\mathrm{T}}}$$

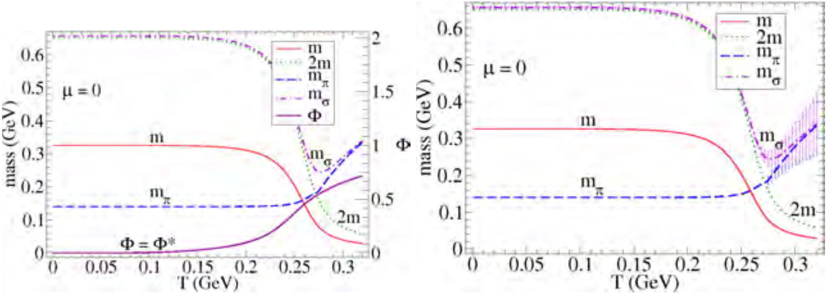
Of course in the above the corresponding PNJL quark mass m, given by the Hartree equation with these modified distribution functions, should be used.

Mesonic correlators



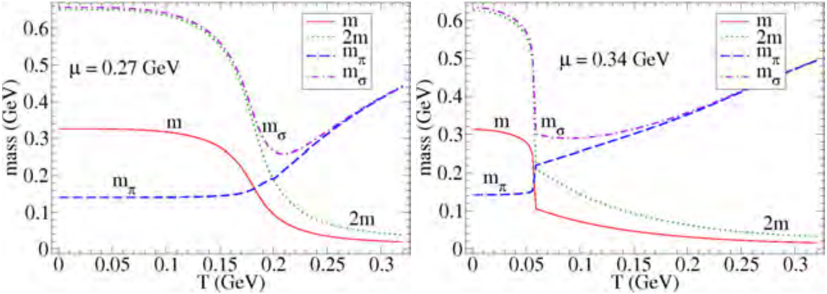
Fermi – Dirac distribution function $f(E_p - \mu)$ (valid for the NJL model) and the corresponding function $f_{\Phi}^{+}(E_p)$ (valid for the PNJL one) as functions of p, for different temperatures. Φ, $\bar{\Phi}$ and m are taken at their mean field values. The upper lines refer to $T = 0.3$ GeV, the lower ones to $T = 0.1$ GeV.

Mesonic correlators



Masses of the σ and π as functions of the temperature, together with the Hartree quark mass and the Polyakov loop, in the PNJL model at $\mu = 0$. The threshold 2m is also plotted to show that the σ mass is close to this value below 0.25 GeV. Right: without the Polyakov loop and adding instead the width of the mesons.

Mesonic correlators

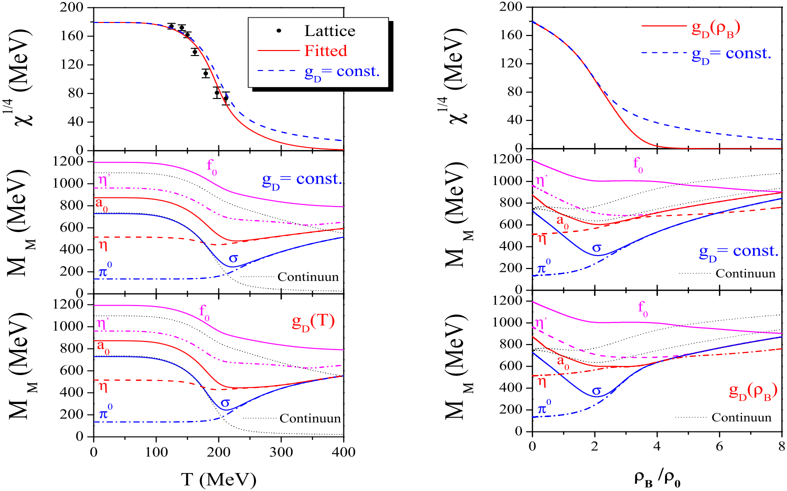


Masses of the σ and π as functions of the temperature, together with the Hartree quark mass, in the PNJL model at $\mu = 0.27$ GeV and $\mu = 0.34$ GeV.

Mesonic correlators

$$\begin{aligned} \mathcal{L} &= \bar{\mathrm{q}}\,(\,\mathrm{i}\,\gamma.\partial - \,\hat{\mathrm{m}})\,\mathrm{q} + \frac{\mathrm{g_S}}{2}\sum_{\mathrm{a}=0}^{8}[\,(\,\bar{\mathrm{q}}\,\lambda^{\mathrm{a}}\,\mathrm{q}\,)^2 + (\,\bar{\mathrm{q}}\,\mathrm{i}\,\gamma_5\,\lambda^{\mathrm{a}}\,\mathrm{q}\,)^2\,] \\ &+ \mathrm{g_D}\,\{\det[\bar{\mathrm{q}}\,(1+\gamma_5)\,\mathrm{q}] + \det[\bar{\mathrm{q}}\,(1-\gamma_5)\,\mathrm{q}]\}. \end{aligned}$$

Mesonic correlators



Задачи

1. Свойства дикварков и барионов при конечной температуре/плотности.
2. "Horn" - эффект для барионов со странностью.
3. Алгоритмы и методы расчета треугольных диаграмм при конечной температуре.
4. Расчет процессов рассеяния адронов в адронном газе.
5. Процессы рассеяния с участием глюонов ($gg \to \pi\pi$).
6. Распад $\rho \to \gamma\gamma$ при конечной температуре/плотности.