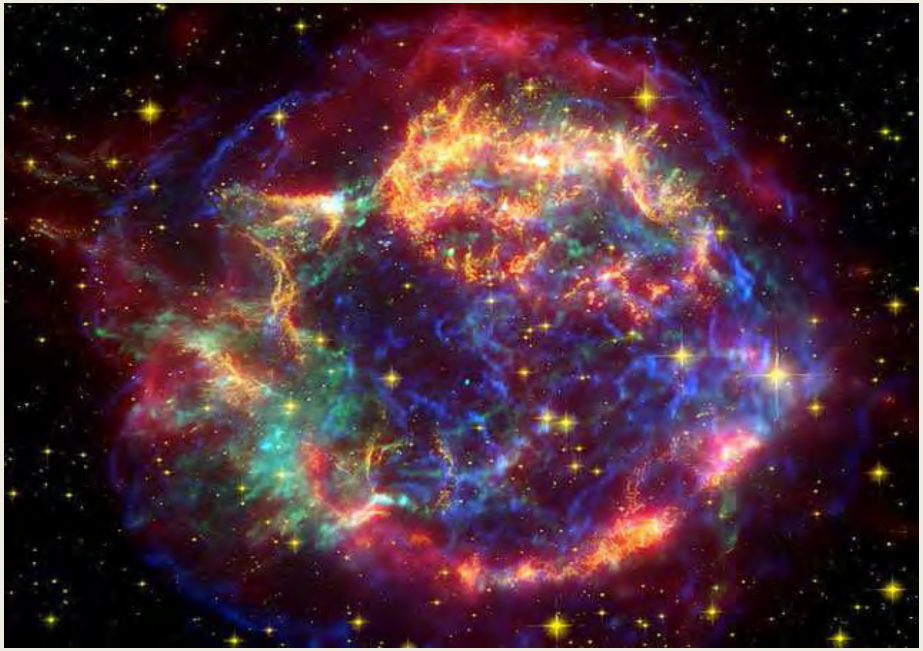
Reconstraction of Neutron Stars Mass Distribution from Cooling Evolution



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Yerevan Armenia

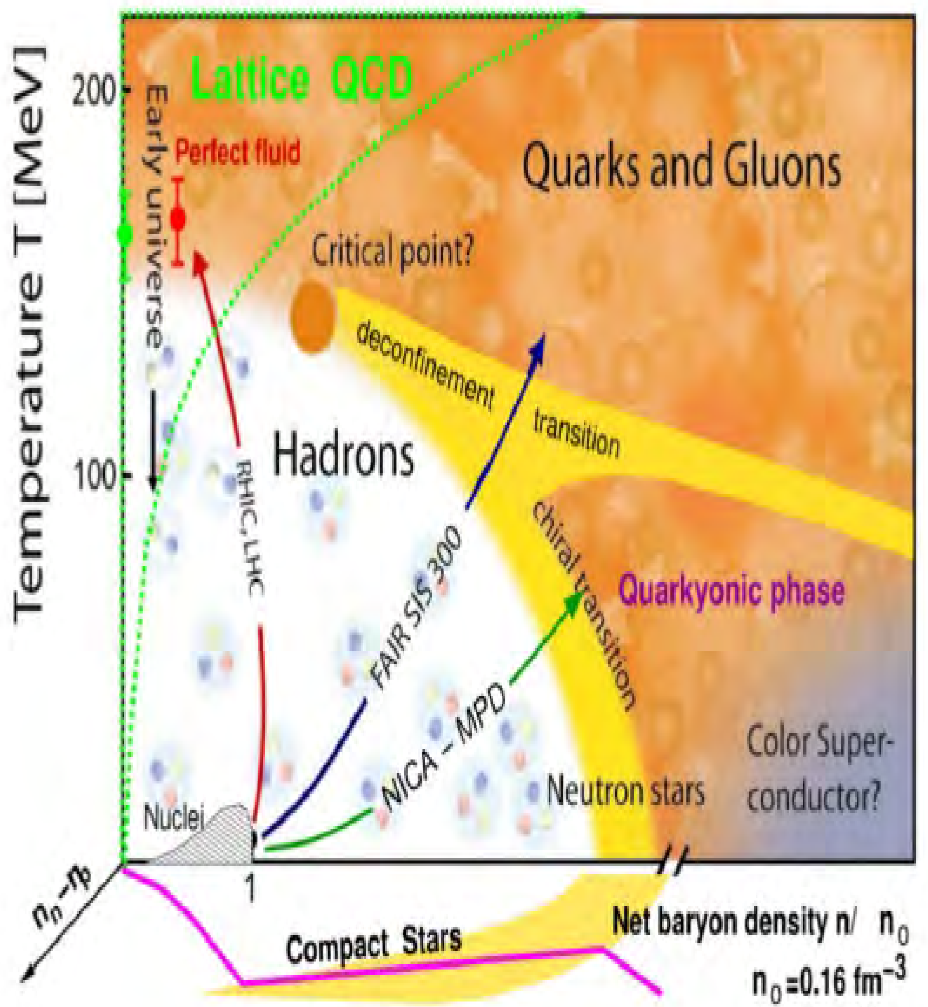
my co-authors: D.Blaschke, D.Voskresensky, A. Ayriyan

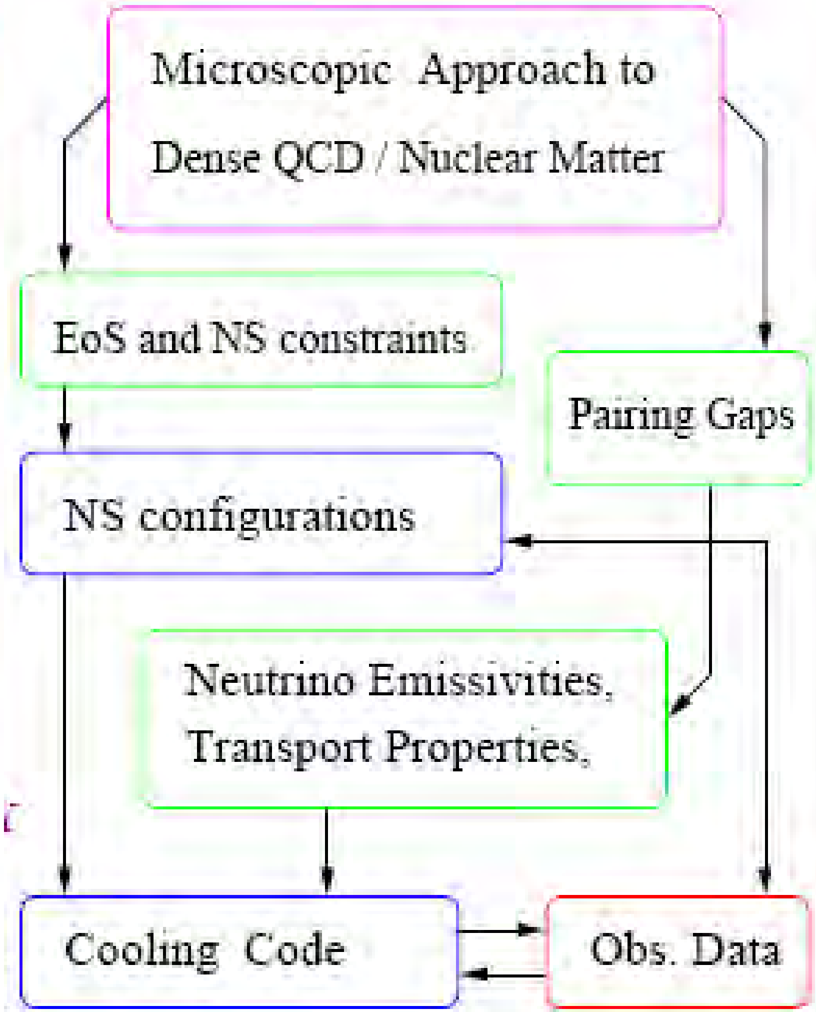
Simulation of Cooling Evolution of Neutron Stars

- Introduction
- Neutron Stars structure
- Neutron Stars cooling problem
- Simulations algorithm
- ***Results for NS cooling***

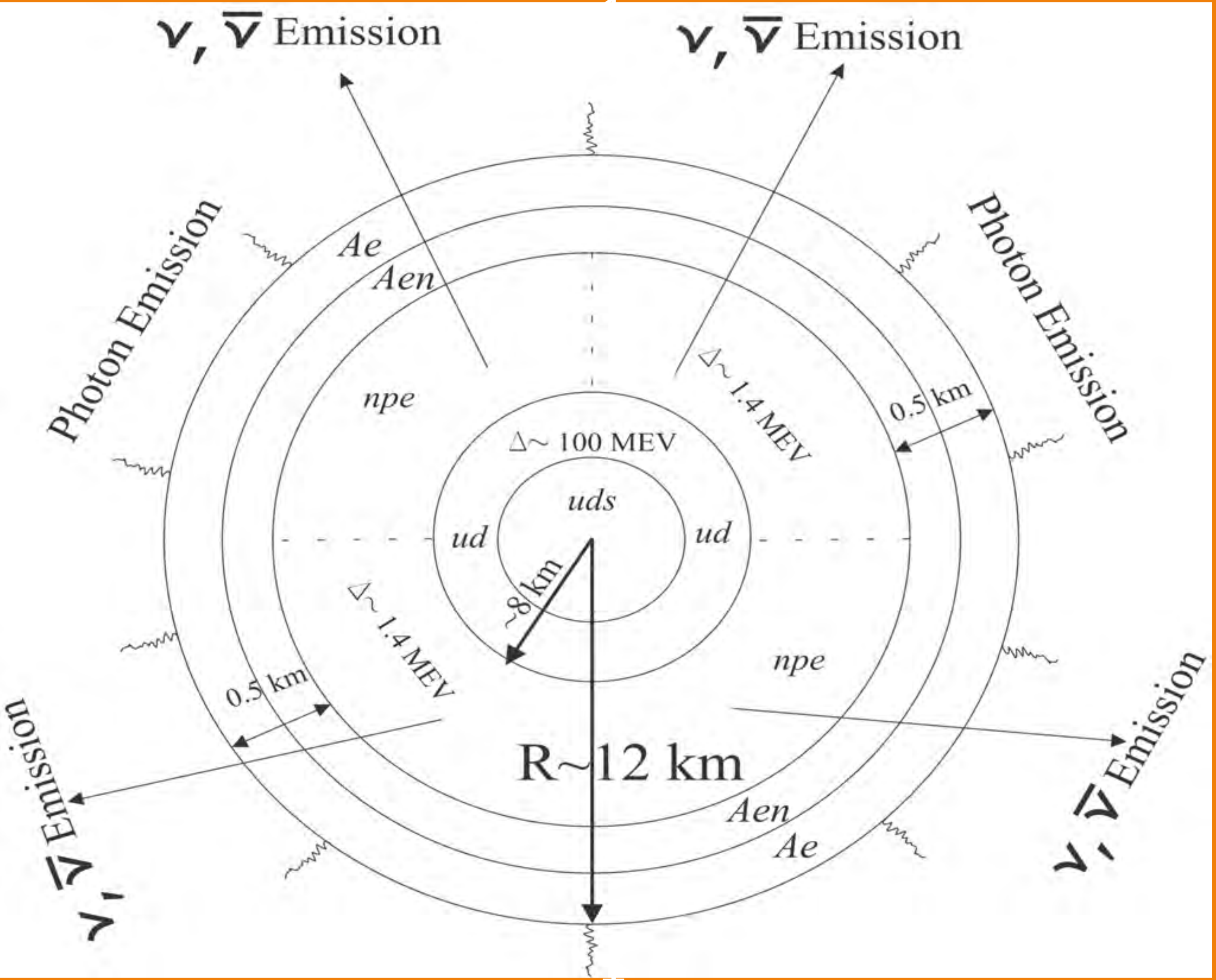
H. Grigorian, D. N. Voskresensky and D. Blaschke
Eur. Phys. J. A **52: 67 (2016).**

Phase Diagramm & Cooling Simulation





Structure Of Hybrid Star



Static neutron star mass and radius

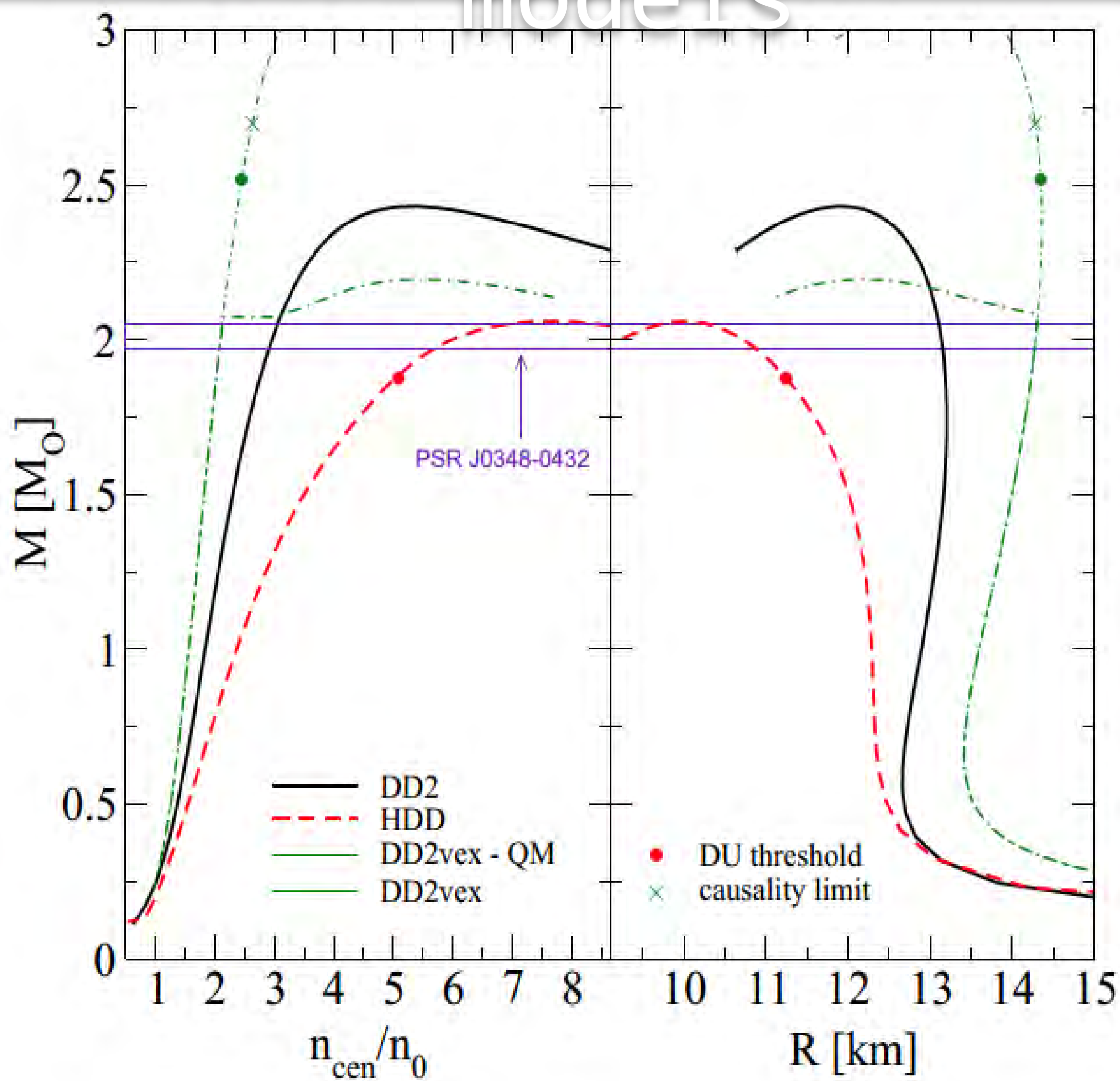
The structure and global properties of compact stars are obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations[1,2]:

$$\begin{cases} \dfrac{dP(r)}{dr} = -\dfrac{GM(r)\varepsilon(r)}{r^2} \dfrac{\left(1+\dfrac{P(r)}{\varepsilon(r)}\right)\left(1+\dfrac{4\pi r^3 P(r)}{M(r)}\right)}{\left(1-\dfrac{2GM(r)}{r}\right)}; \\ \dfrac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r); \\ \dfrac{dN_B(r)}{dr} = 4\pi r^2 \left(1-\dfrac{2GM(r)}{r}\right)^{-1/2} n(r). \end{cases}$$

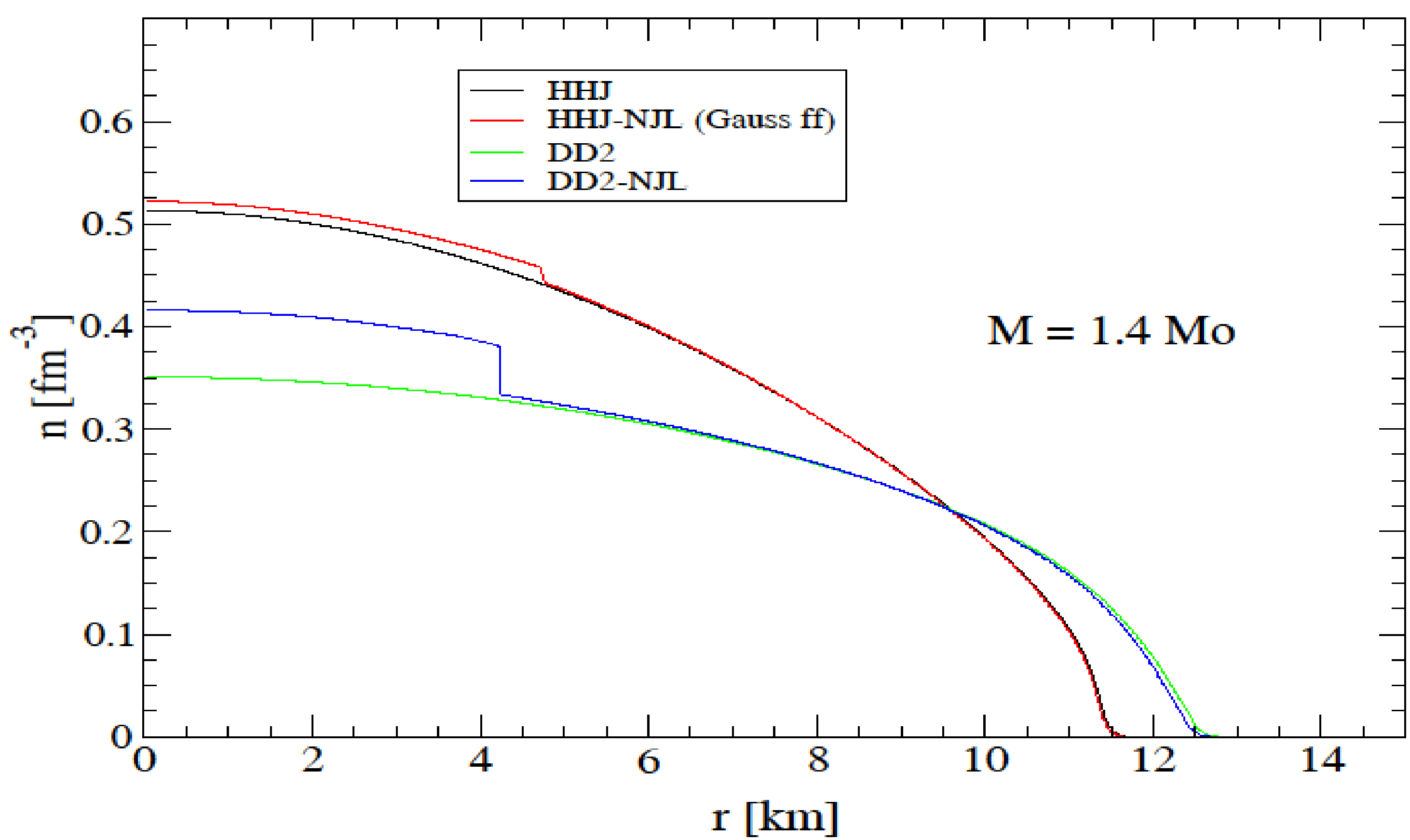
[1] R. C. Tolman, Phys. Rev. **55**, 364 (1939).

[2] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. **55**, 374 (1939).

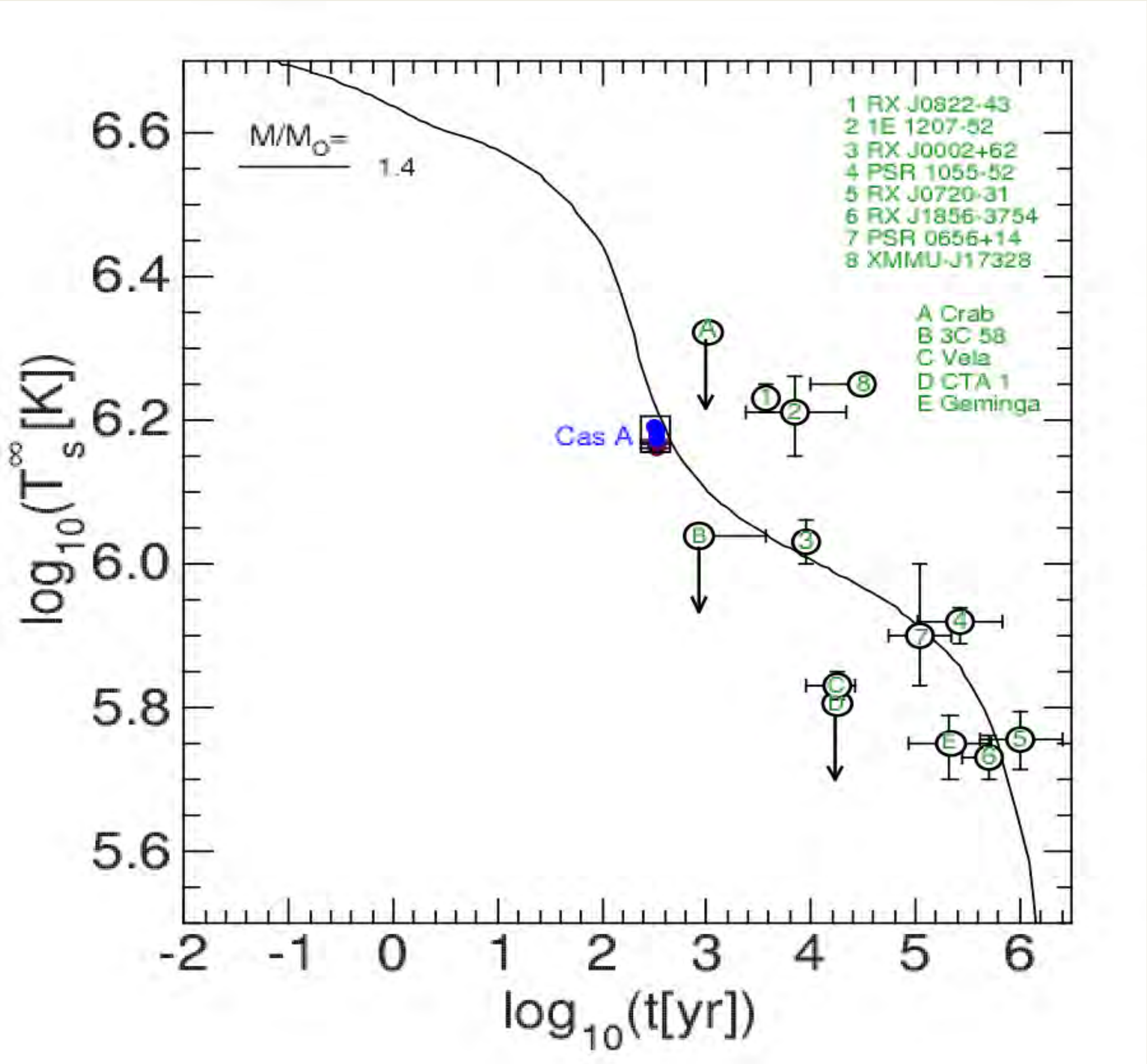
Stability of stars HDD, DD2 & DDvex-NJL EoS models



Different Configurations with the same NS mass



Surface Temperature & Age Data



Cooling Mechanism

$$\frac{dU}{dt} = \sum_i C_i \frac{dT}{dt} = -\varepsilon_\gamma - \sum_j \varepsilon_\nu^j$$

Cooling Processes

- Direct Urca: $n \rightarrow p + e + \bar{\nu}_e$
- Modified Urca: $n + n \rightarrow n + p + e + \bar{\nu}_e$
- Photons: $\rightarrow \gamma$
- Bremsstrahlung: $n + n \rightarrow n + n + \nu + \bar{\nu}$

Cooling Evolution

The energy flux per unit time l(r) through a spherical slice at distance r from the center is:

$$l(r) = -4\pi r^2 k(r) \frac{\partial (T e^{\Phi})}{\partial r} e^{-\Phi} \sqrt{1 - \frac{2M}{r}}.$$

