

Algorithms and Methods for High Precision Computer Simulations of Cyclotrons for Proton Therapy: The Case of SC202

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Abstract

Effective and accurate computer simulations are highly important in the design and production of an accelerator. The most difficult and important task in the cyclotron development is magnetic field simulations. It is necessary to achieve the accuracy of the model that is higher than the tolerance for the magnetic field in a real magnet. An accurate model of the magnet and other systems of the cyclotron allows us to perform beam tracking through the whole accelerator from ion source to extraction. While high accuracy is necessary at late stages of research and development, high performance of simulations and the ability to swiftly analyze and apply changes to the project play a key role at early stages of the project. Techniques and algorithms for high accuracy and performance of magnet simulations have been created and used for the development of the SC202 cyclotron for proton therapy, which is under production by the collaboration between JINR (Dubna, Russia) and ASIPP (Hefei, China).

Introduction

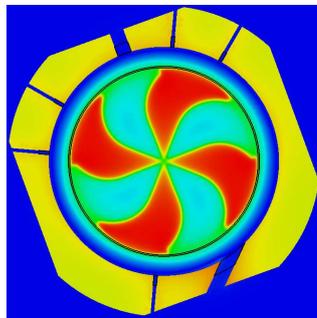
Two cyclotrons are planned to be manufactured in China: one will operate in the Hefei cyclotron medical center, the other will replace the Phasotron in the JINR Medico-technical complex in Dubna and be used to treat cancer with protons. We developed computer programs for the cyclotron design and implemented them in the MATLAB/Simulink environment. MATLAB software is widely used in accelerator physics. We used MATLAB to create input data sets and access them via Simulink block diagrams during simulation. We performed thousands of particle tracking simulations in the cyclotron in parallel, analyzed and visualized the data in MATLAB.

Magnet design and beam dynamics simulations

Beam dynamics simulations solve the system of differential equations of particle motion with field maps from FEM (finite element method) codes or field measurements with different initial coordinates and velocities. The fourth-order Runge-Kutta method is usually used to solve the system. Systems of differential equations of motion of charged particles in the electromagnetic field are derived from Newton-Lorentz equations:

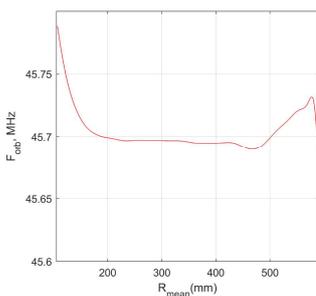
$$\frac{d\vec{V}}{dt} = \frac{q}{m} \sqrt{1 - \frac{V^2}{c^2}} \left(\vec{E} + [\vec{V}\vec{B}] - \frac{1}{c^2} \vec{V}(\vec{V}\vec{E}) \right),$$

$$\begin{cases} r'' - \frac{2r'r'}{r} - r = \frac{q\sqrt{r^2 + r'^2 + z'^2}}{mcV} (rB_z - z'B_\phi - r'z'B_r + \frac{r^2}{r}B_z) + \frac{q}{m} \frac{r^2 + r'^2 + z'^2}{V^2} \left(E_r - \frac{r'}{r}E_\phi \right), \\ z'' - \frac{2z'z'}{z} = \frac{q}{mc} \frac{\sqrt{r^2 + r'^2 + z'^2}}{V} (r'B_\phi - rB_z - z'^2B_r + \frac{r^2}{r}B_z) + \frac{q}{m} \frac{r^2 + r'^2 + z'^2}{V^2} \left(E_z - \frac{z'}{r}E_\phi \right), \\ t' = \frac{1}{c\beta} \sqrt{r^2 + r'^2 + z'^2} = \frac{1}{V} \sqrt{r^2 + r'^2 + z'^2}. \end{cases}$$

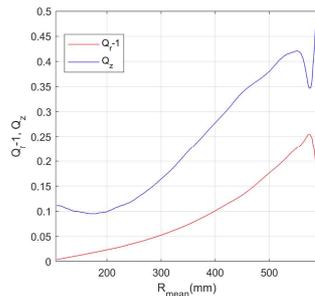


Magnetic field calculated by means of the CST Studio package

The CYCLOPS program [1] is one of the most reliable tools for dynamics analysis in cyclotrons. We implemented a version of CYCLOPS using MATLAB R2018a. The program has an equilibrium orbit search algorithm and performs calculations of orbital and betatron frequencies. The calculation results can be used for the isochronisation of the magnetic field map.