The equations for energy balance and thermal energy transport are:

$$\frac{\partial}{\partial N_B}(l e^{2\Phi}) = -\frac{1}{n}(\epsilon_\nu e^{2\Phi} + c_V \frac{\partial}{\partial t}(T e^{\Phi}))$$

$$\frac{\partial}{\partial N_B}(T e^{\Phi}) = -\frac{1}{k} \frac{l e^{\Phi}}{16\pi^2 r^4 n}$$

where n = n(r) is the baryon number density, NB = NB(r) is the total baryon number in the sphere with radius r

$$\frac{\partial N_B}{\partial r} = 4\pi r^2 n (1 - \frac{2M}{r})^{-1/2}$$

F.Weber: Pulsars as Astro. Labs ... (1999);

D. Blaschke Grigorian, Voskresensky, A& A 368 (2001)561.

Neutrino emissivities in hadronic matter:

- Direct Urca (DU) the most efficient processes

$$\epsilon_{DU} = M_{DU} \quad *(m_p^*)(m_n^*) * \Gamma_{wN}^2 * (n_e)^{1/3}(T_9)^6 * R_D;$$
$$M_{DU} = \quad 4 \times 10^{27} \quad erg/s/cm^3$$

- Modified Urca (MU) and Bremsstrahlung

$$\epsilon_{MUp} = F_M * M_p * (m_p)^3(m_n^*)(T_9)^8(n_e)^{1/3} * R_{MUp}(v_n, v_p);$$
$$\epsilon_{nnBS} = P_{nnBS} * R_{BS}^{nn}(v_n) * \Gamma_w^2\Gamma_s^4(n_b)^{4/3}(T_9)^8(m_n^*)^4/(\omega)^3;$$

- Suppression due to the pairing

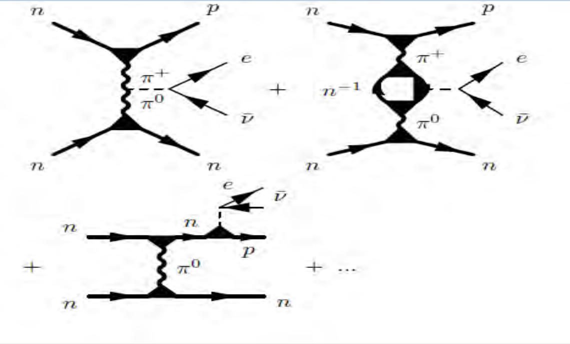
$$v_N = \Delta_N(T)/T = \sqrt{1-\tau_N}\left(1.456 - \frac{0.157}{\sqrt{\tau_N}} + \frac{1.766}{\tau_N}\right)$$

- Enhanced cooling due to the pairing

$$\epsilon_\nu^{\mathrm{NPBF}} = 6.6 \times 10^{28}(m_n^*/m_n)(\Delta_n(T)/\mathrm{MeV})^7 \ u^{1/3}$$
$$\times\xi \ I(\Delta_n(T)/T) \ \mathrm{erg\ cm^{-3}s^{-1}},$$
$$\epsilon_\nu^{\mathrm{PPBF}} = 0.8 \times 10^{28}(m_p^*/m_p)(\Delta_p(T)/\mathrm{MeV})^7 \ u^{2/3}$$
$$\times \ I(\Delta_p(T)/T) \ \mathrm{erg\ cm^{-3}s^{-1}},$$

Medium Effects In Cooling Of Neutron Stars

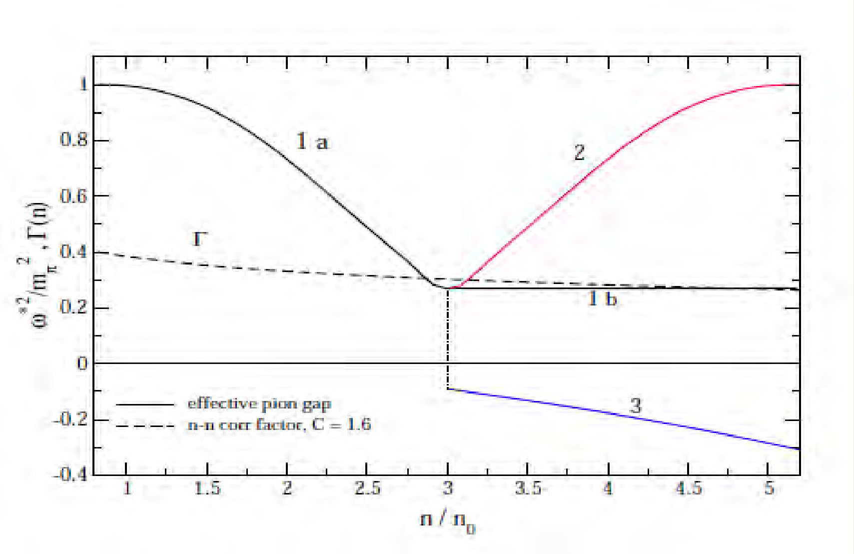
- Based on Fermi liquid theory (Landau (1956), Migdal (1967), Migdal et al. (1990))
- MMU – insted of MU



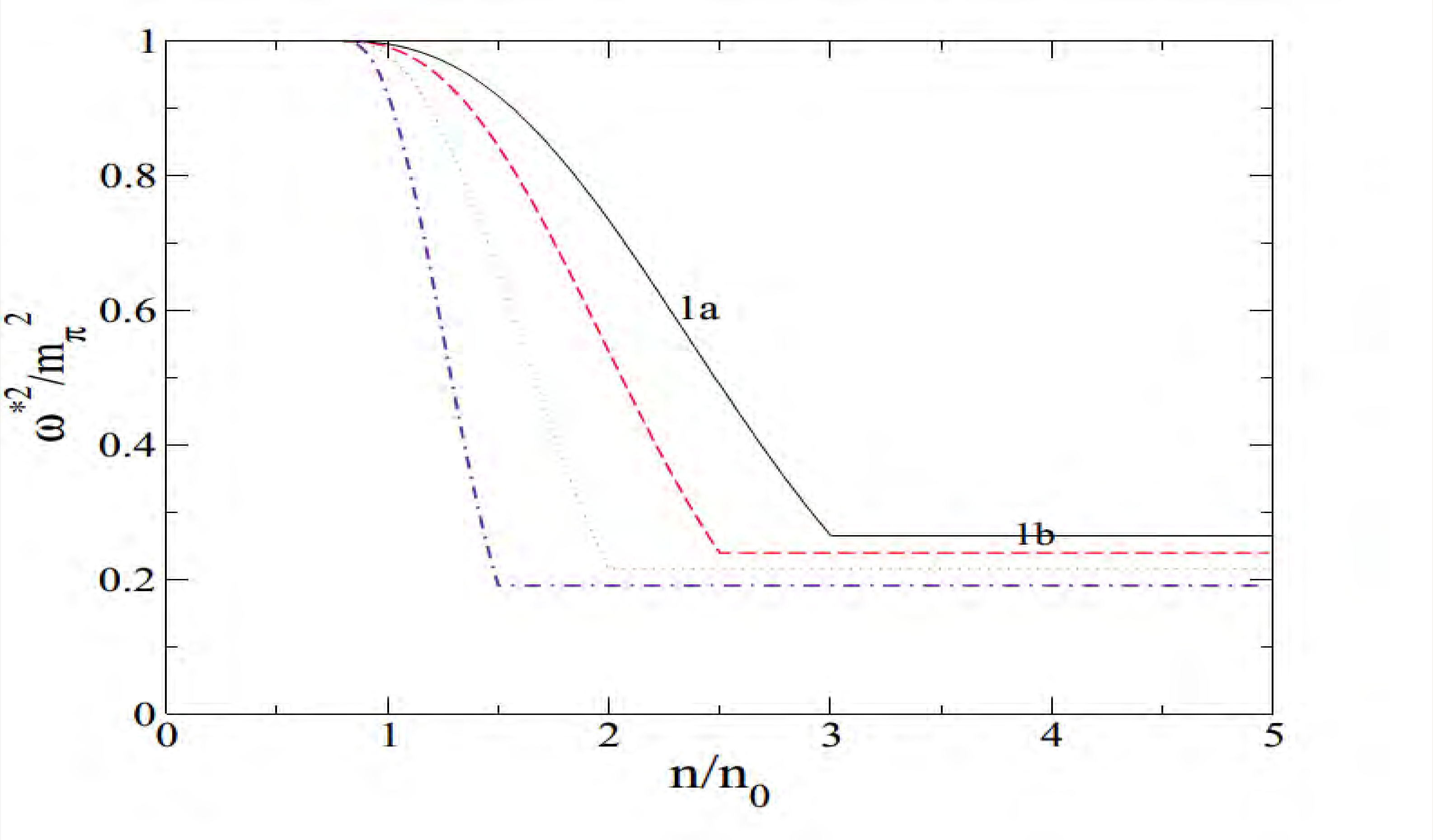
$$\frac{\varepsilon_\nu[\mathrm{MMU}]}{\varepsilon_\nu[\mathrm{MU}]} \sim 10^3\ (n/n_0)^{10/3} \frac{\Gamma^6(n)}{[\omega^*(n)/m_\pi]^8},$$

- Main regulator in Minimal Cooling

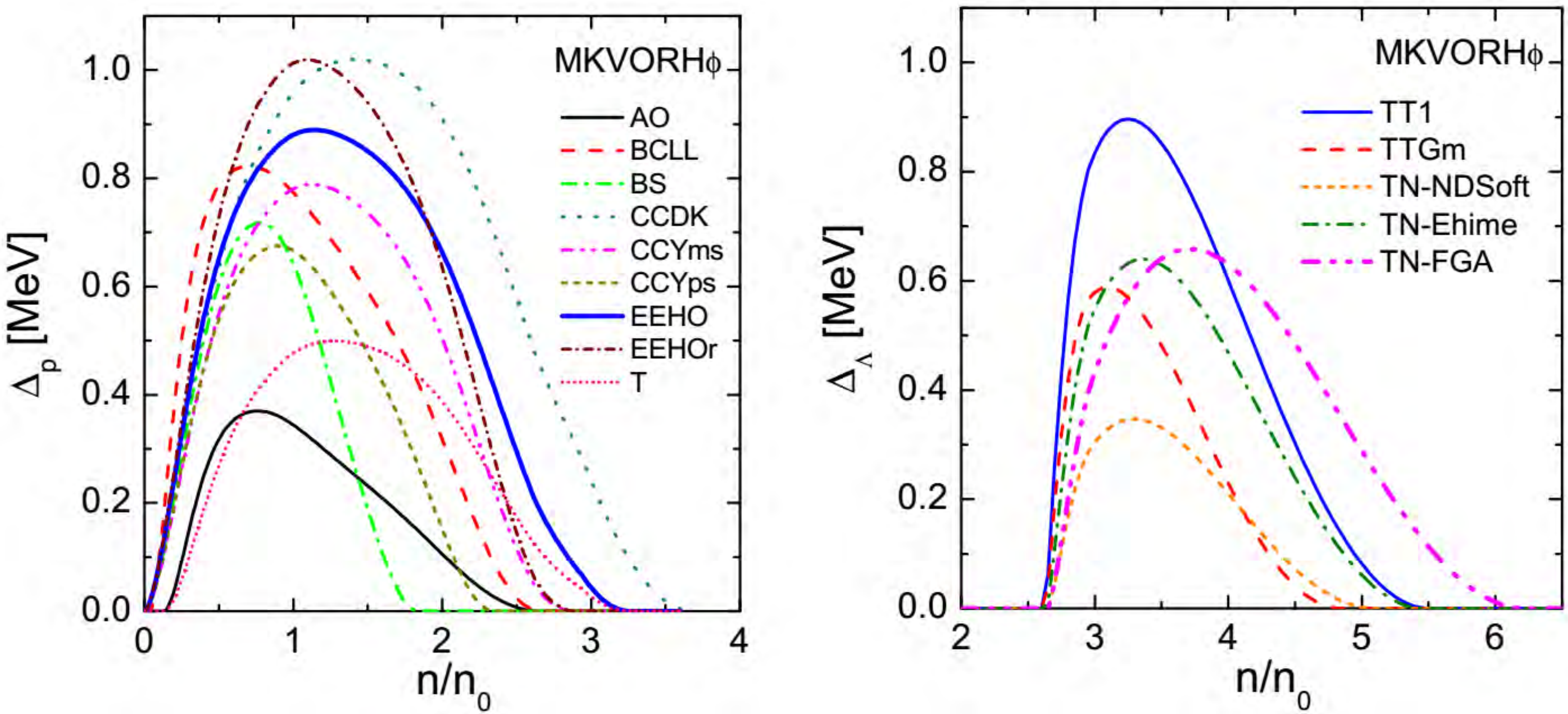
$$\varepsilon_\nu[\mathrm{MpPBF}] \sim 10^{29}\ \frac{m_N^*}{m_N} \left[\frac{p_{Fp}}{p_{Fn}(n_0)}\right] \left[\frac{\Delta_{pp}}{\mathrm{MeV}}\right]^7 \times \left[\frac{T}{\Delta_{pp}}\right]^{1/2} \xi_{pp}^2\ \frac{\mathrm{erg}}{\mathrm{cm}^3\ \sec},\quad T < T_{cp}.$$



Medium Effects In Cooling Of Neutron Stars



MKVORHp – Gap models



Equations for Cooling Evolution

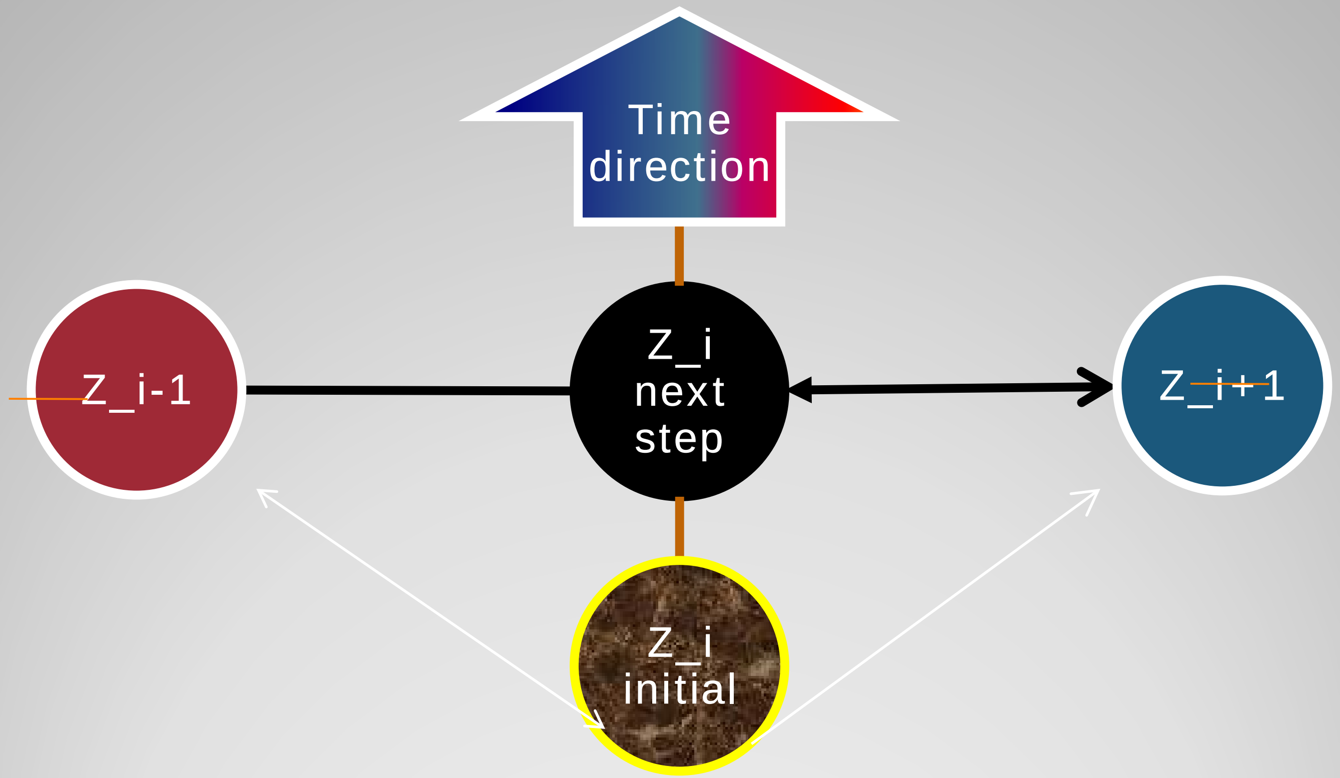
$$\begin{cases} \dfrac{\partial z(\tau,a)}{\partial \tau} = A(z,a)\dfrac{\partial L(\tau,a)}{\partial a} + B(z,a) \\ L(\tau,a) = C(z,a)\dfrac{\partial z(\tau,a)}{\partial a} \end{cases}$$

$$z(\tau,a) = \log T(\tau,a)$$

$$L_{i\pm1/2} = \pm\frac{C_i + C_{i\pm1}}{2}\,\frac{z_{i\pm1} - z_i}{\Delta a_{i-1/2(1\mp1)}}$$

$$\frac{\partial L_i}{\partial a} = 2\frac{L_{i+1/2} - L_{i-1/2}}{\Delta a_i + \Delta a_{i-1}}$$

Finite difference scheme



$$\alpha_{i,j-1} z_{i+1,j} + \beta_{i,j-1} z_{i,j} + \gamma_{i,j-1} z_{i-1,i} = \delta_{i,j-1}$$

Finite difference scheme

$$\begin{pmatrix} \beta_{0,j-1} & \alpha_{0,j-1} & & & 0 \\ \gamma_{1,j-1} & * & * & & \\ & * & * & * & \\ & & * & * & \alpha_{N-1,j-1} \\ 0 & & & \gamma_{N,j-1} & \beta_{N,j-1} \end{pmatrix} \begin{pmatrix} z_{0,j} \\ z_{1,j} \\ * \\ * \\ z_{N,j} \end{pmatrix} = \begin{pmatrix} \delta_{0,j-1} \\ \delta_{1,j-1} \\ * \\ * \\ \delta_{N,j-1} \end{pmatrix}$$

$$\alpha_{i,j-1} z_{i+1,j} + \beta_{i,j-1} z_{i,j} + \gamma_{i,j-1} z_{i-1,i} = \delta_{i,j-1}$$

Crust Model

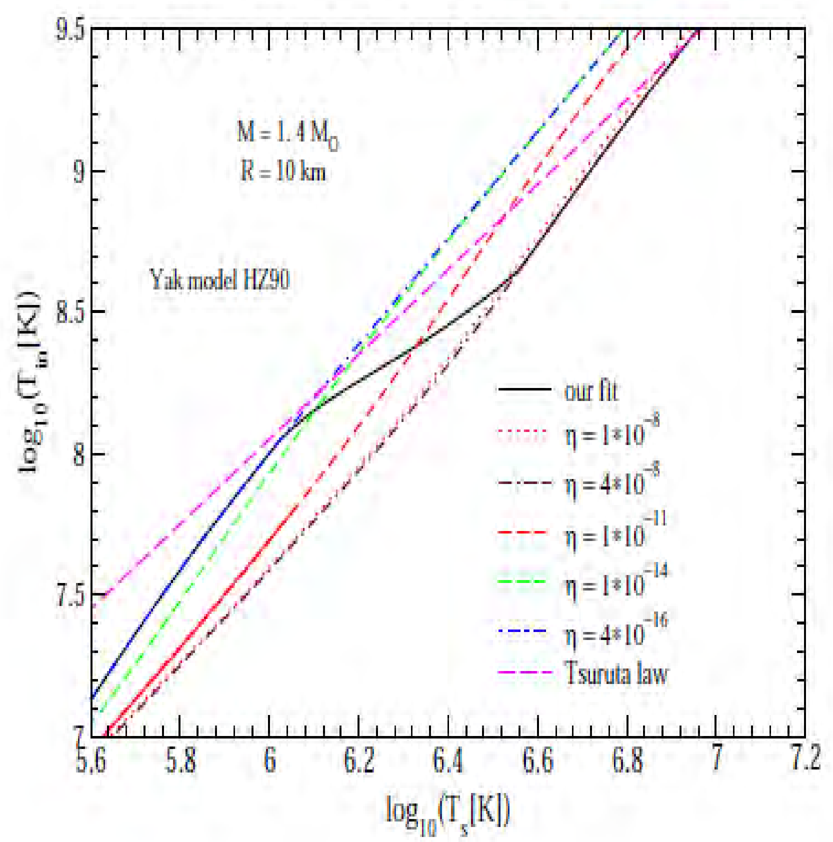
Time dependence of the light element contents in the crust

$$\Delta M_L(t) = e^{-t/\tau} \Delta M_L(0)$$

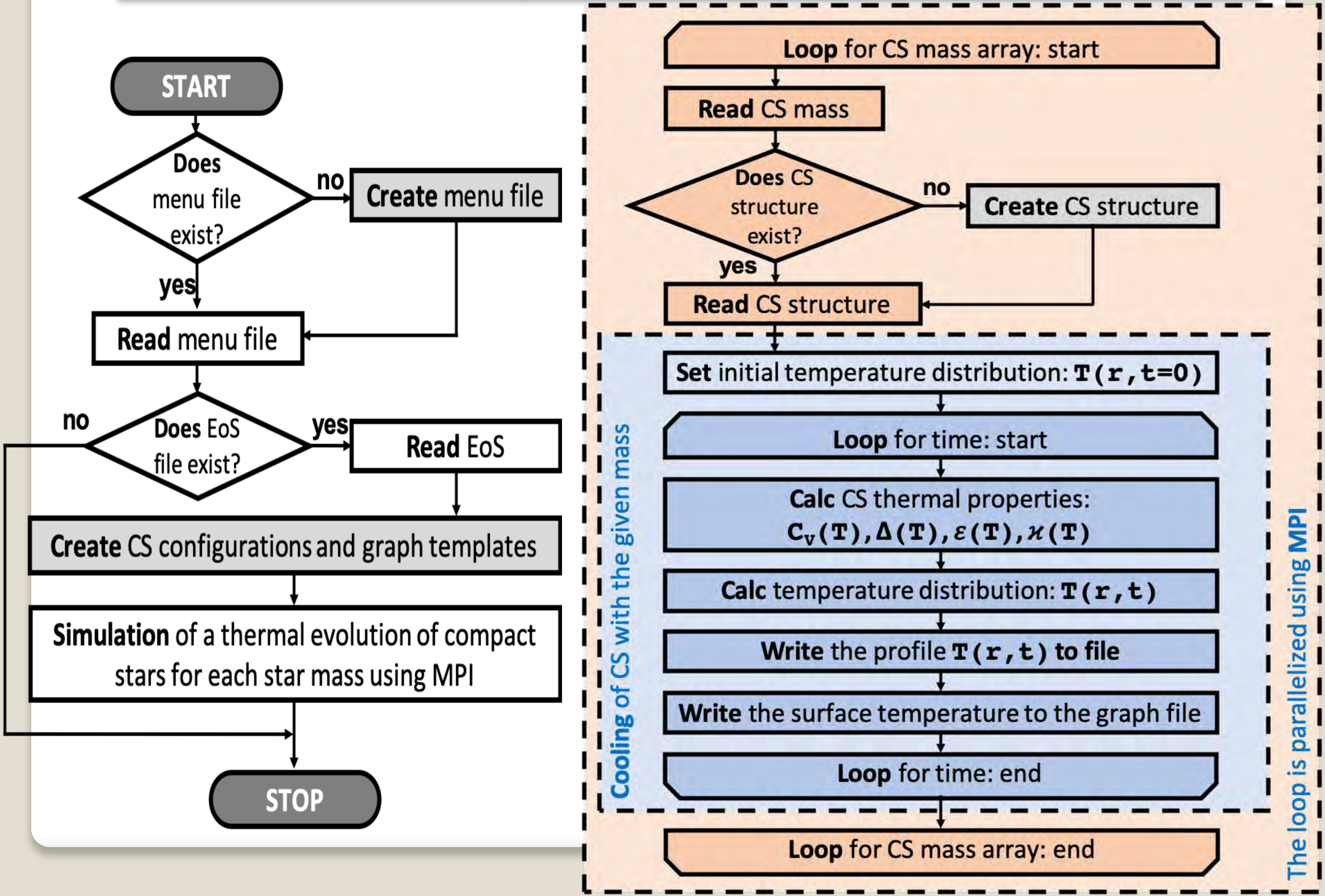
Blaschke, Grigorian, Voskresensky, A& A 368 (2001)561.

Page,Lattimer,Prakash & Steiner, Astrophys.J. 155,623 (2004)

Yakovlev, Levenfish, Potekhin, Gnedin & Chabrier , Astron. Astrophys , 417, 169 (2004)



Program Algorithm

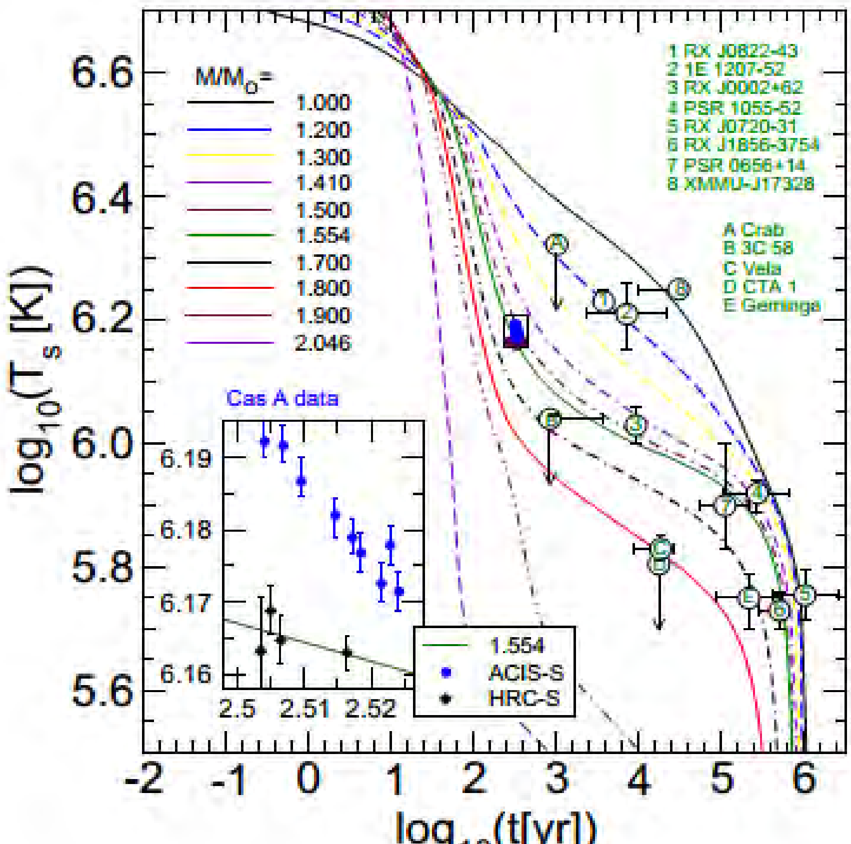


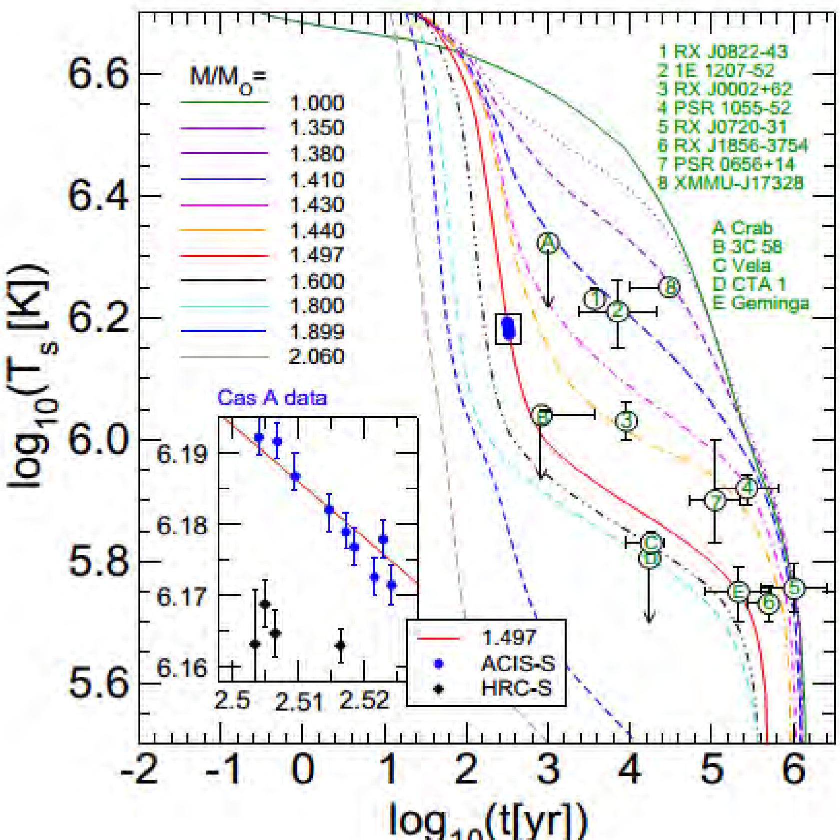
Model parameters – DD2

```
                              Menu_dd2_2017n.dat
Model Parametrs

The HOME directory is : .\Data\DD2\Configs-2
The EV UOTPUT directory : .\Data\DD2\17-12-2019\EV-DD2-pi-F4-o3-D
Make EoS file : 0
Make new config. file : 0
Read full EoS from a file : 1
Read from : .\EoS\DD2_HG
Hadronic EoS
LWalecka (0)  NLW (1) HDD (3) BSk20 (4): 3
Normal Shell : 0
Quark EoS SM model (1) Bag model (0) : 0
In case of SM  GF (0)  GL(1)  NJL (2) : 0
with Quark core : 1
without Mixed phase : 1
Superconducting Quark core : 1
Quark Star : 0
Medium effects : 1
Pion condensate : 1
Crust Model (Yakovlev - Y Tsuruta - T our - G)  : G
Gaps in Hadrons Model (Yakovlev - Y AV18 - A Schwenk - U Armen-fit - F)  : F
for F-fit p-Gap
1-AO
2-BCLL
3-BS
4-CCDK
5-CCYms
6-CCYps
7-EEHO
8-EEHOr
9-T
: 4
for F-fit n-Gap
2-AWP2
3 - AWP3
4 - CCDK
5 - CLS
6 - GIPSF
7 - MSH
8 - SCLBL
9 - SFB
0 - WAP : o
XGaps in 2SC QModel constant 0 - 0
 constant 0.1 MeV - 1
constant 0.05 MeV - 5
constant 0.03 MeV - 3
 rising 0.03 +  MeV - A
 incrising 0.03 - MeV - B
       constant 0.03 ++ MeV - C
 constant 0.03 -- MeV - D
 : C
```

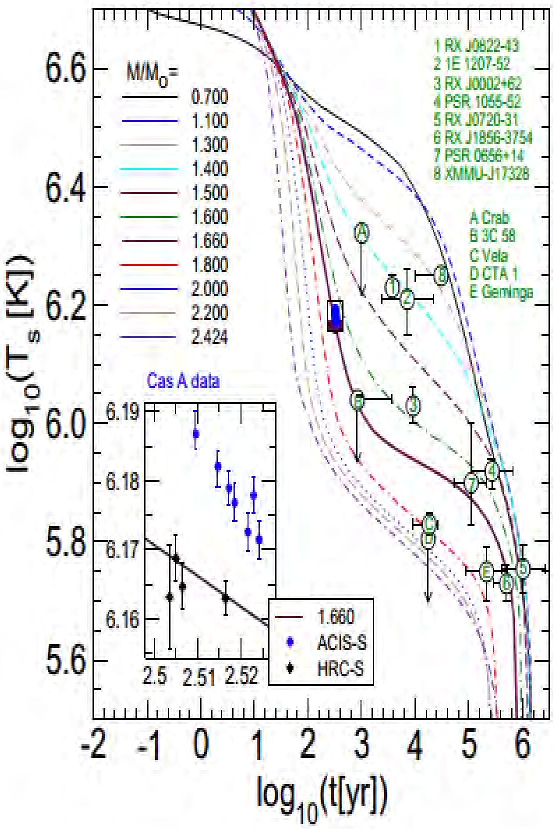
```
                              Menu_dd2_2017n.dat
Gap factors in HM
Protons  1S0p : 1
Neutrons 1S0n : 1
Neutrons 3P2n : 0.1

End time point log10(t/yr) : 8

initial temperatur in MeV : 0.5

minimal value of log Temperature : 5.5

Print output files for LogN-LogS : 0

Print profiles for the time points : 0

Number of points : 7
0 0 0 0 0 0 0

 The Masses [Mo] of Configurations to be Cooled

Number of points : 51
1.450
0.5 0.51 0.52 0.53 0.54 0.55 0.56 0.57 0.58 0.59
0.6 0.61 0.62 0.63 0.64 0.65 0.66 0.67 0.68 0.69
0.7 0.71 0.72 0.73 0.74 0.75 0.76 0.77 0.78 0.79
0.8 0.81 0.82 0.83 0.84 0.85 0.86 0.87 0.88 0.89
0.9 0.91 0.92 0.93 0.94 0.95 0.96 0.97 0.98 0.99



1.0 1.01 1.02 1.03 1.04 1.05 1.06 1.07 1.08 1.09
1.10 1.11 1.12 1.13 1.14 1.15 1.16 1.17 1.18 1.19
1.20 1.21 1.22 1.23 1.24 1.25 1.26 1.27 1.28 1.29
1.30 1.31 1.32 1.33 1.34 1.35 1.36 1.37 1.38 1.39
1.40 1.41 1.42 1.43 1.44 1.45 1.46 1.47 1.48 1.49
1.50 1.51 1.52 1.53 1.54 1.55 1.56 1.57 1.58 1.59
1.60 1.61 1.62 1.63 1.64 1.65 1.66 1.67 1.68 1.69
1.70 1.71 1.72 1.73 1.74 1.75 1.76 1.77 1.78 1.79
1.80 1.81 1.82 1.83 1.84 1.85 1.86 1.87 1.88 1.89
1.90 1.91 1.92 1.93 1.94 1.95 1.96 1.97 1.98 1.99
2.00 2.01 2.02 2.03 2.04
2.05 2.06 2.07 2.08 2.09
2.10 2.11 2.12 2.13 2.14 2.15 2.16 2.17 2.18 2.19
2.20
2.21 2.22 2.23 2.24 2.25 2.26 2.27 2.28 2.29
2.30 2.31 2.32 2.33 2.34 2.35 2.36 2.37 2.38 2.39
2.40 2.41

1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2
```