Orbital frequency of the equilibrium particle along the radius



Betatron frequency of the equilibrium particle along the radius

Representation of the electromagnetic field

When calculating particle trajectories, the electromagnetic field can be specified analytically in the form of a field map obtained as a result of measurements (2D maps) or as a result of model calculations (3D field maps). Using a 2D field map in the median plane for beam dynamics simulations is a traditional method that is widely used in cyclotron physics. Vertical and other components of the magnetic field at the distance z from the median plane can be calculated by:

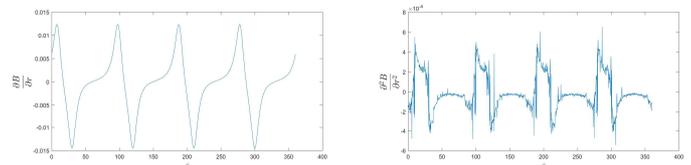
$$B_r = z \frac{\partial B}{\partial r} + \frac{\partial}{\partial r} \left[\sum_{n=1}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!} \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right)^n B \right],$$

$$rB_\phi = z \frac{\partial B}{\partial \phi} + \frac{\partial}{\partial \phi} \left[\sum_{n=1}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!} \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right)^n B \right],$$

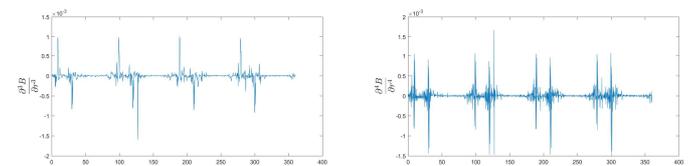
$$B_z = B + \left[\sum_{n=1}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!} \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right)^n B \right].$$

A rigorous proof of these expansions is presented in [2]. We used the expansion to the 4th order. It is important to be able to calculate all derivatives for correct beam dynamics simulations during research and development, and especially at the commissioning stage, when the real magnetic field is measured only on the median plane.

The biggest problem of these calculations is that the derivatives must be taken using the measured or calculated field map, which already contains errors. Special smoothing algorithms were applied in order to obtain smooth and realistic derivatives of the magnetic field on the median plane. In particular, we used a MATLAB function SMOOTHN, which provides a fast, unsupervised and robust discretized spline smoother for data of arbitrary dimension [3].



Derivatives of the magnetic field in the median plane taken without smoothing algorithms



Derivatives of the magnetic field in the median plane taken with smoothing algorithms. Such results were obtained by combining the fitting of the field map with the spline surface together with smoothing algorithms.

Conclusion

The codes for beam dynamics simulations were improved with new algorithms where the magnetic field components outside the median plane were calculated up to the fourth order. Scripts for producing 3D magnetic field maps, 3D electric and magnetic field maps from RF cavity simulations in CST Studio and for reading them into the MATLAB workspace were written. MATLAB, being a matrix-oriented tool, allows us to implement an equilibrium orbit search algorithm for a large number of particles with different energies with a good calculation speed. High accuracy and high efficiency of simulations will help at the commissioning stage during the shimming of the magnet.

References:

- [1] Gordon, M.M., 1984. Computation of closed orbits and basic focusing properties for sector-focused cyclotrons and the design of "cyclotrons". Part. Accel., 16, pp. 39-62.
- [2] Hart, T.L. et al., Magnetic Field Expansion Out of a Plane: Application to Inverse Cyclotron Muon Cooling, in arXiv:1105.2754 [physics.acc-ph] UMISS-HEP-2010-02
- [3] Garcia D. Robust smoothing of gridded data in one and higher dimensions with missing values. Comput Statist Data Anal, 2010; 54: 1167-1178