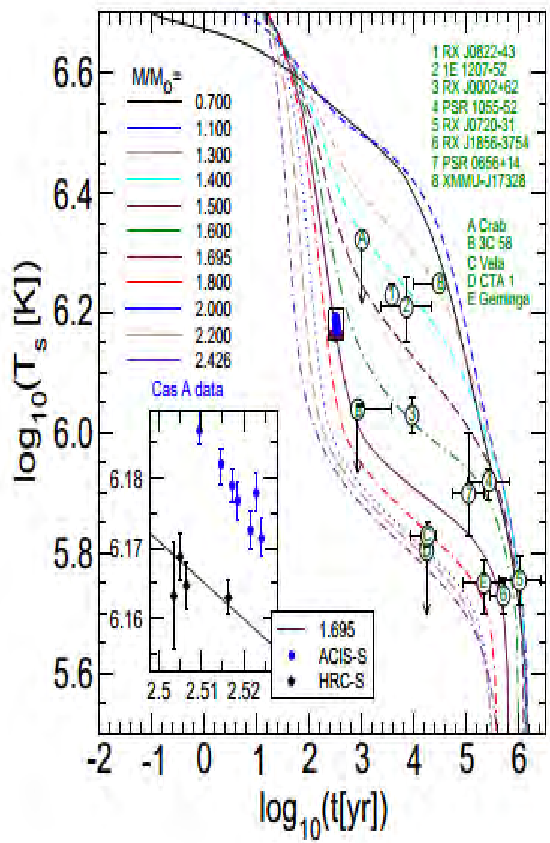
HDD – AV18 , Yak.
ME nc = 3 n0

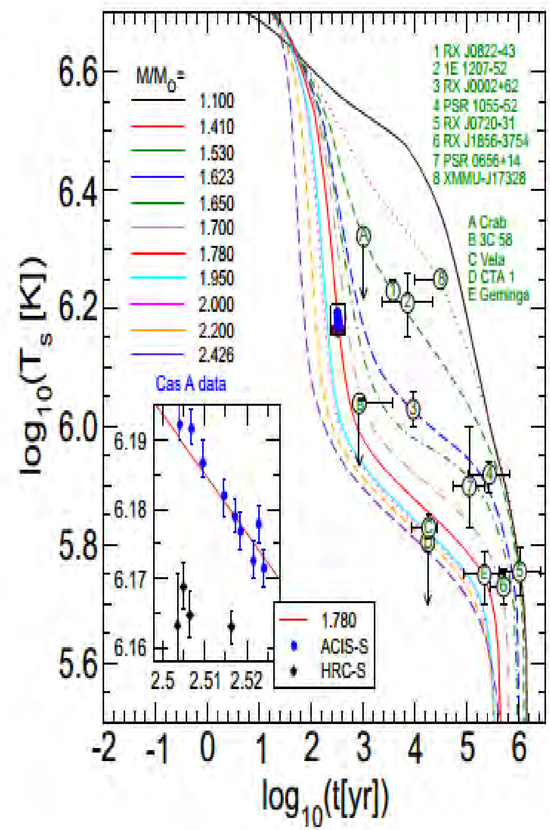




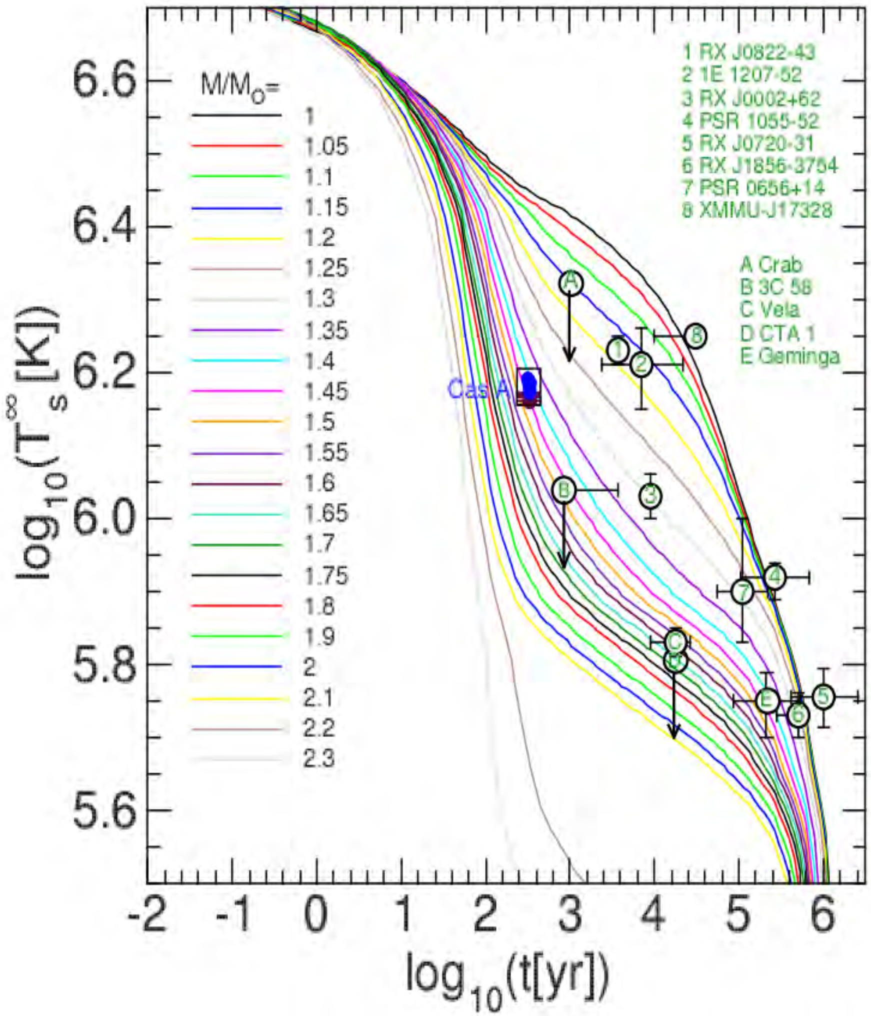
DD2 – EEHOr
ME-nc=1.5,2.0,2.5n0

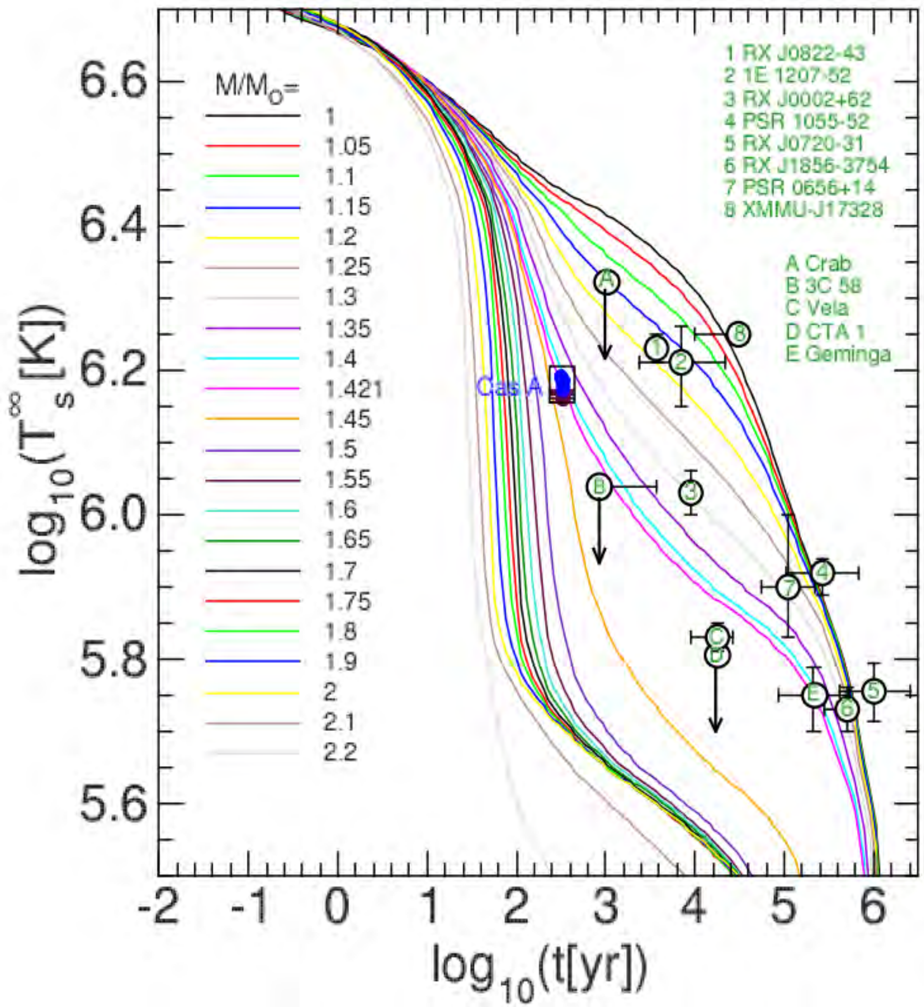




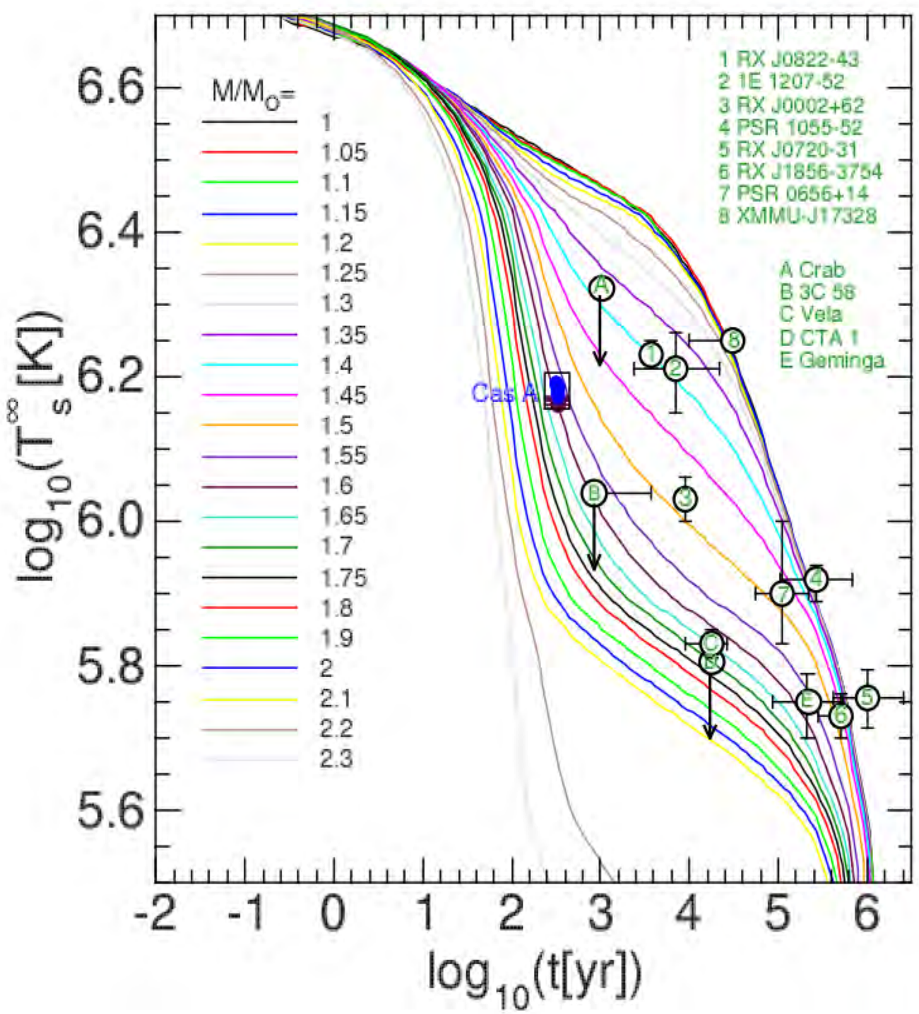


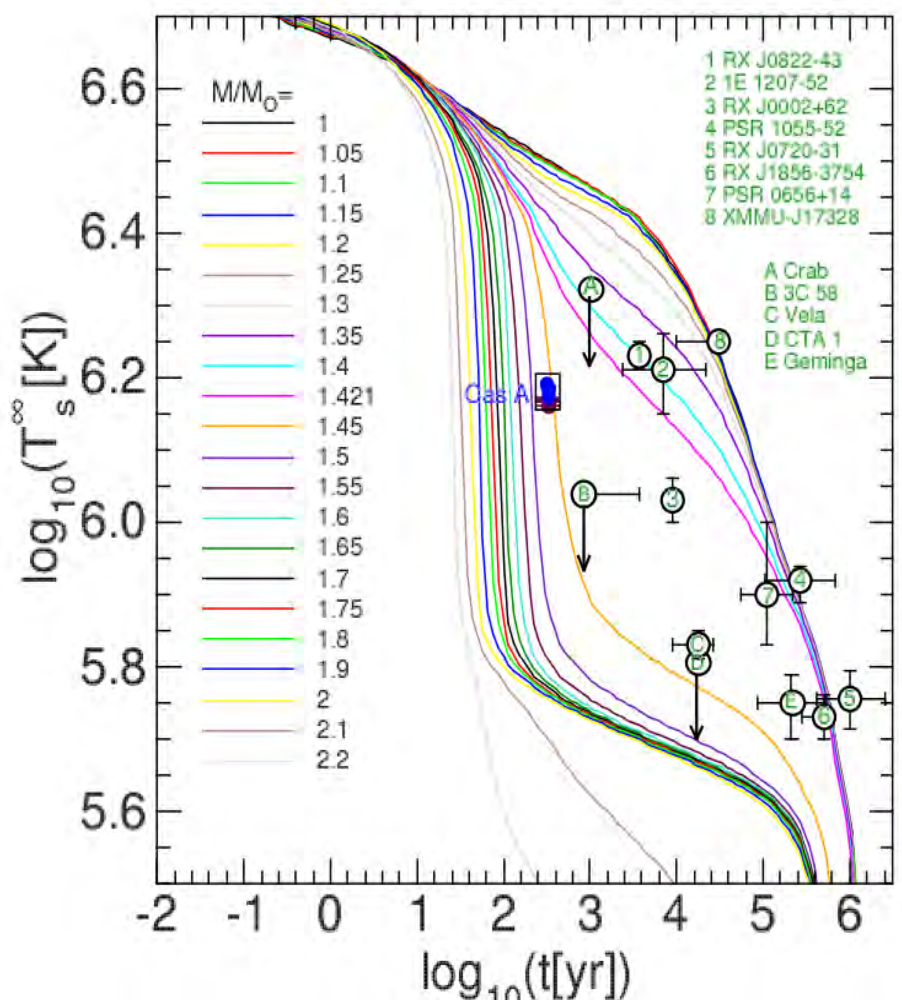
MKVOR – BCLL, TN-FGA ME-nc=3.0n0



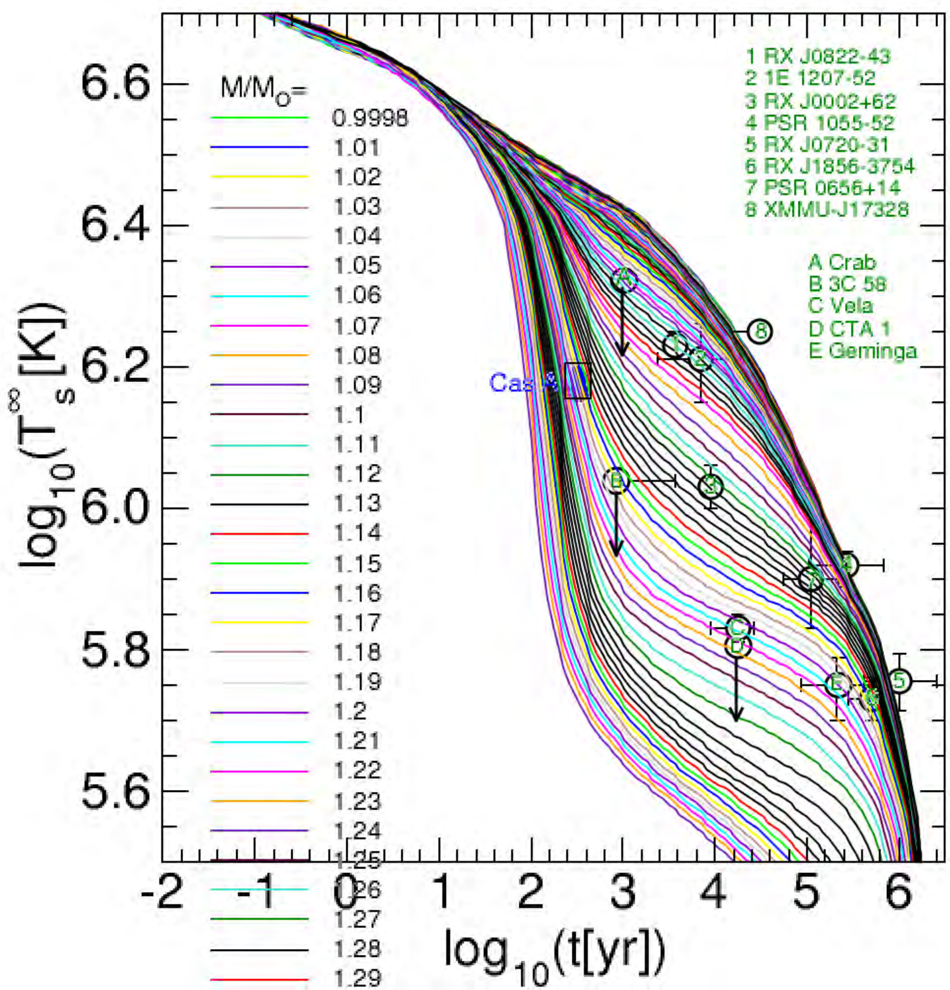


MKVOR Hyp – EEHOr, TN-FGA ME-nc=3.0n0

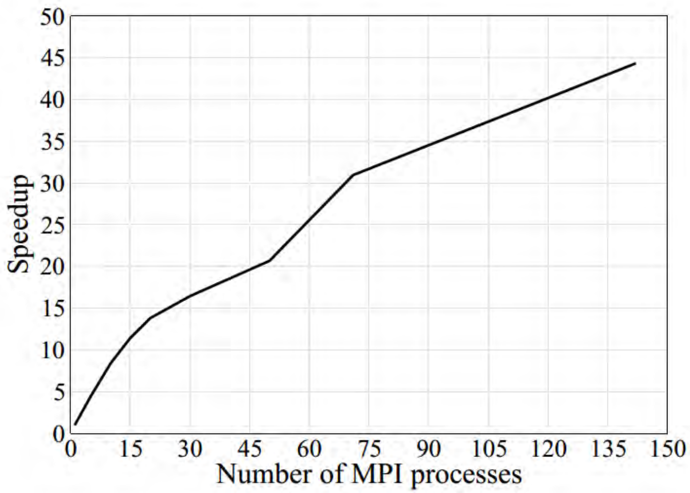




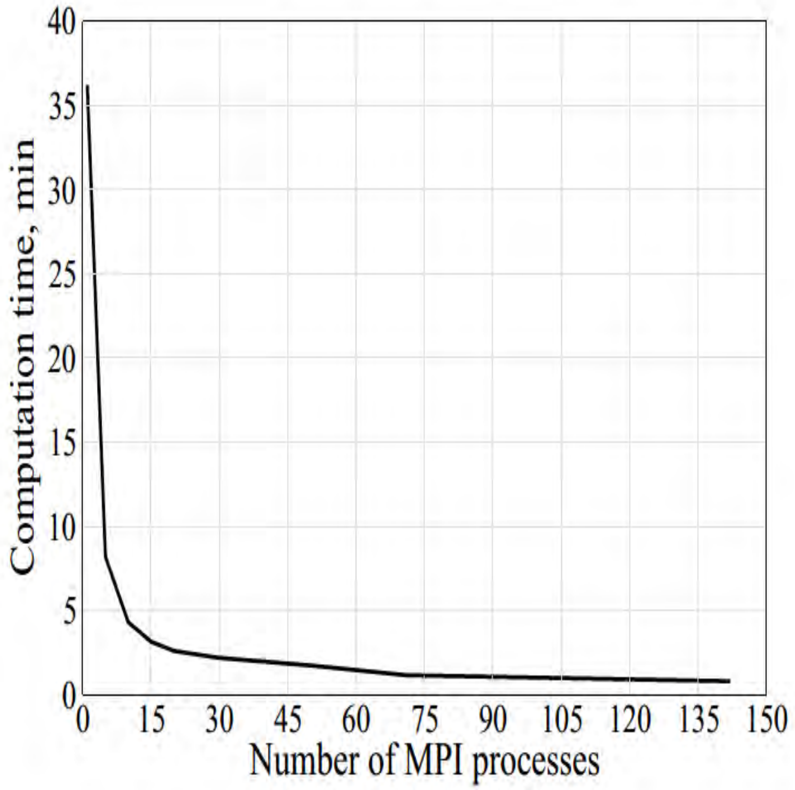
Results produced with use of MPI Technology

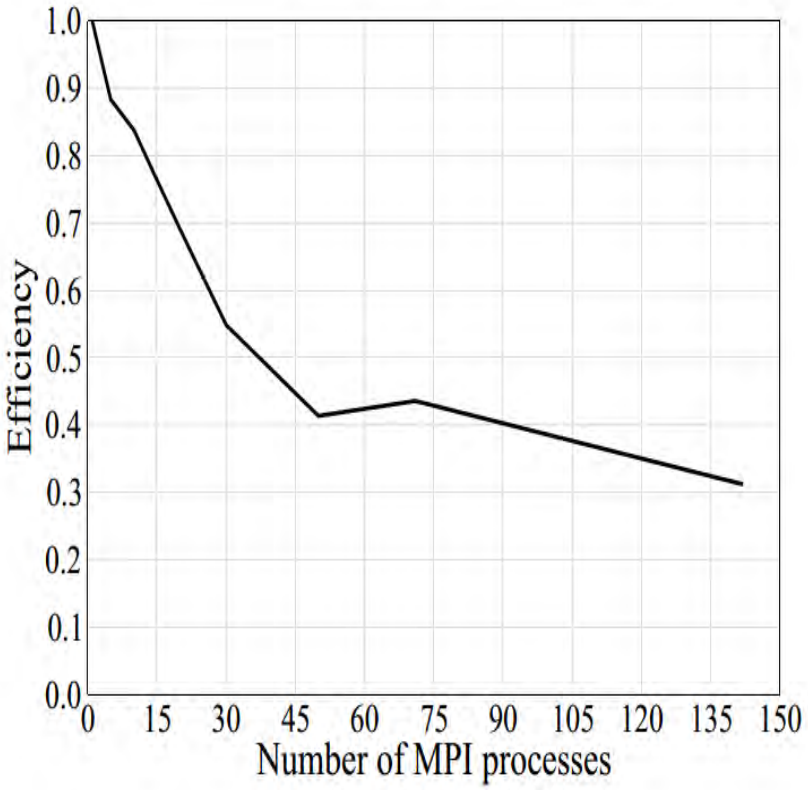


142 configurations has been calculated in 0m49s on the 142 processes.
On 1 process it takes 36m14s
– acc is ~ 44 times

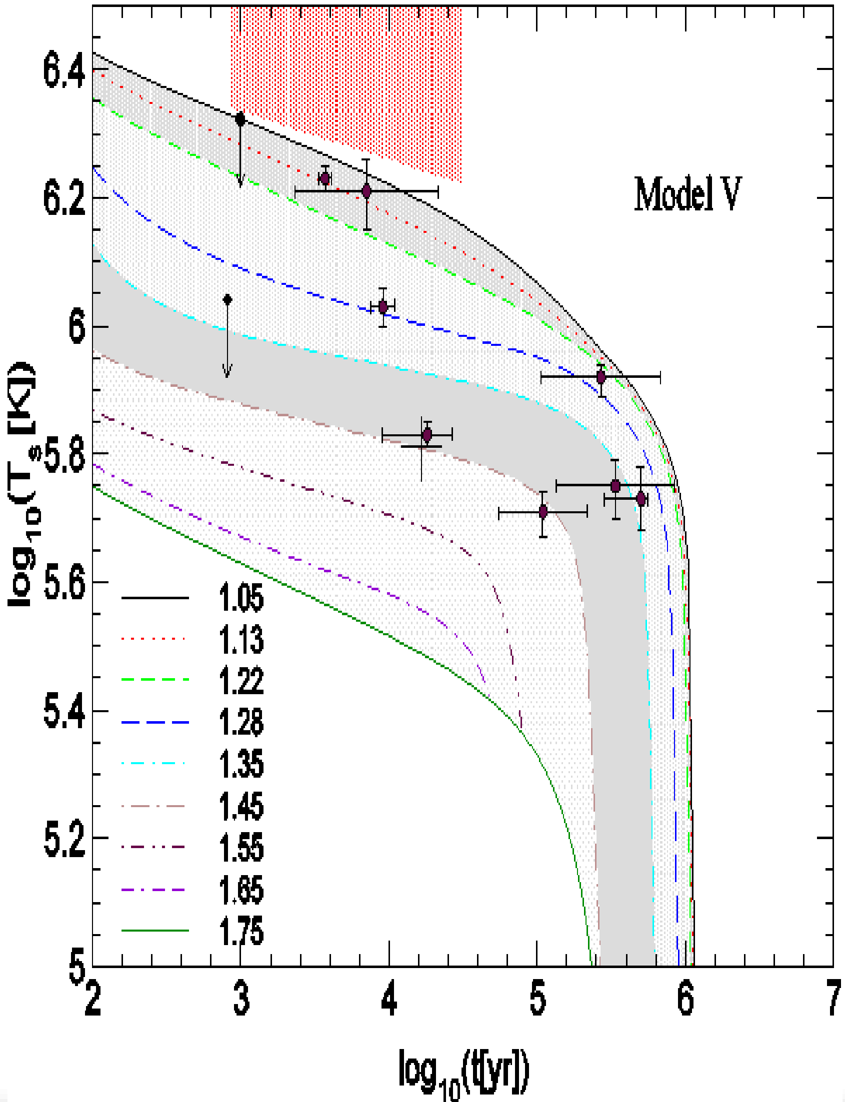


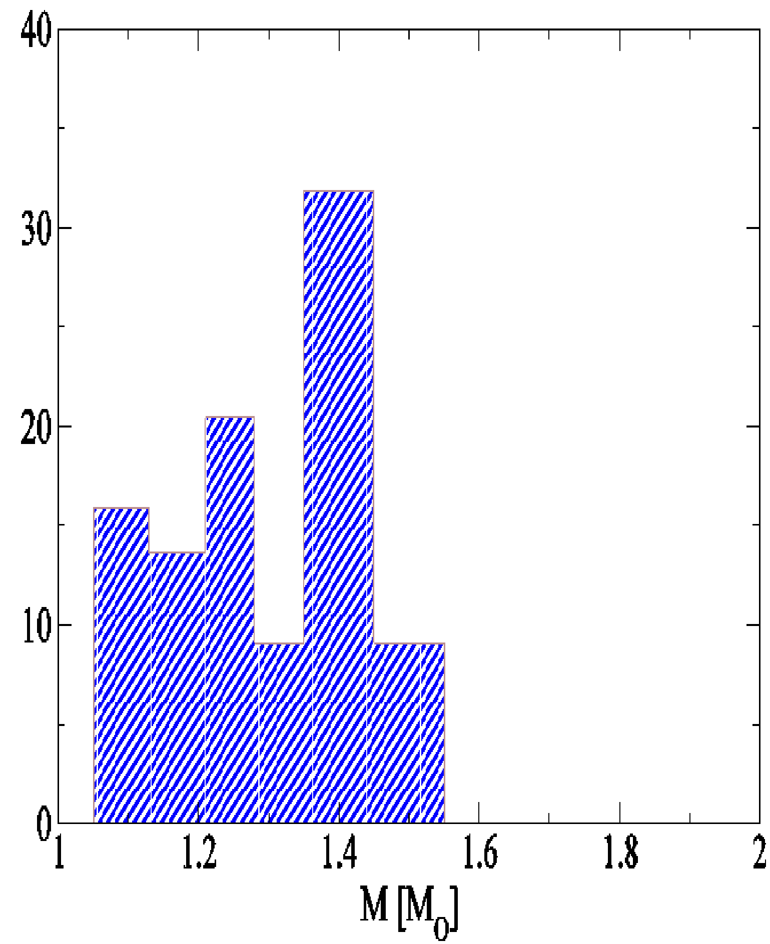
Calculation Time and efficiency



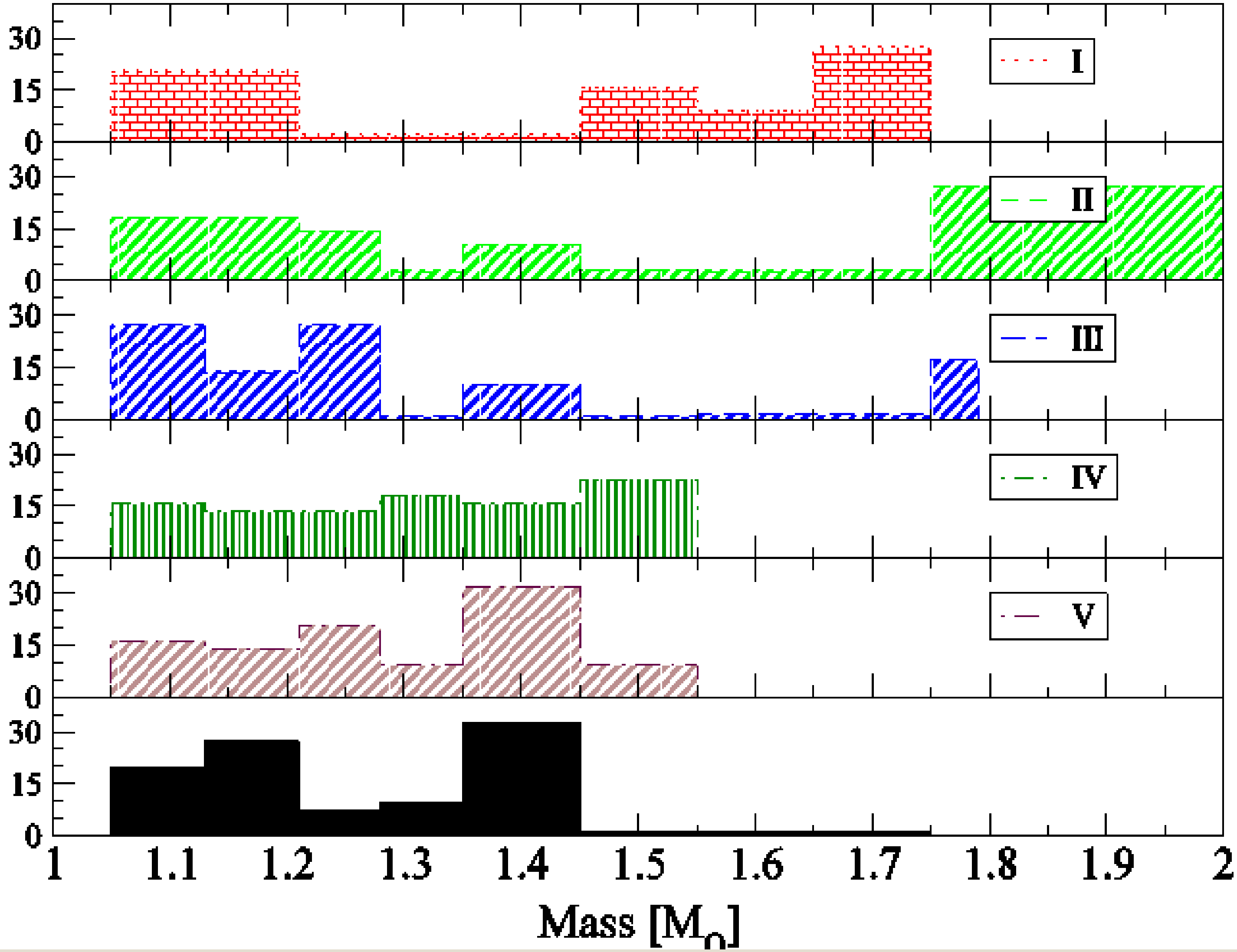


Mass Distribution of NS

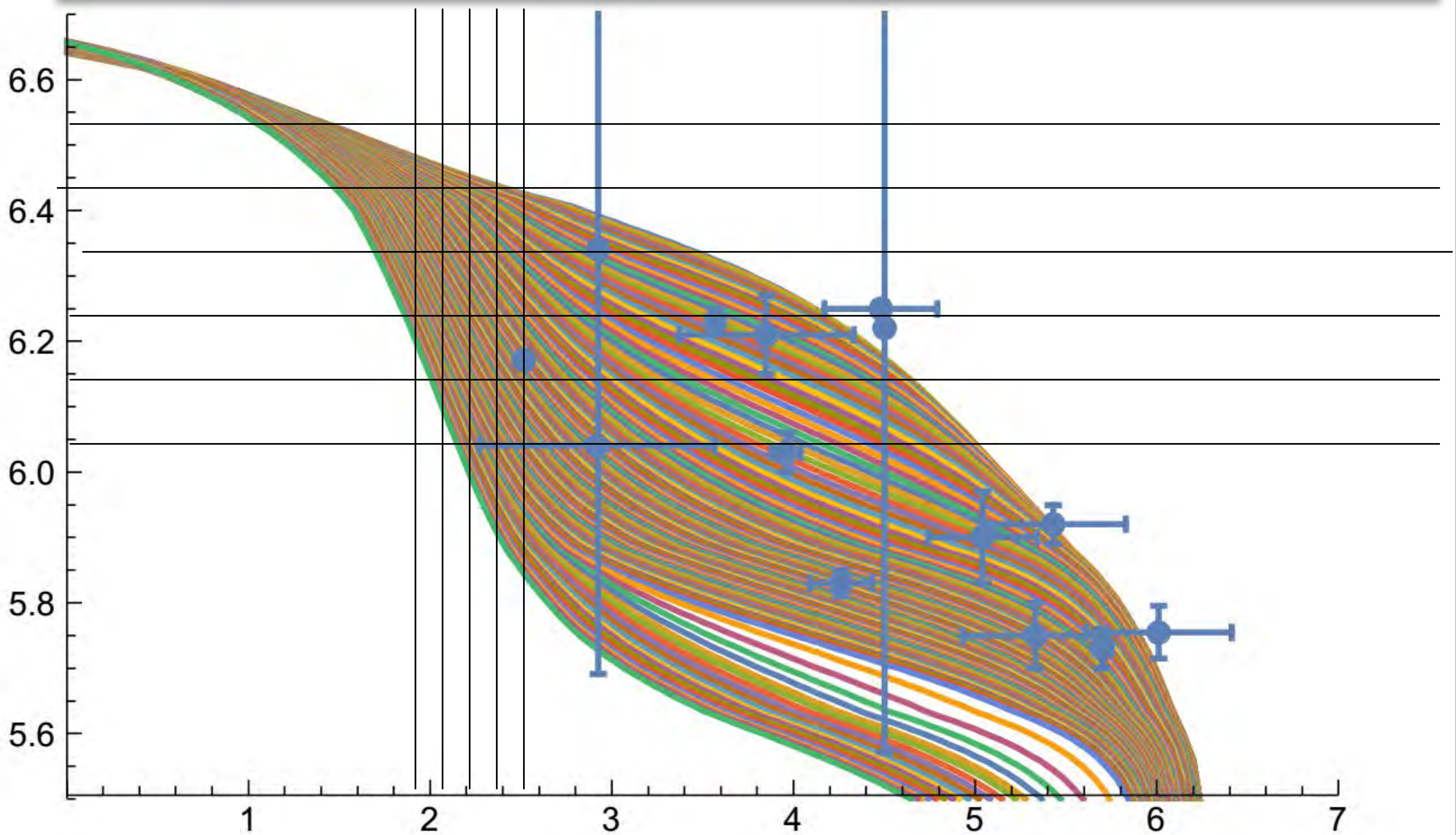




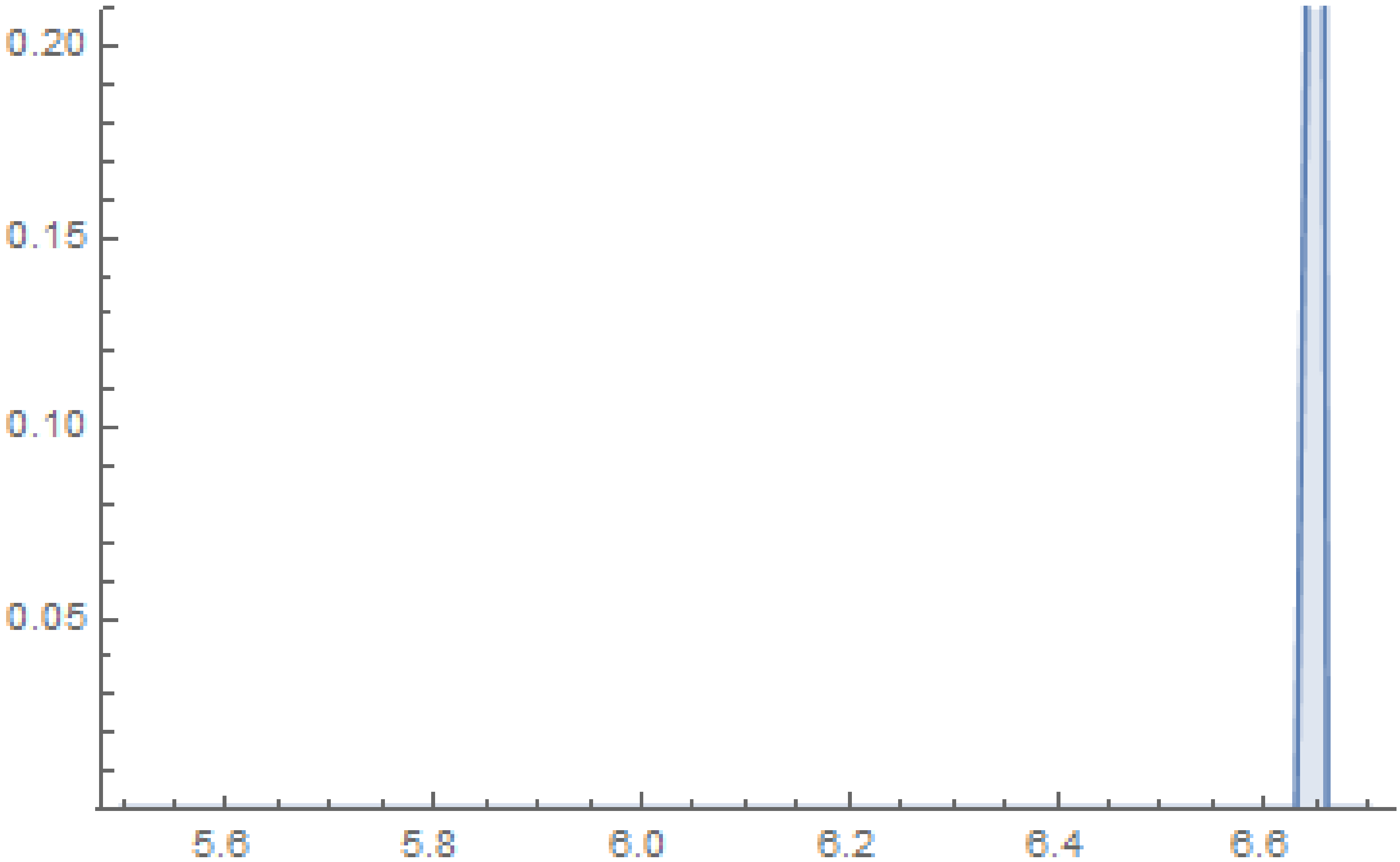
EXTRACTED MASS DISTRIBUTION



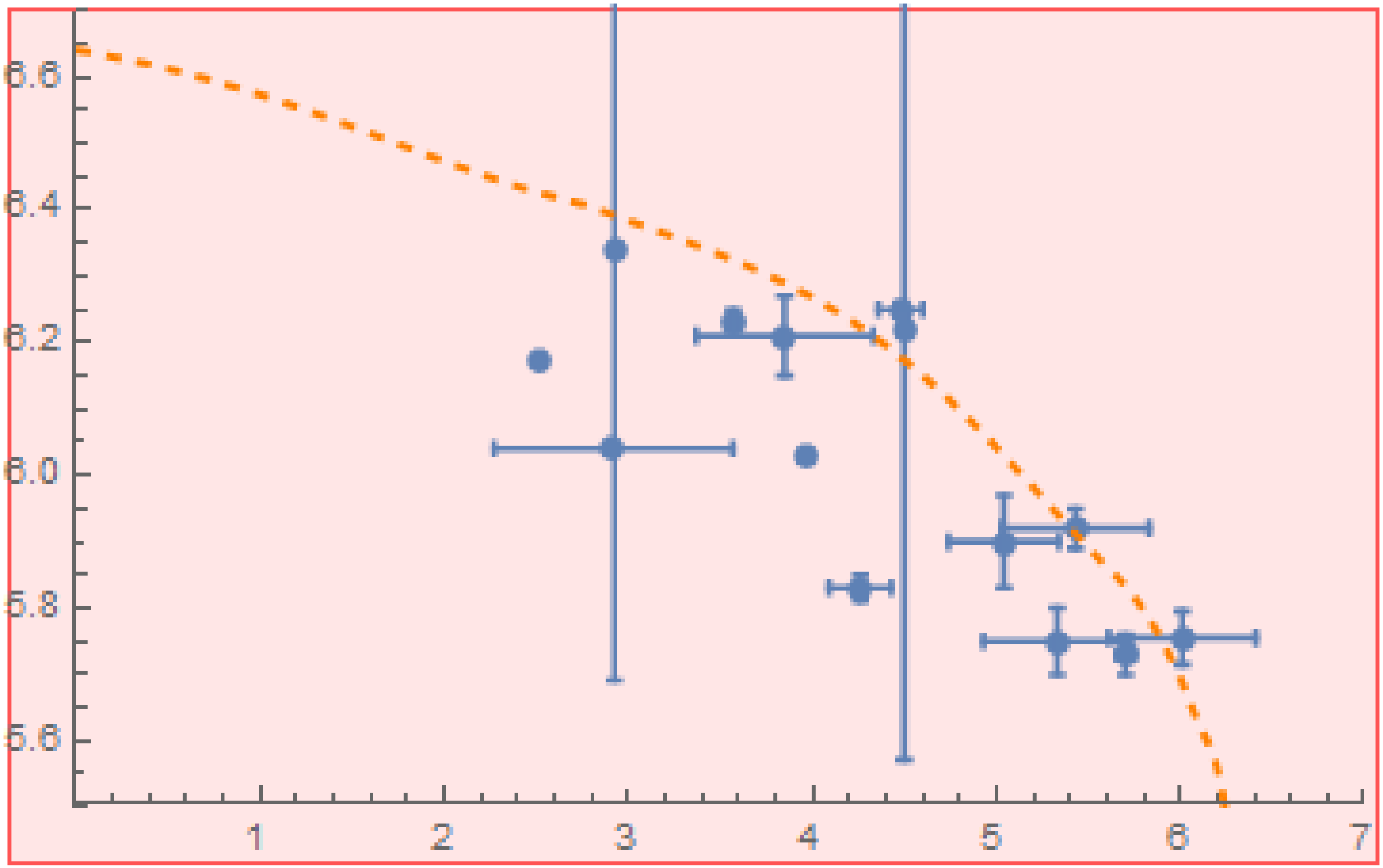
Distribution of Evolution tracks via Temperature at given Time



Distribution of Evolution tracks via Temperature at given Time

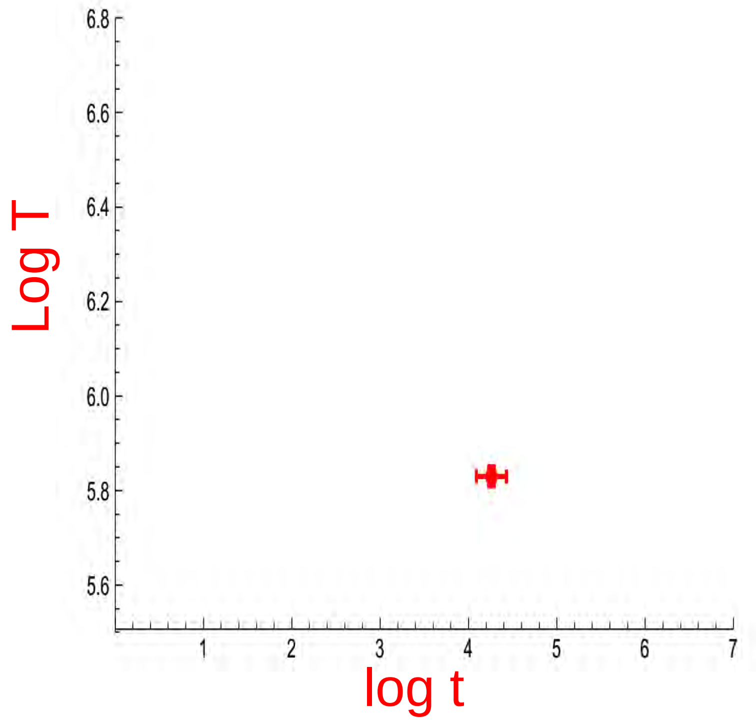


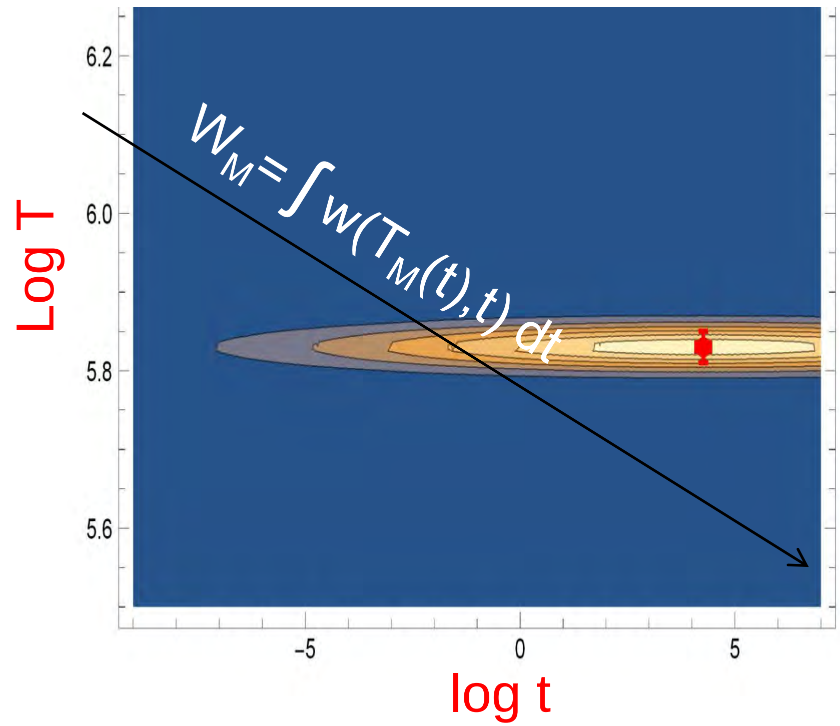
Evolution tracks for different NS Masses



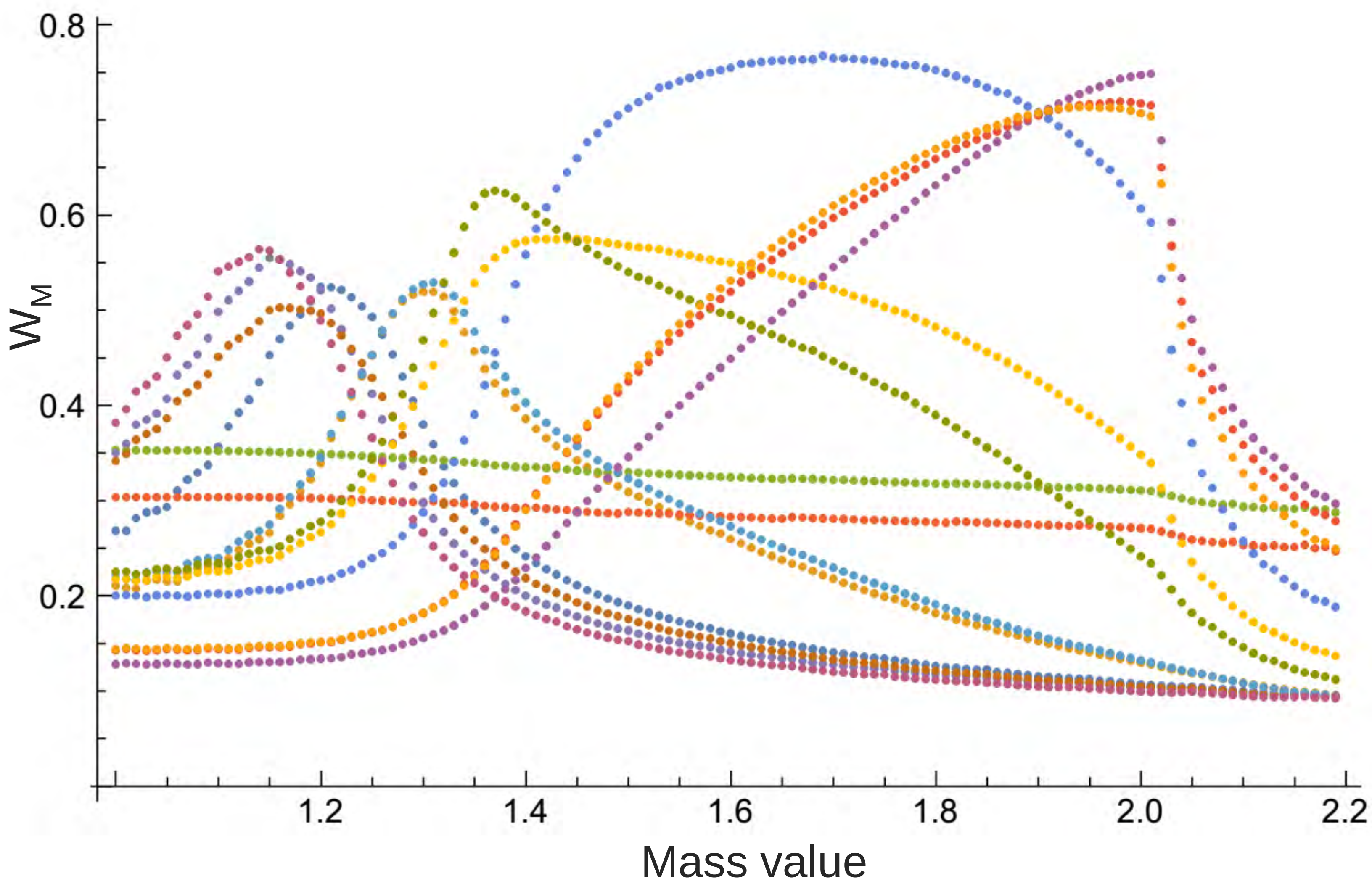
Weighting of Data point on the Temperature - Age Diagram

$$w(T,t) = \mathrm{Exp}\{ - (\log T - \log T_D)^2/\sigma_T - (\log t - \log t_D)^2/\sigma_t \}$$





Expected Mass value for the Data points on the T - t Diagram



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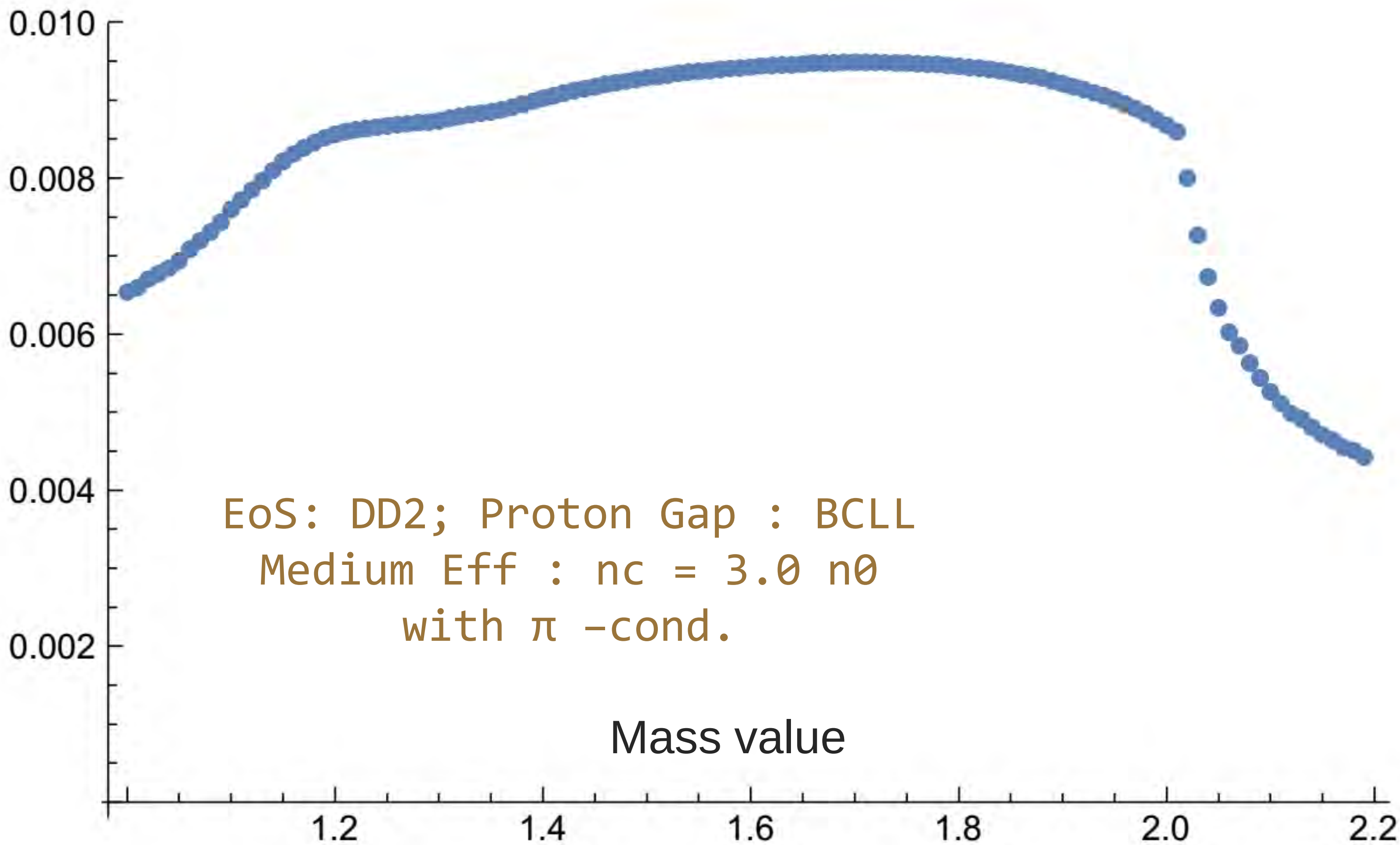

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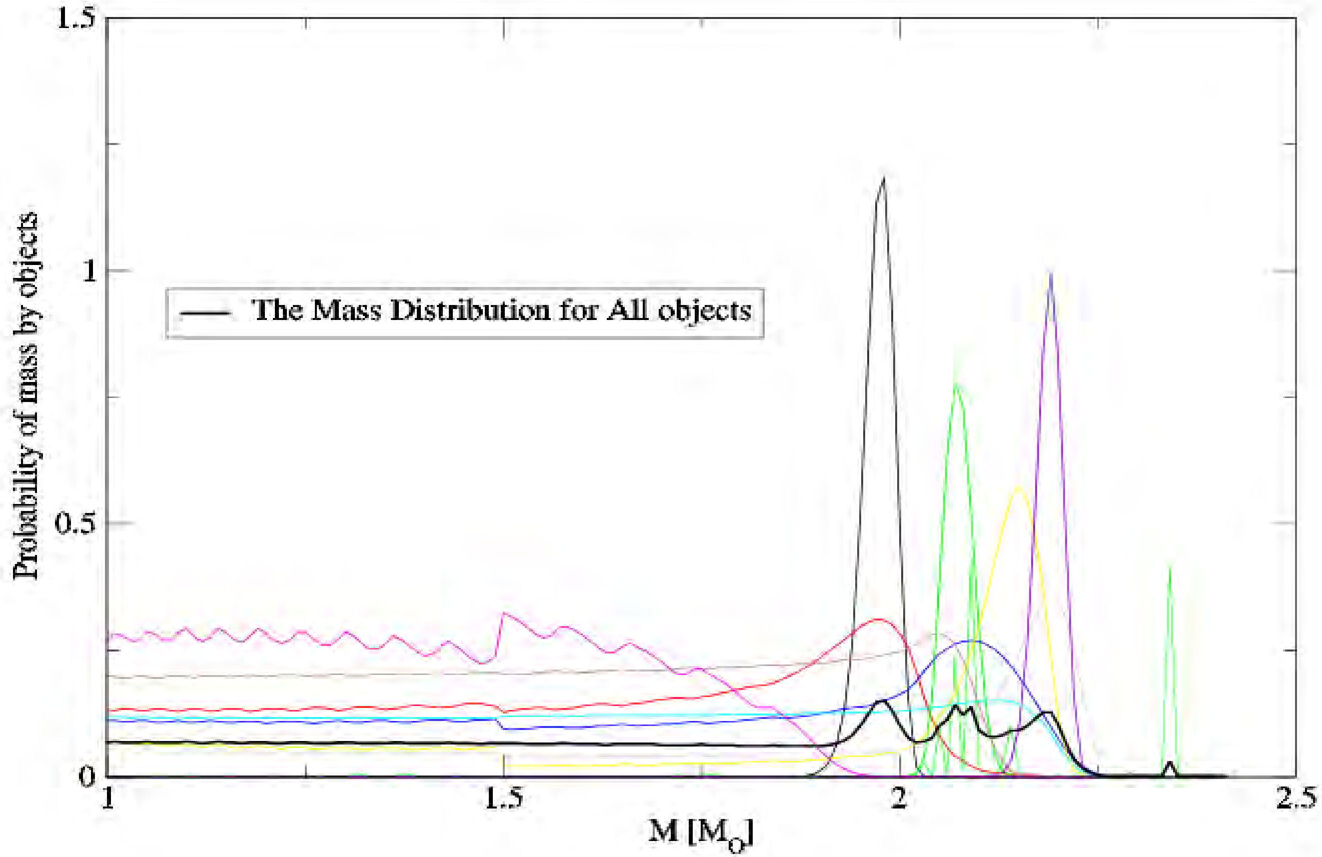
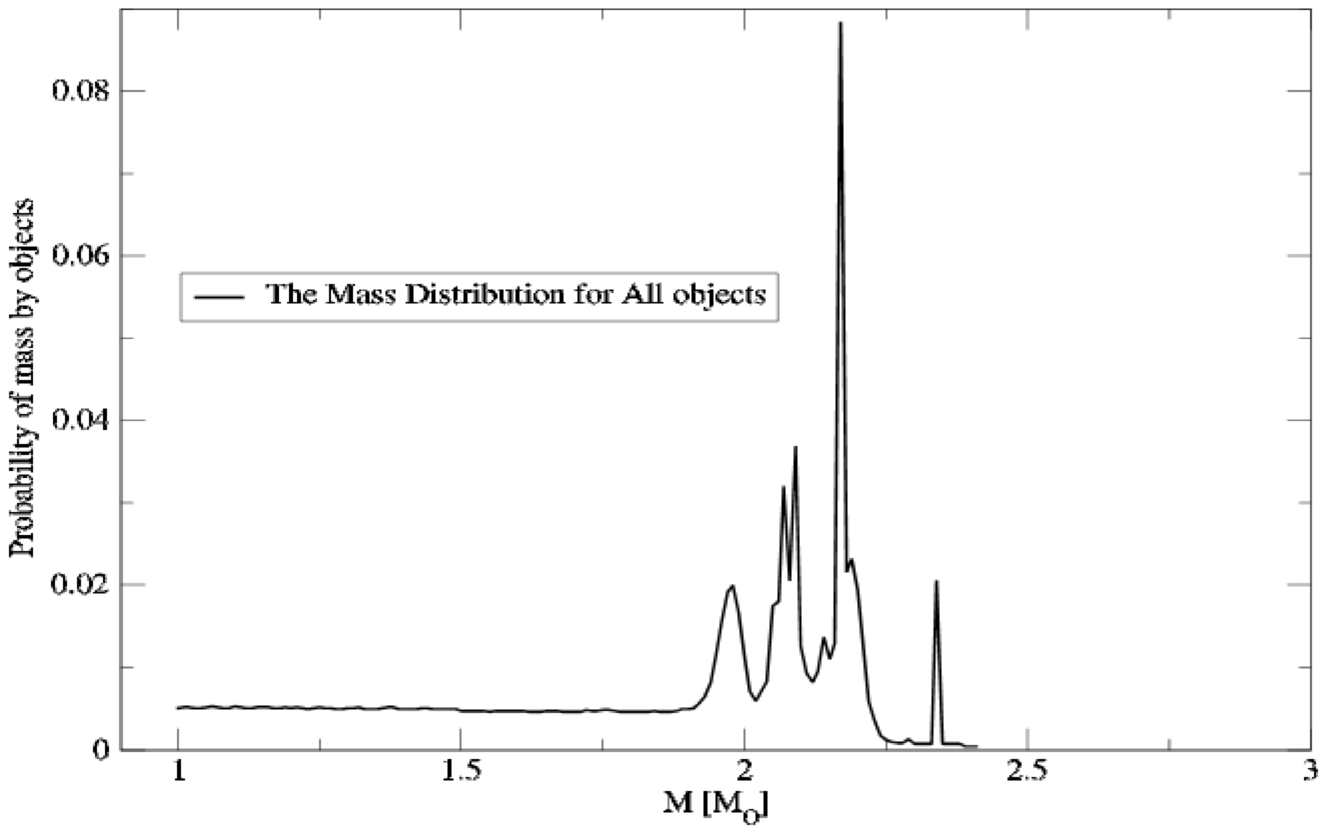
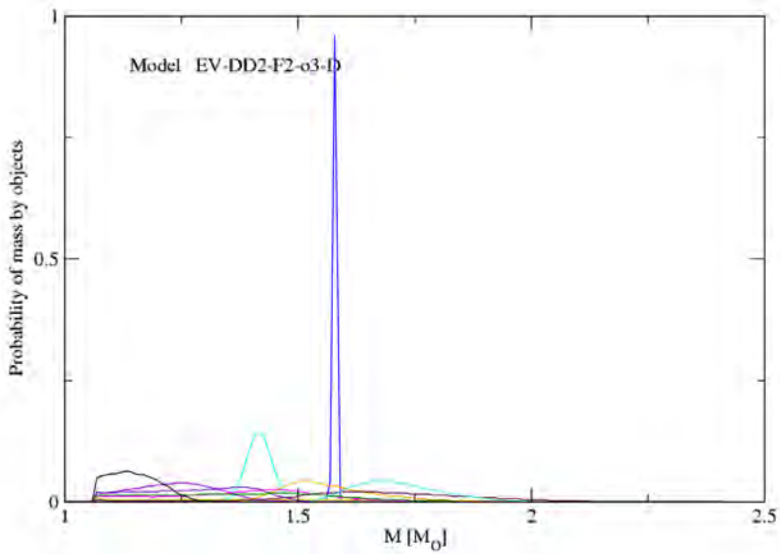


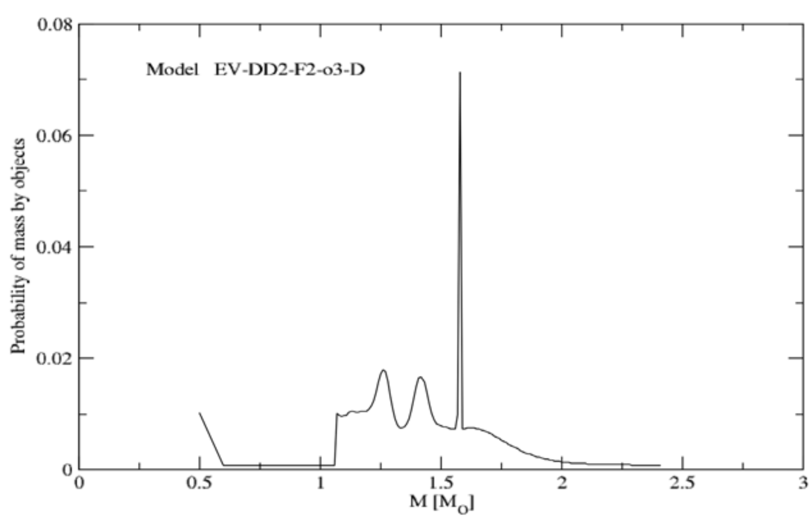
Expected Mass value for the Data points on the T - t Diagram

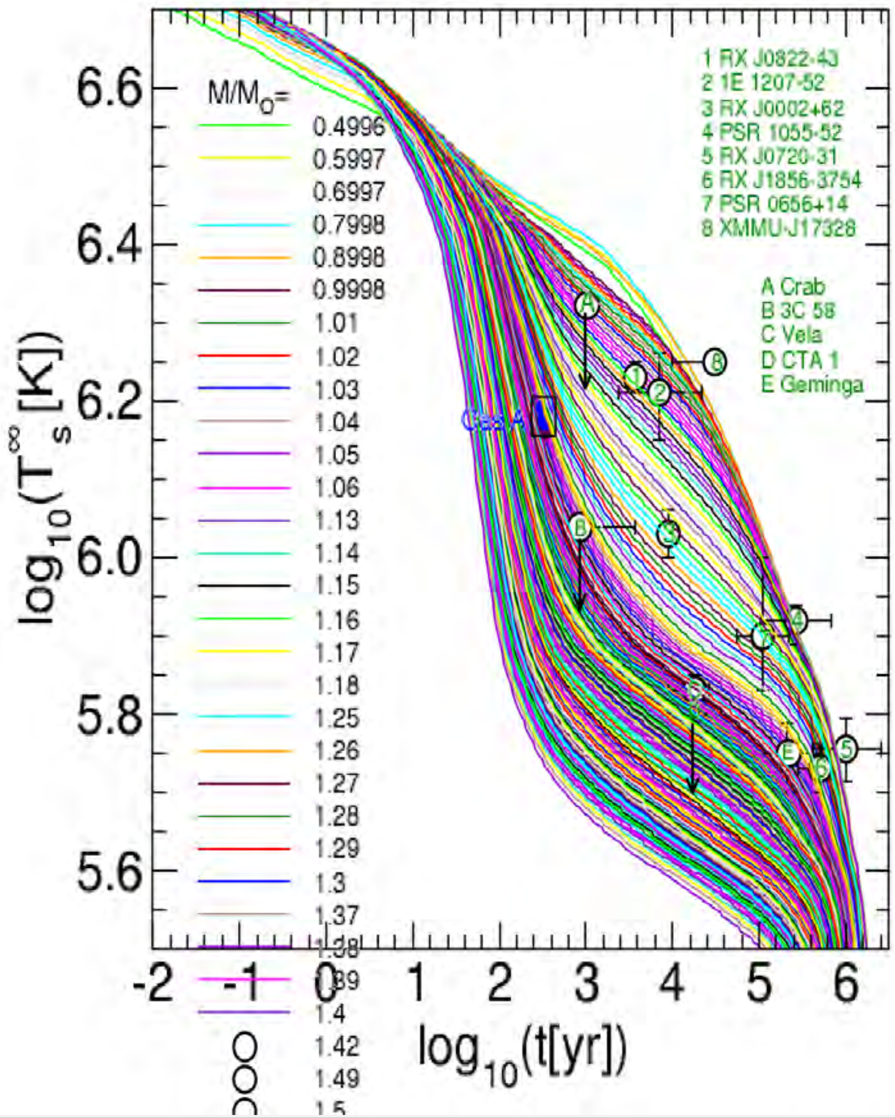


EoS: DD2; Proton Gap : BCLL Medium Eff : nc = 3.0 n0 with π – cond.

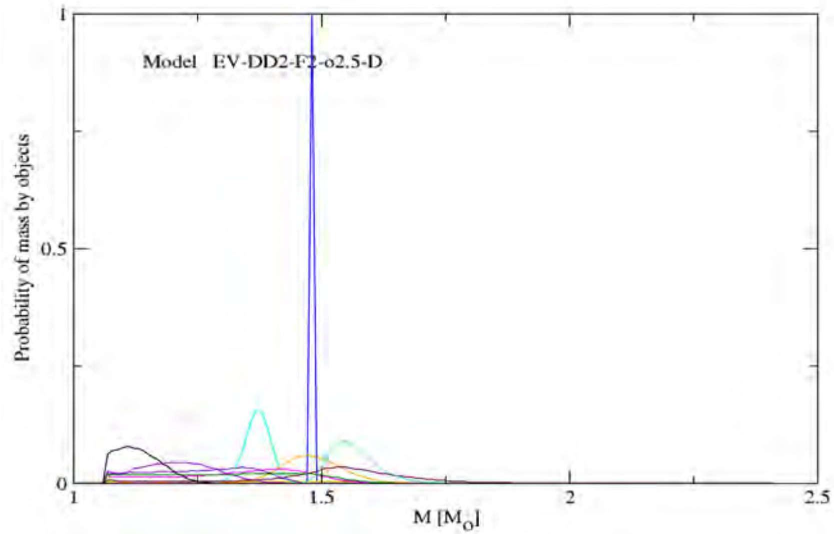
Model DD2 F2 o 3.0

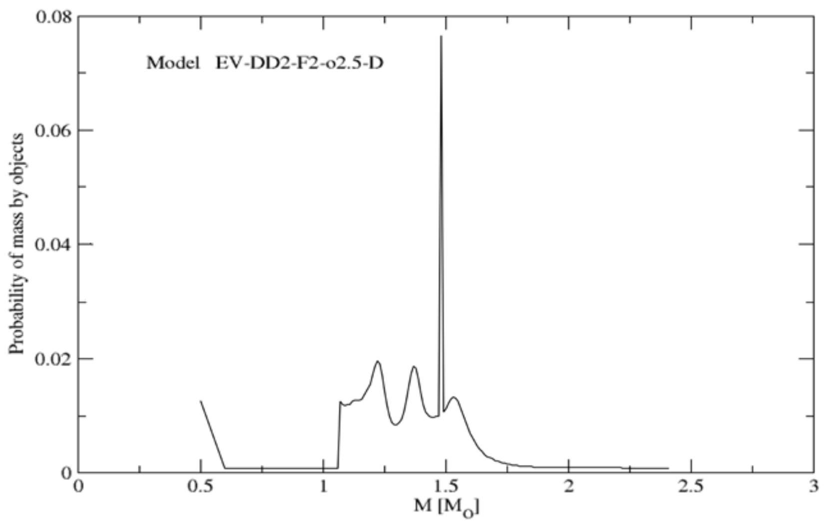


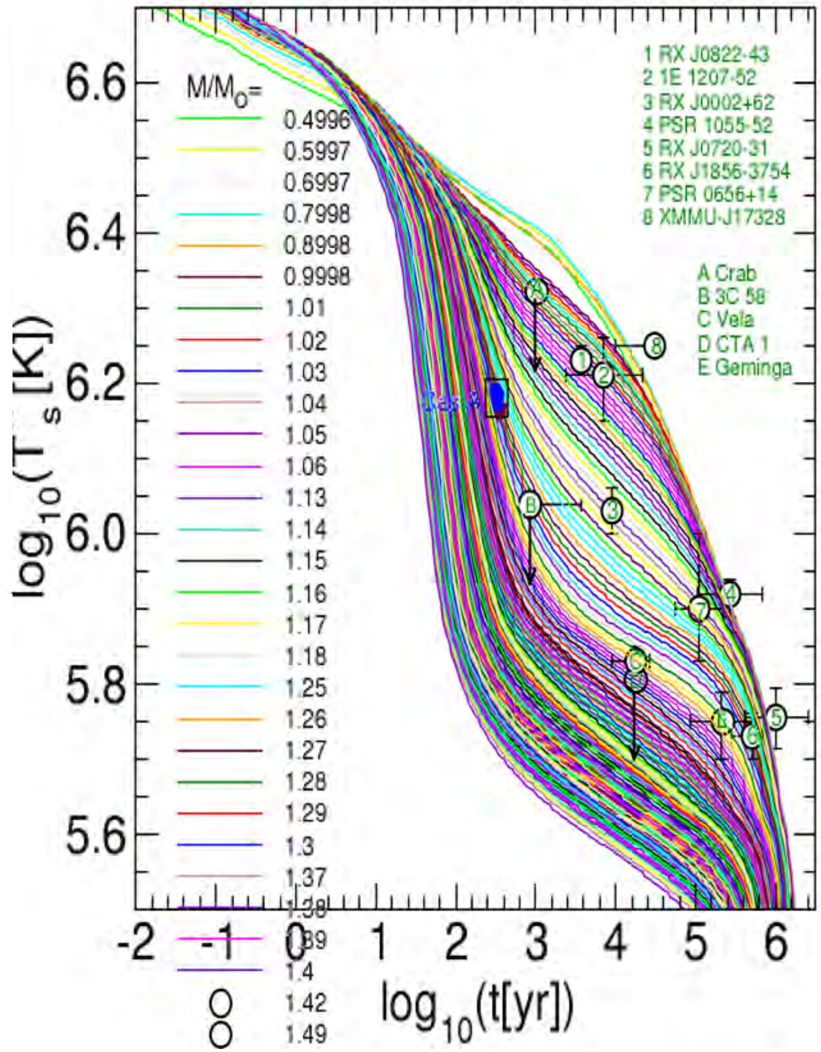




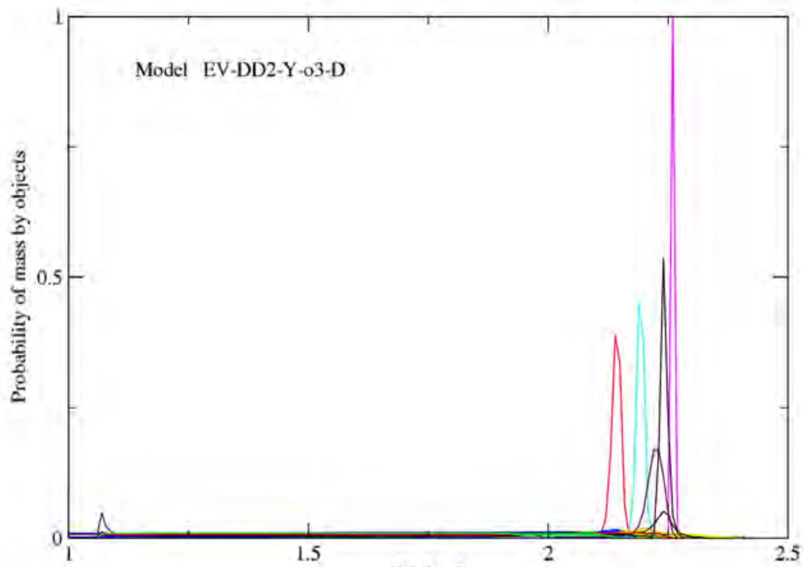
Model DD2 F2 o 2.5

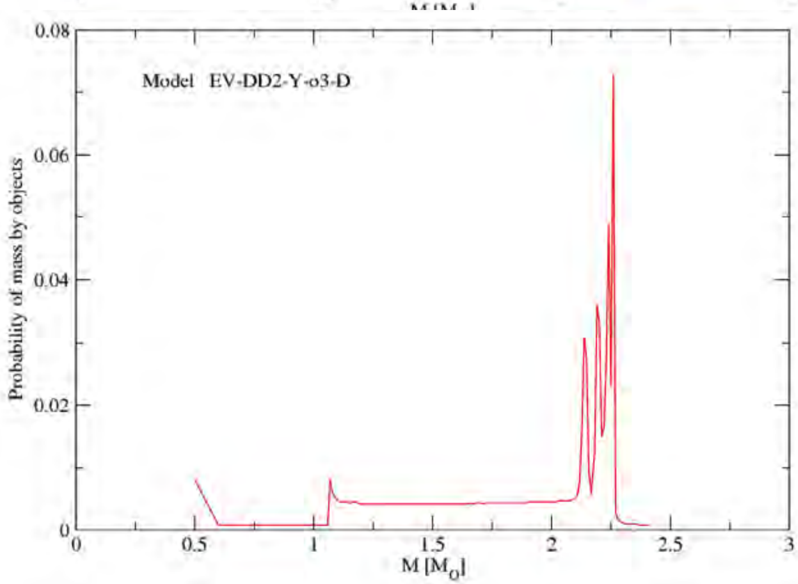


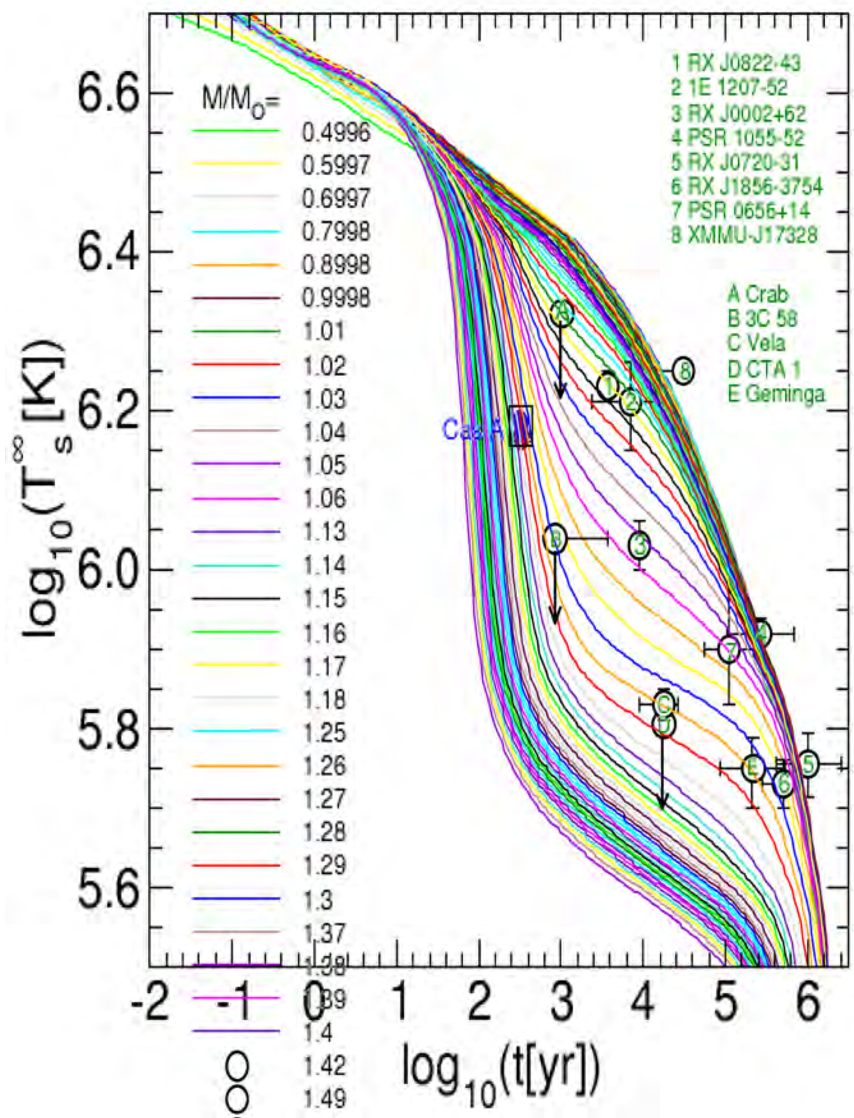




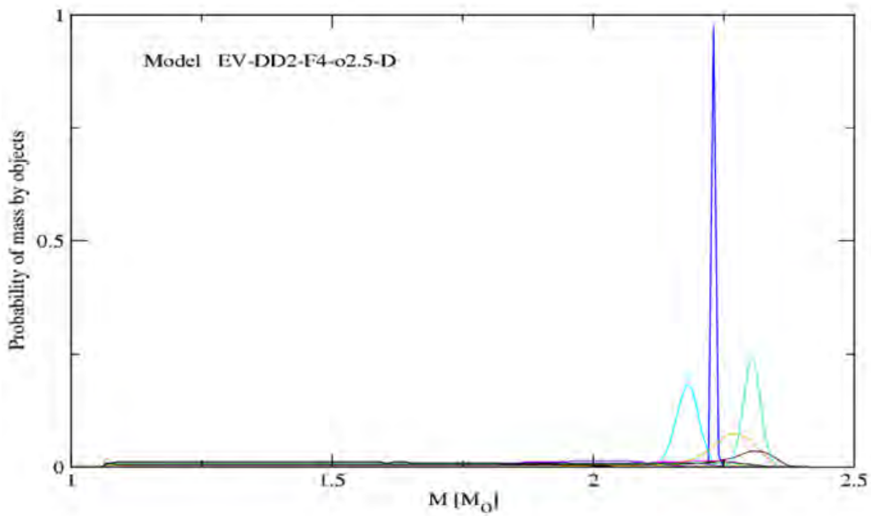
Model DD2 Y o 3.0

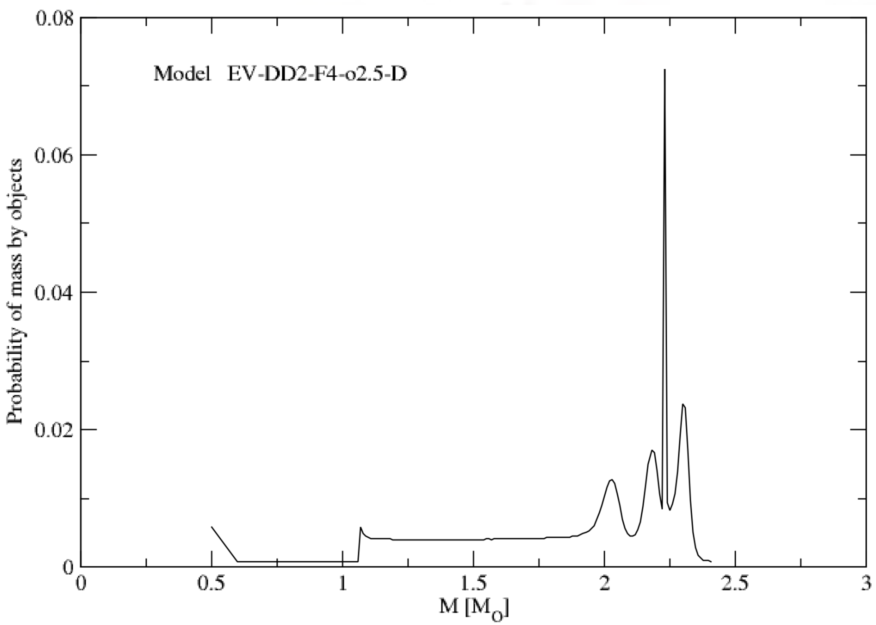


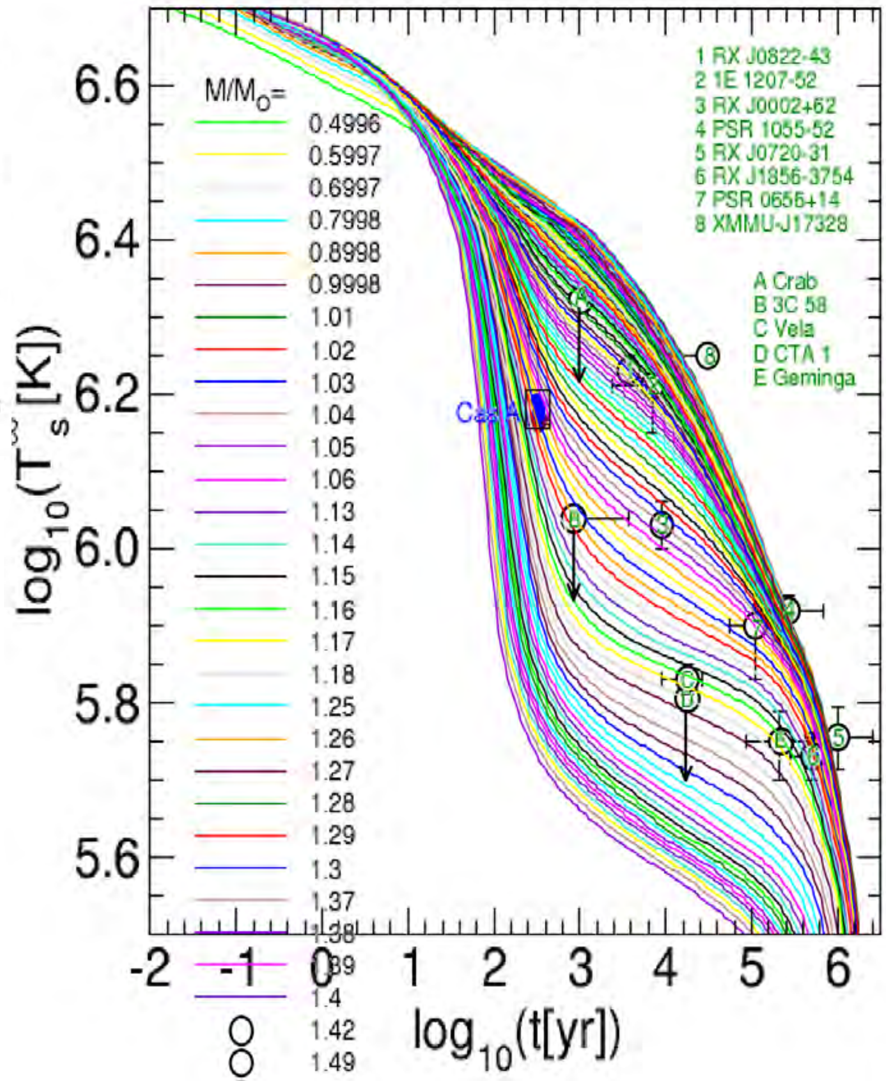




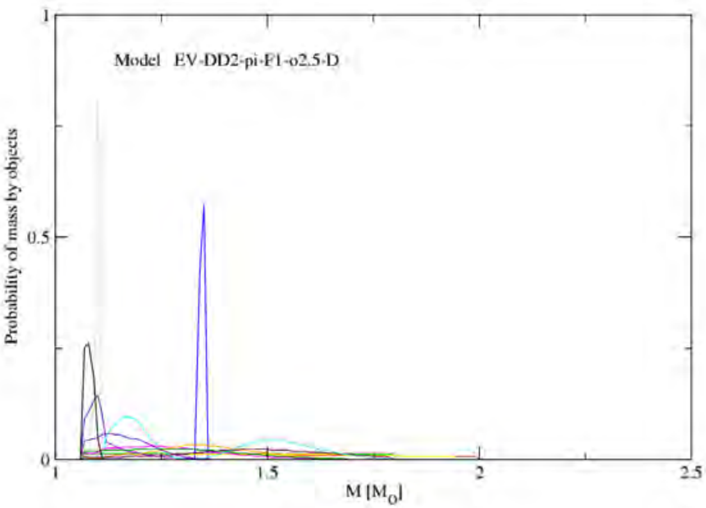
Model DD2 F4 o 2.5

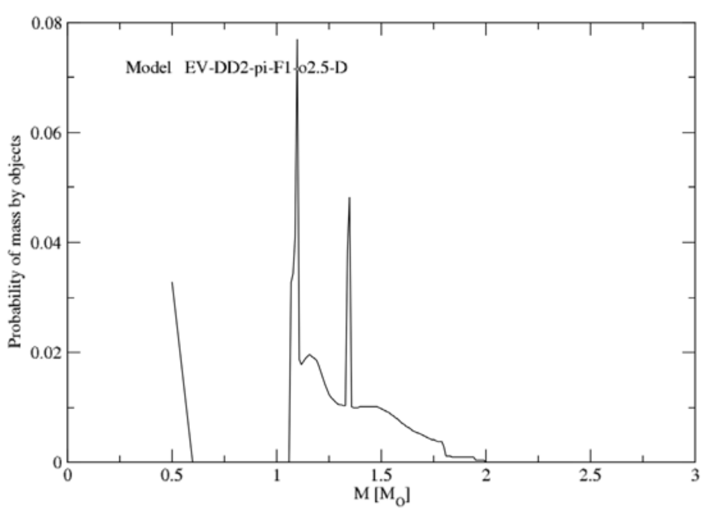


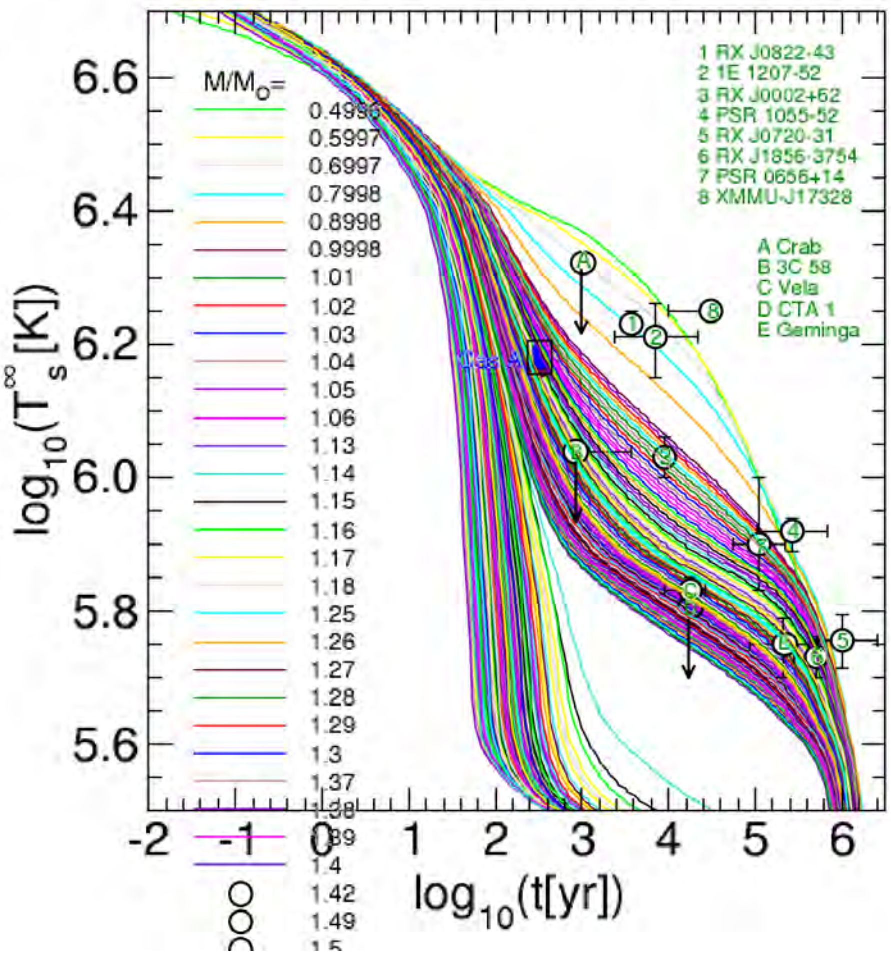




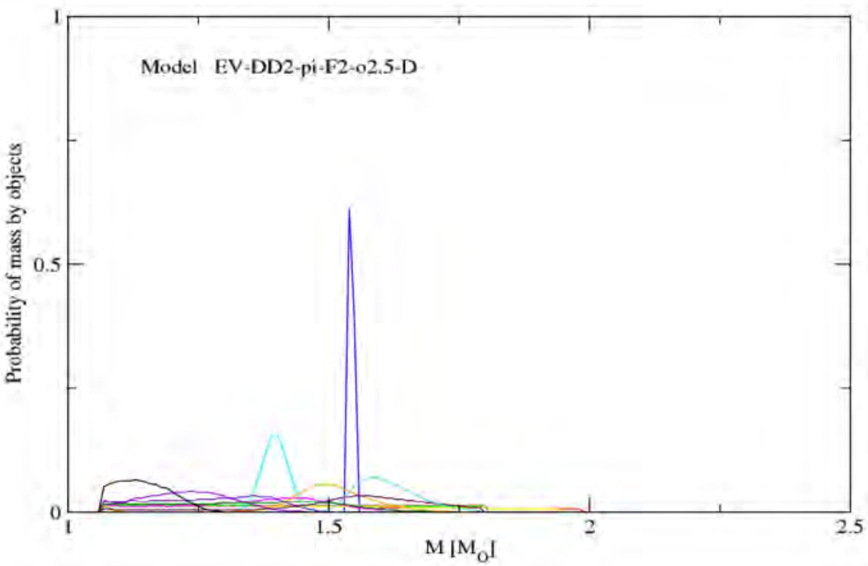
Model DD2 pi F1 o 2.5

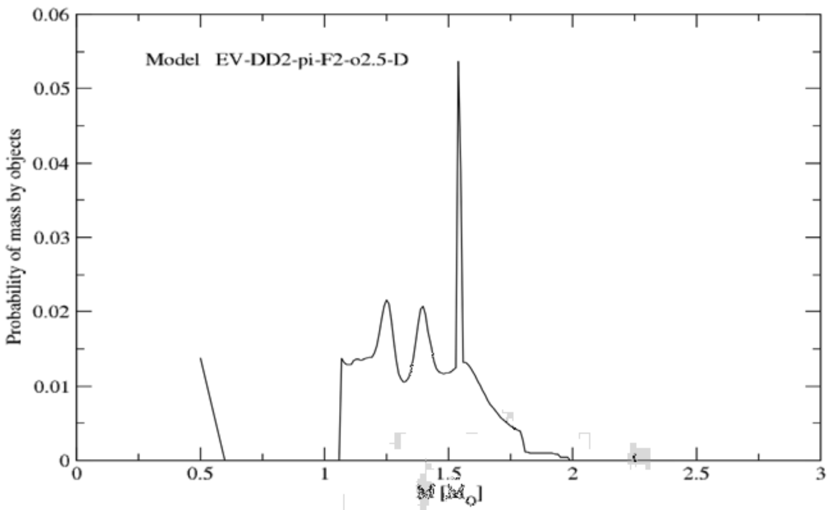


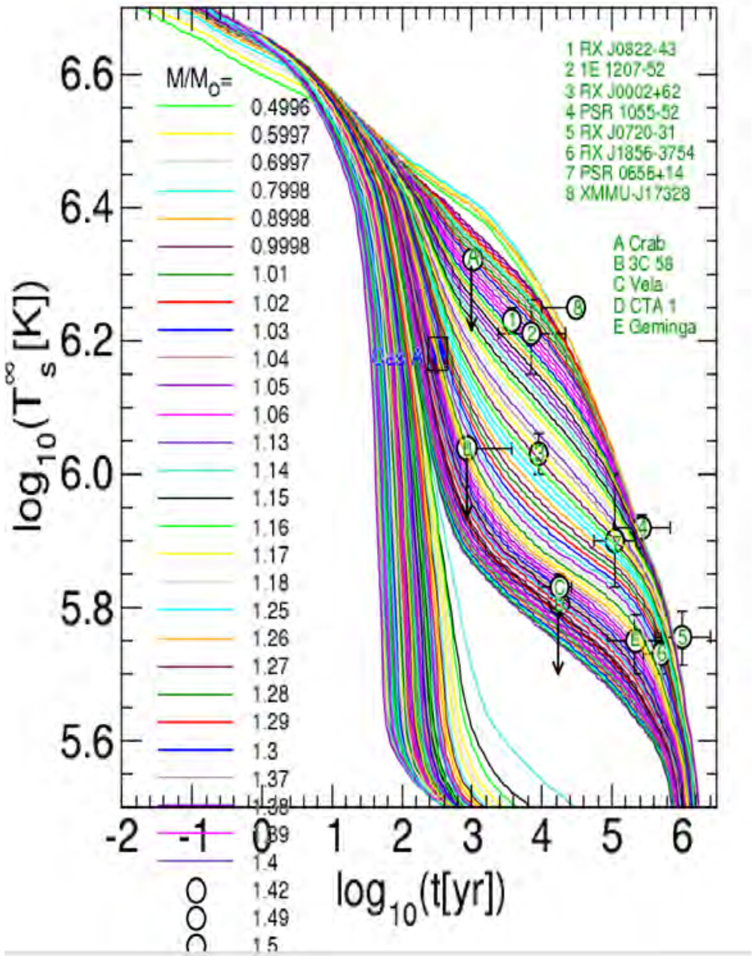




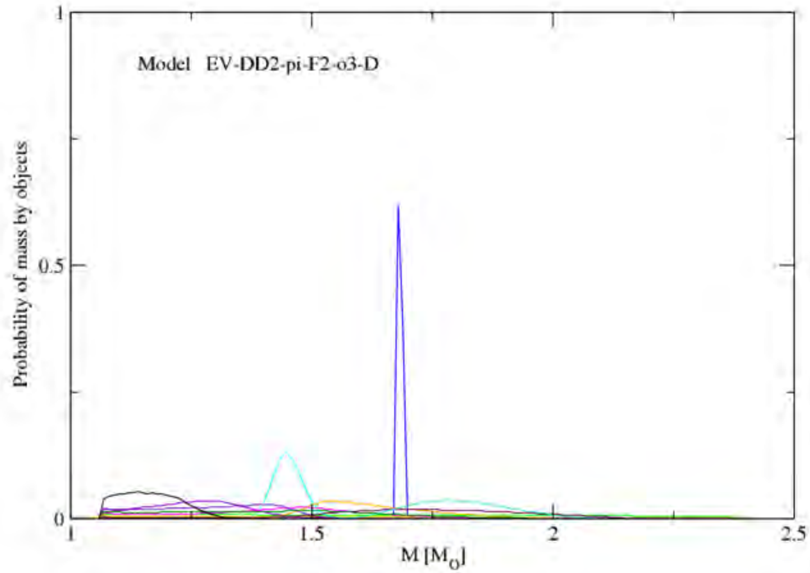
Model DD2 pi F2 o 2.5

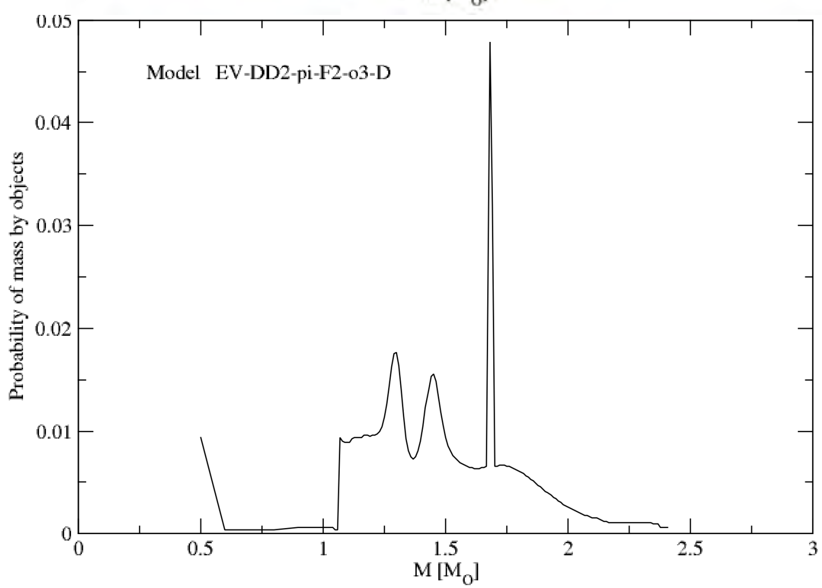


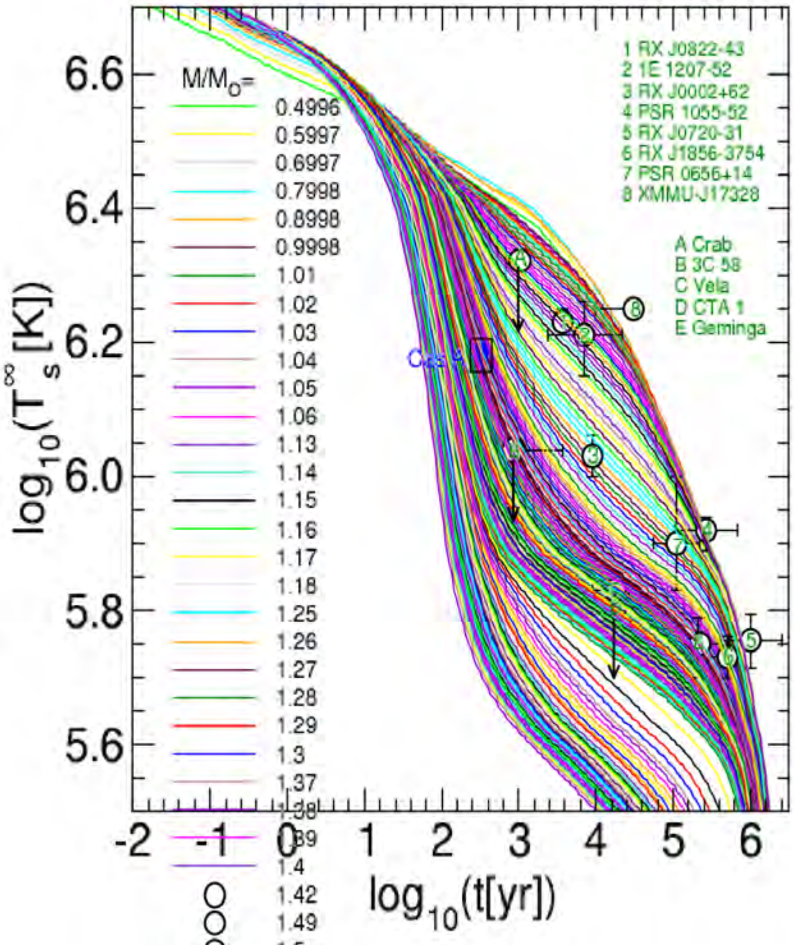




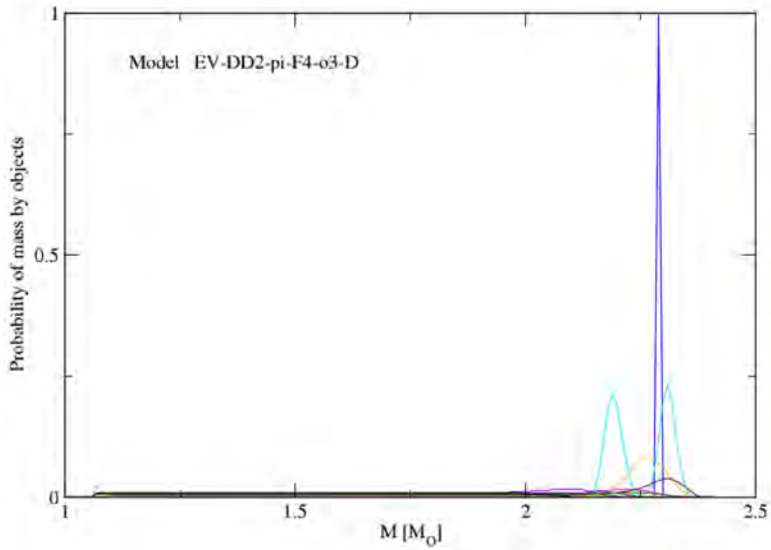
Model DD2 pi F2 o 3.0

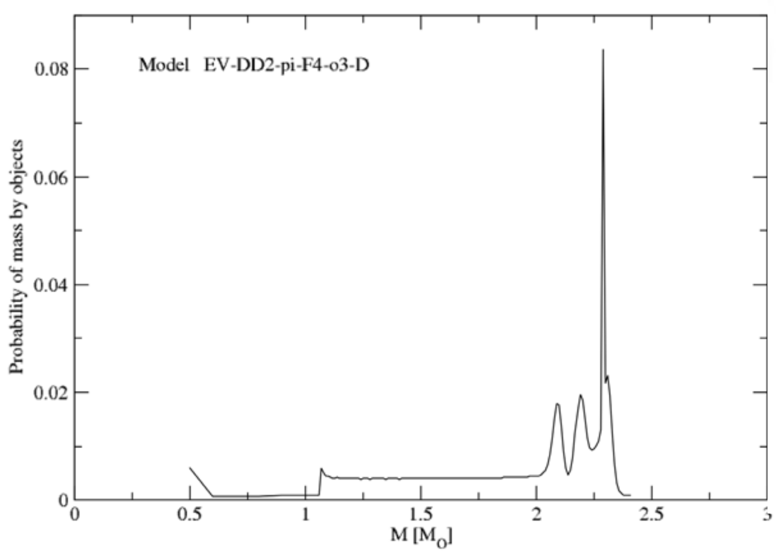


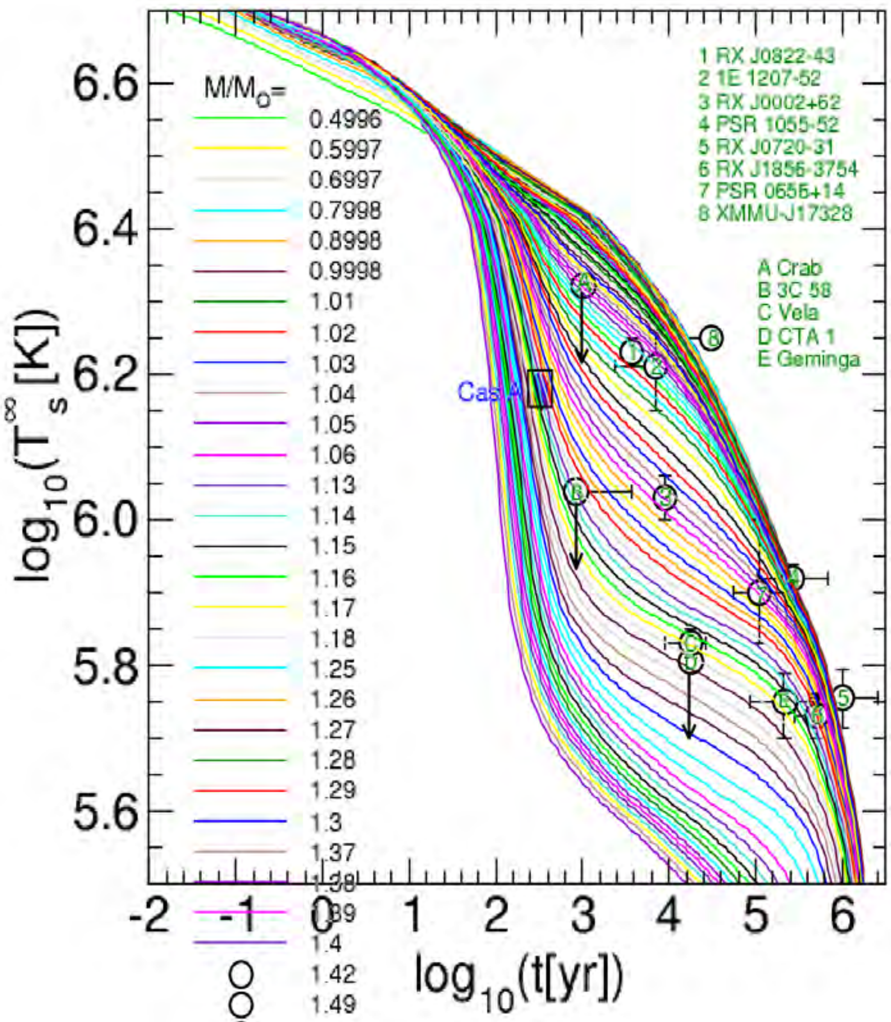




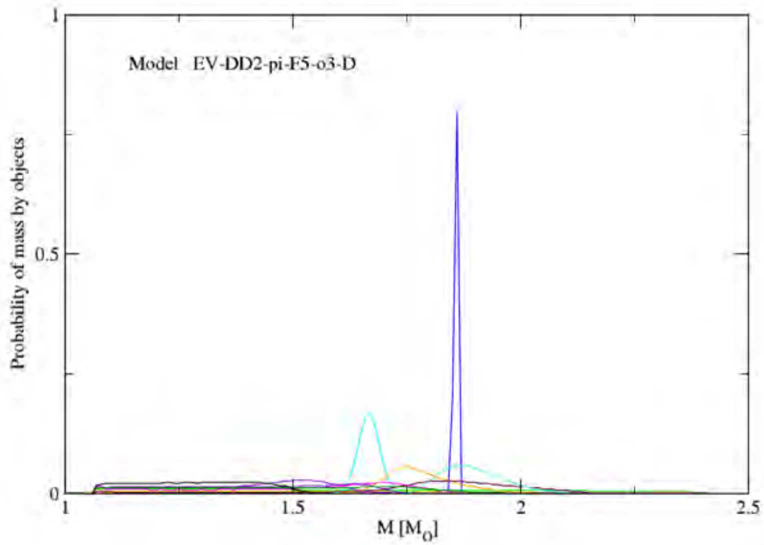
Model DD2 pi F4 o 3.0

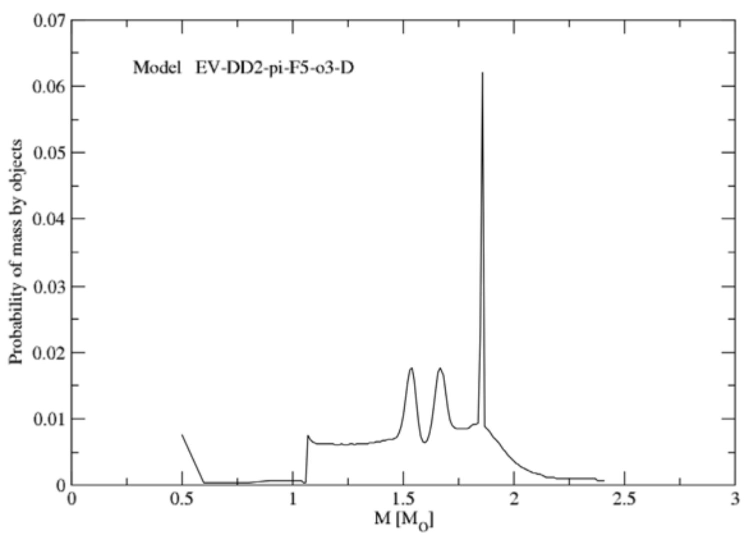


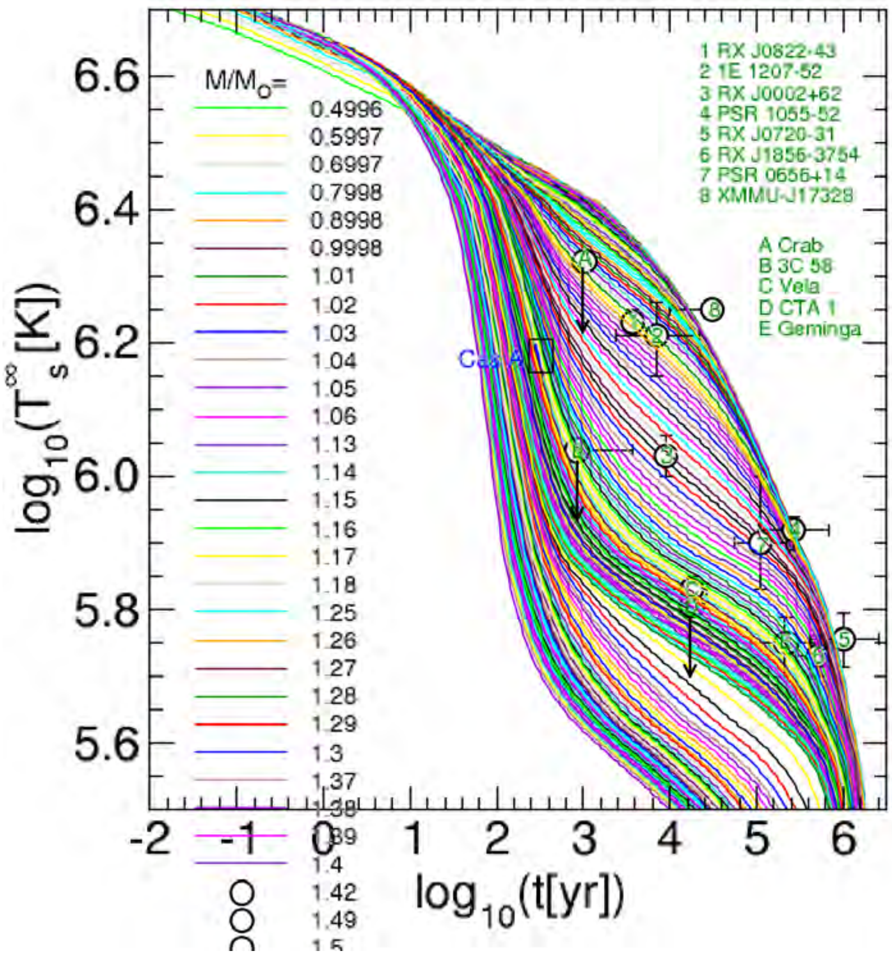




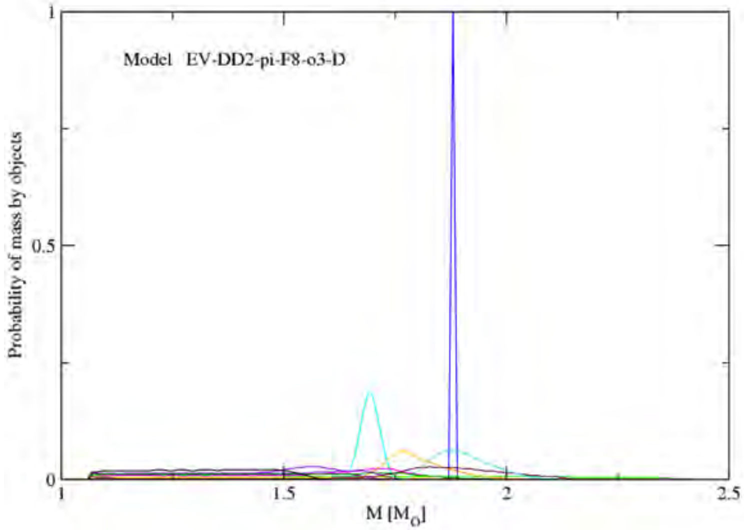
Model DD2 pi F5 o 3.0

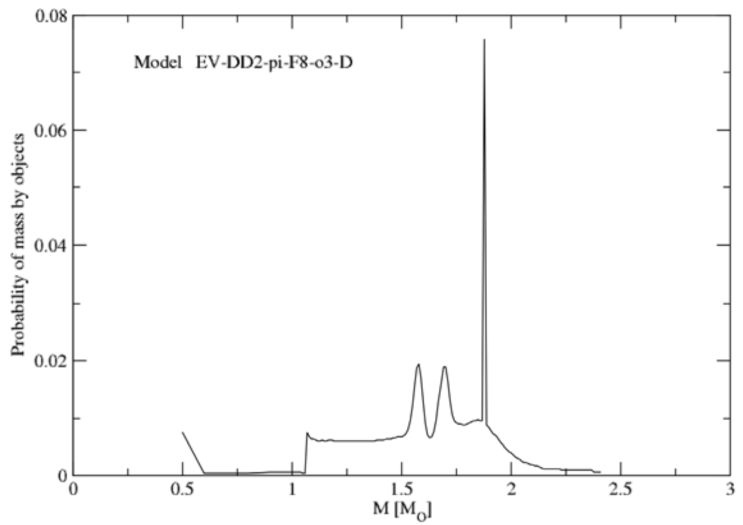


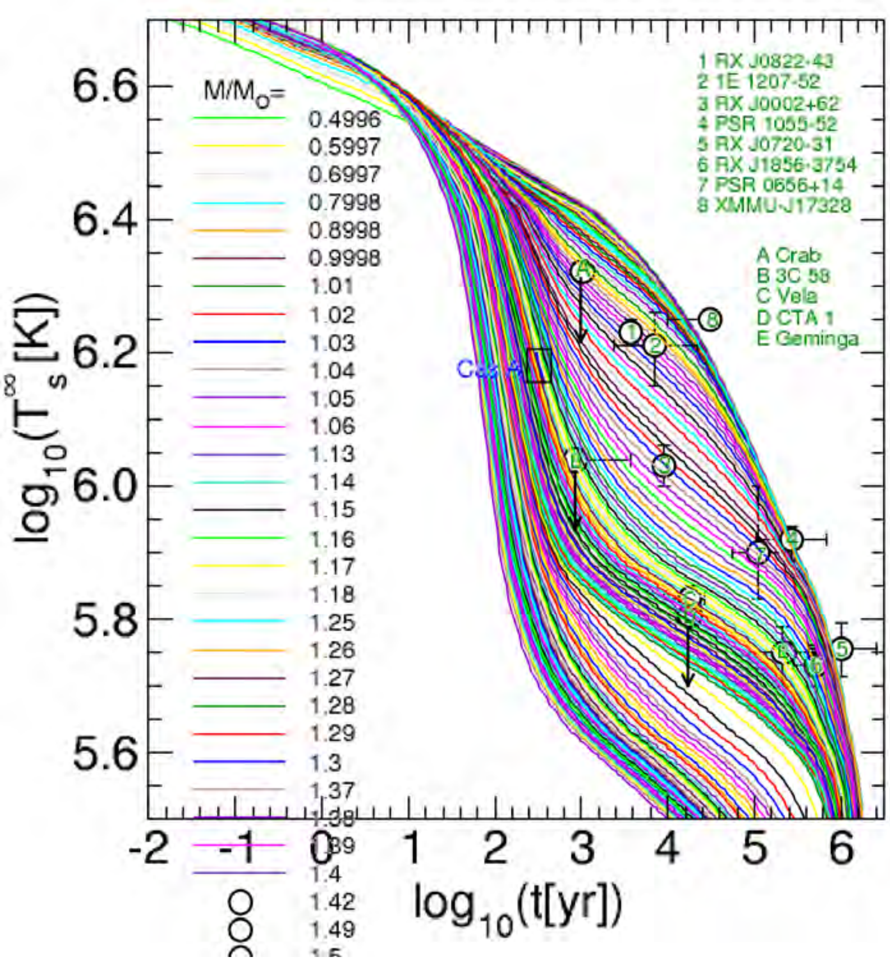




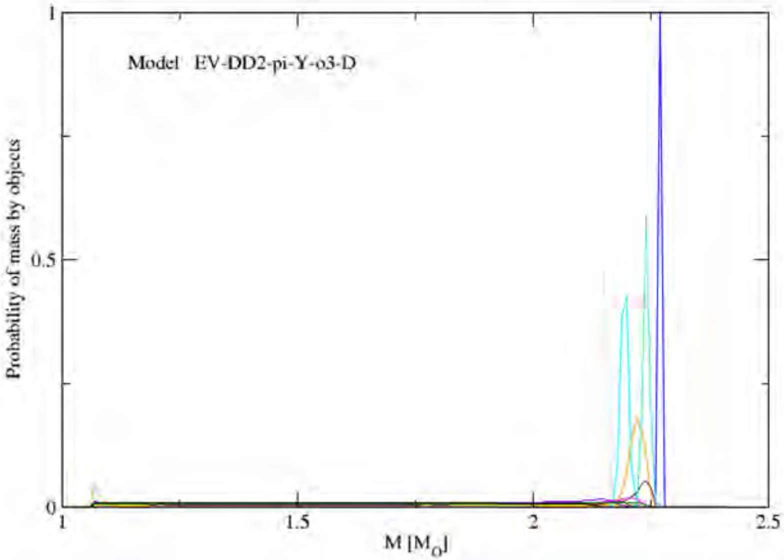
Model DD2 pi F8 o 3.0

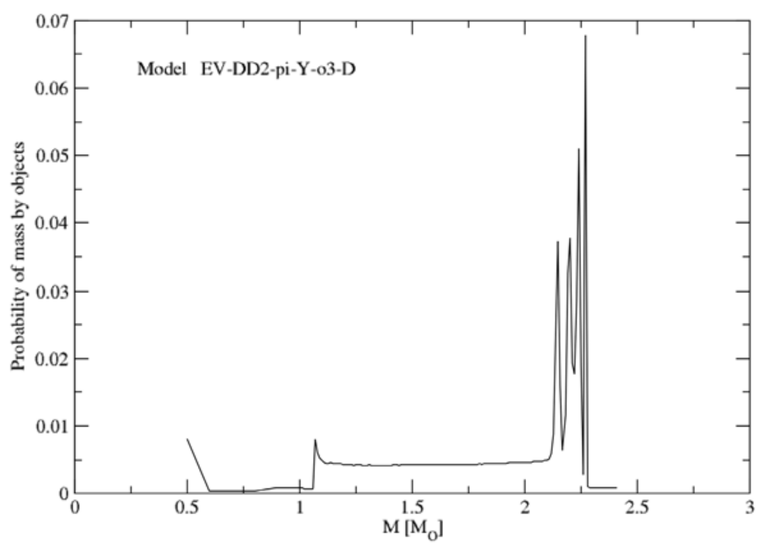


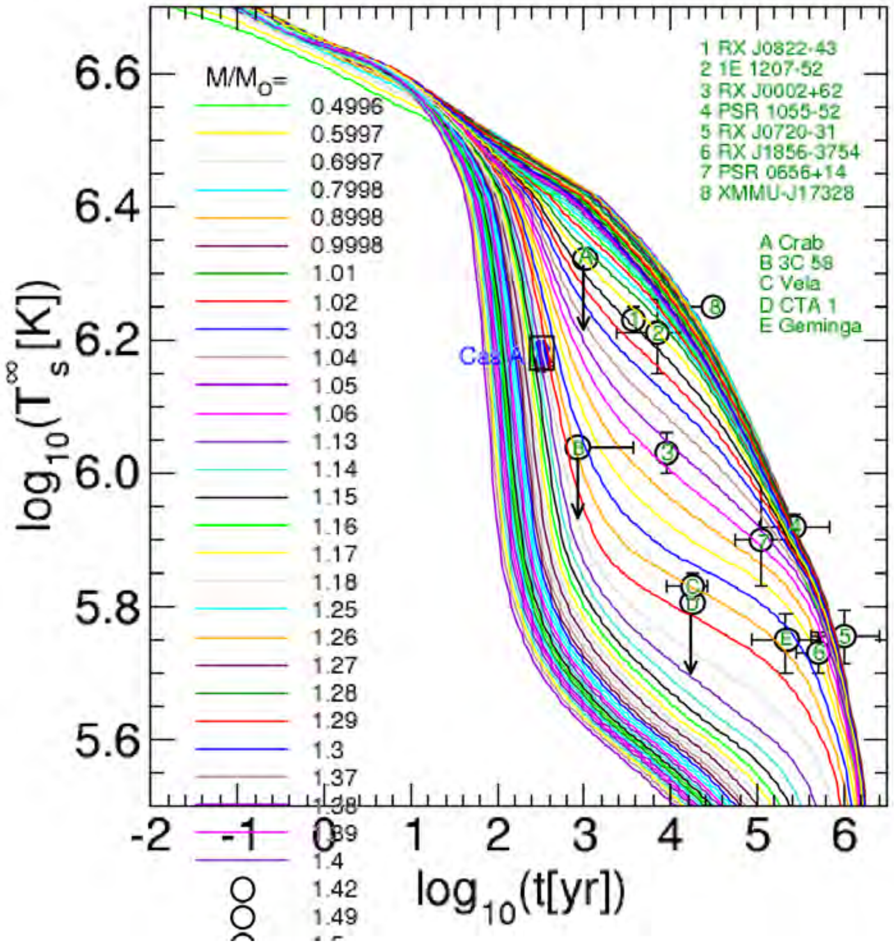




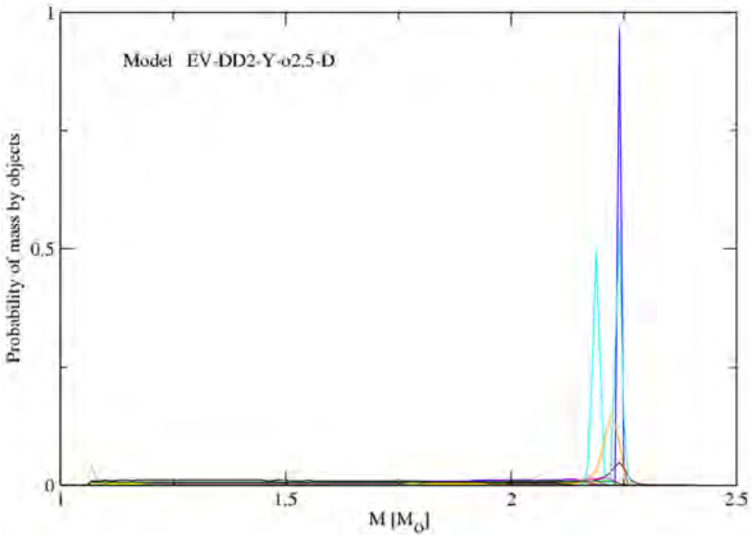
Model DD2 pi Y o 3.0

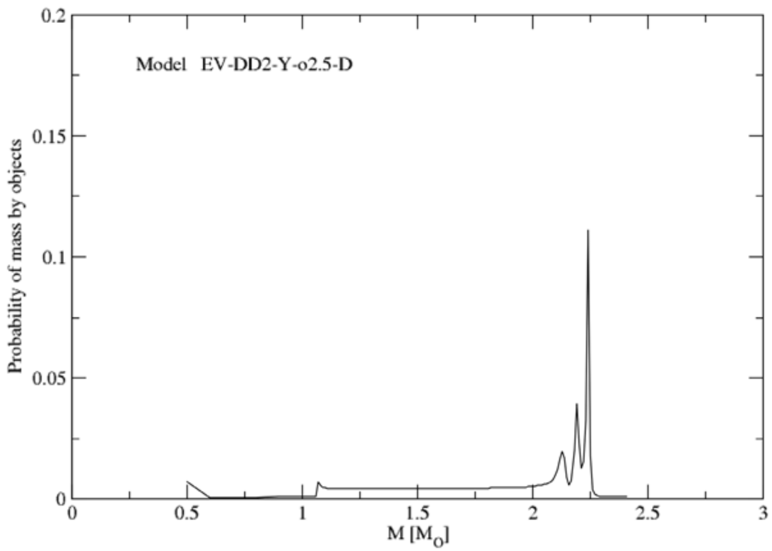


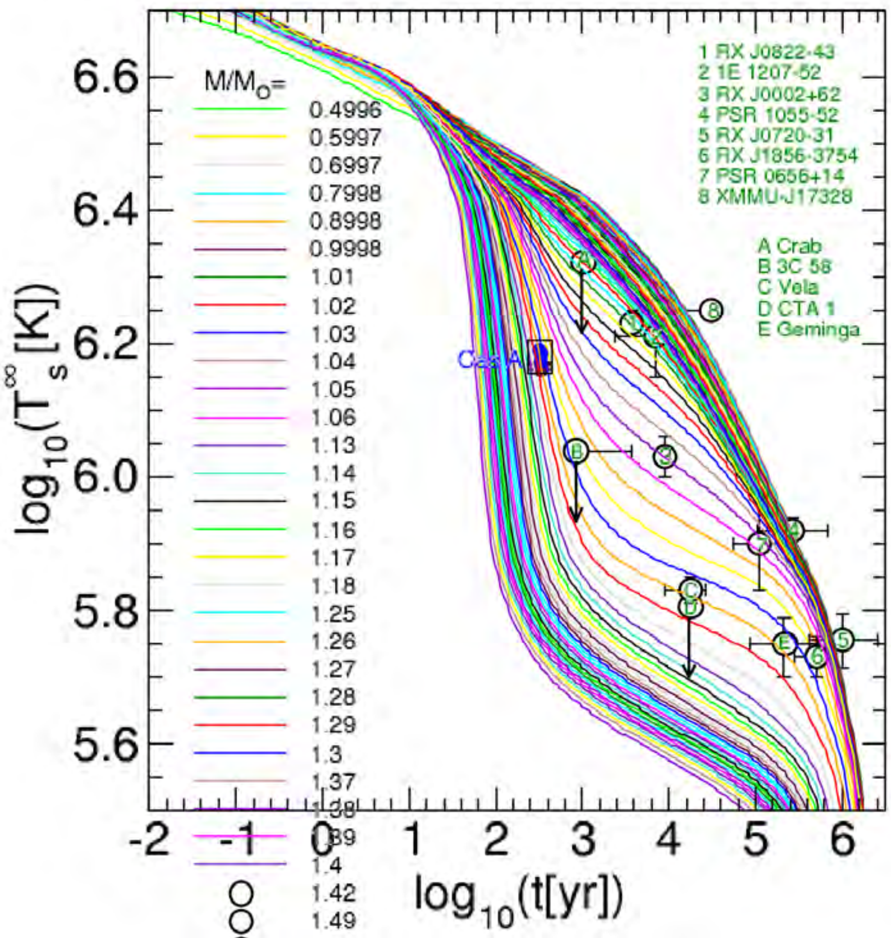




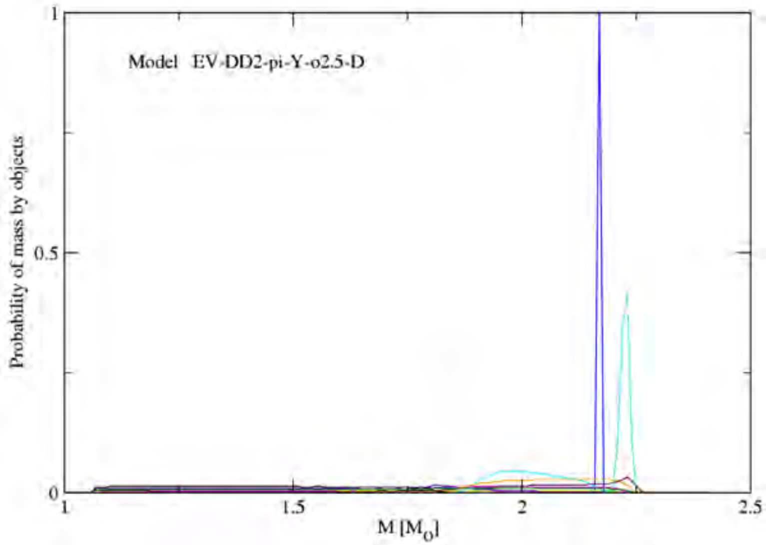
Model DD2 Y o 2.5

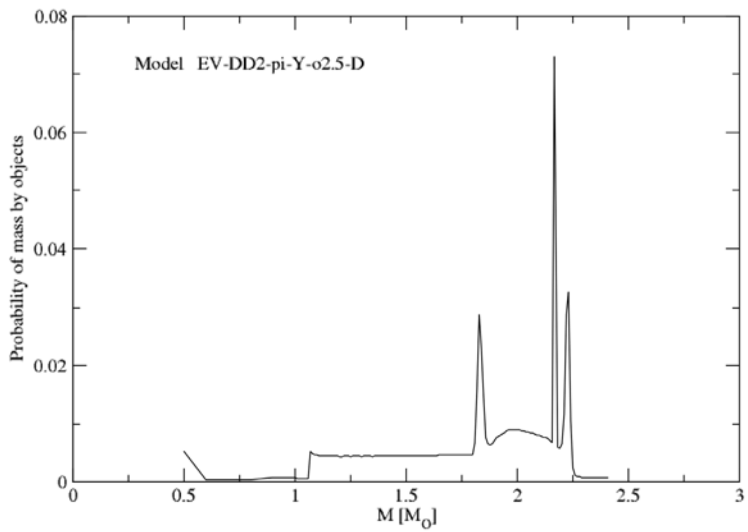


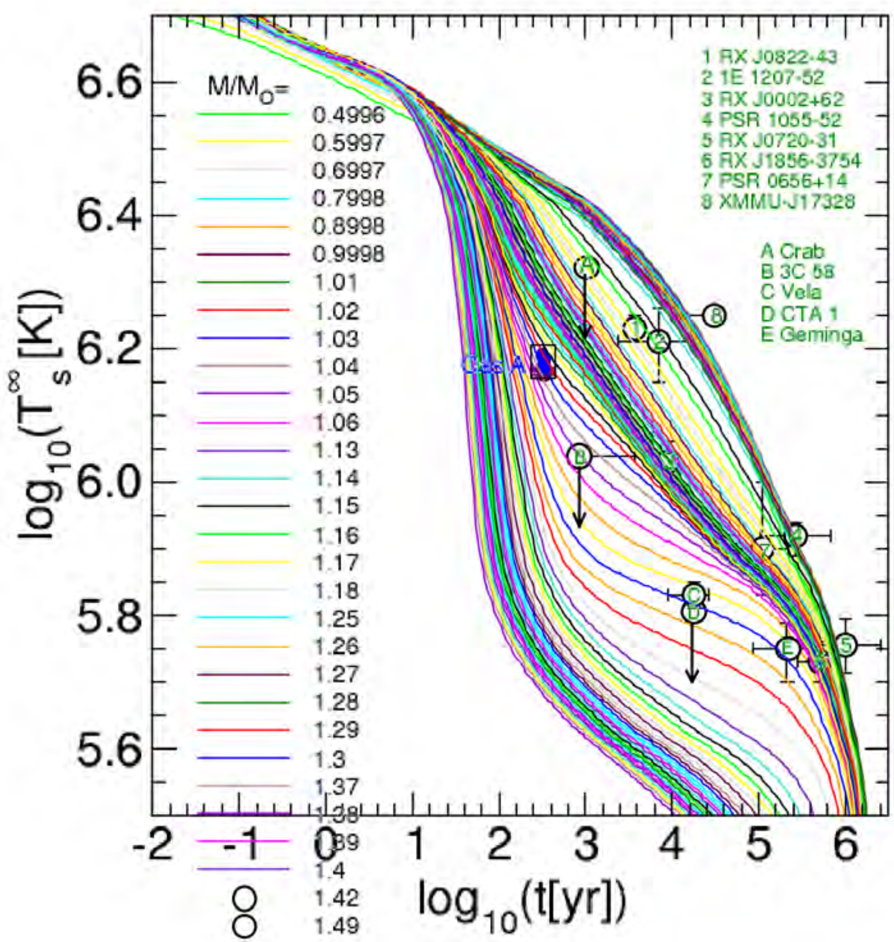




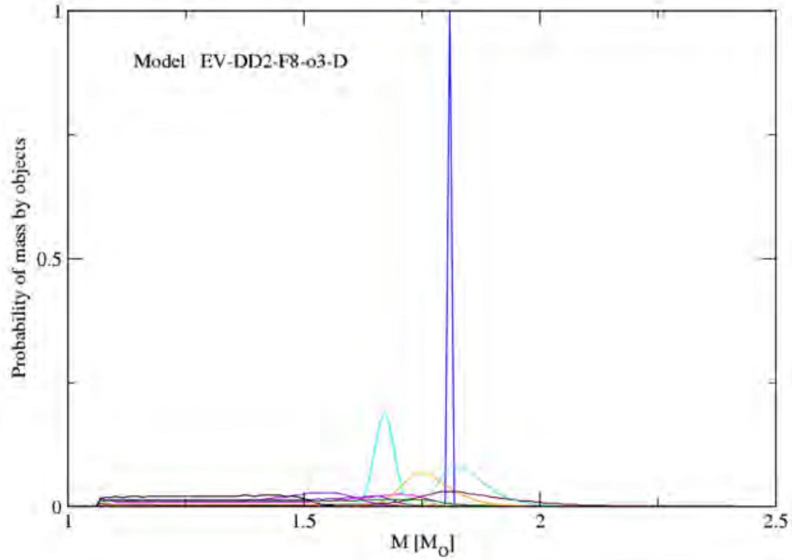
Model DD2 pi Y o 2.5

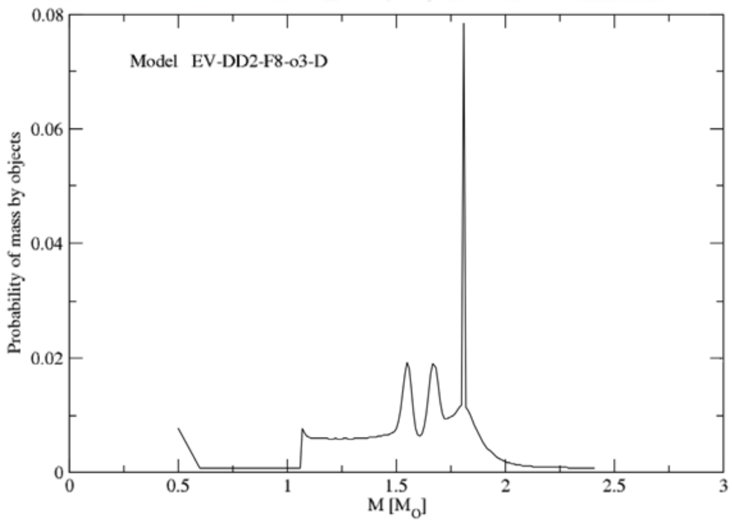


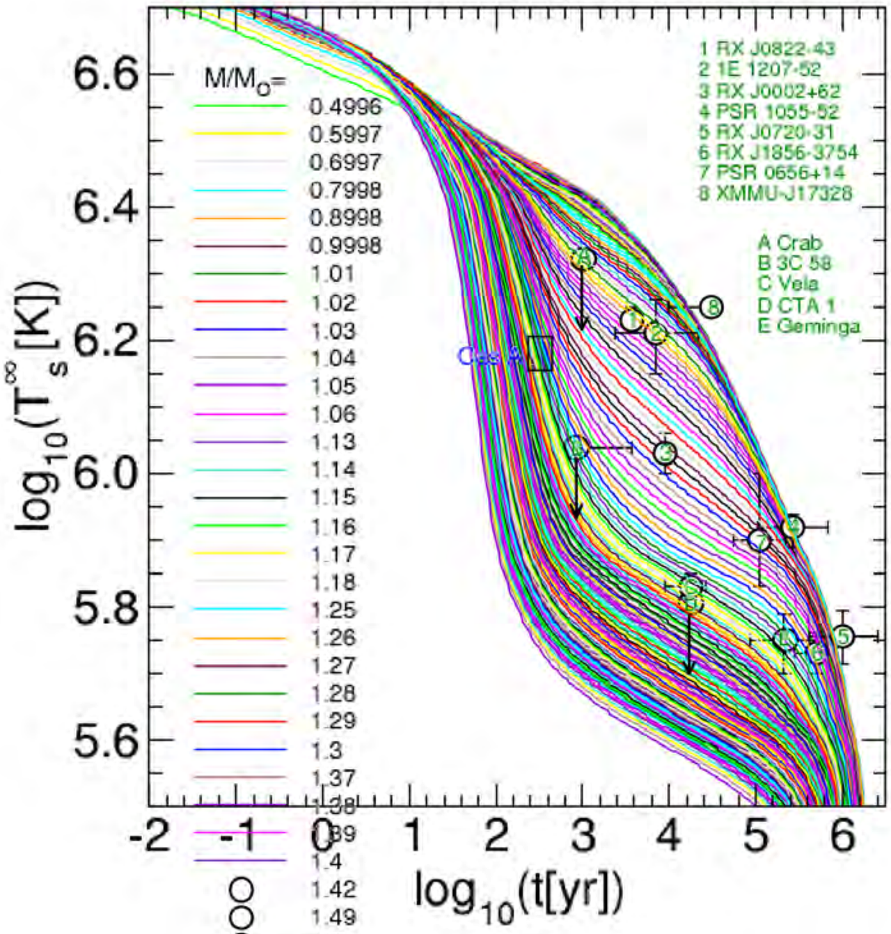




Model DD2 F8 o 3.0







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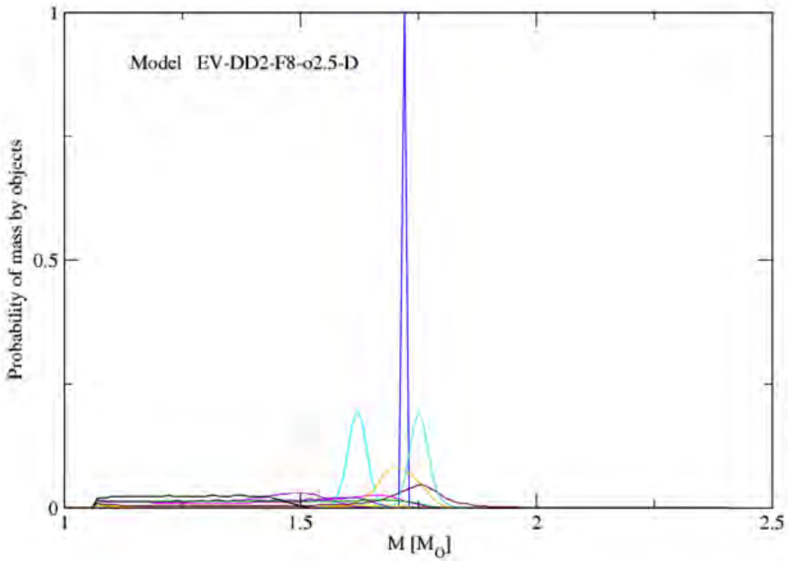
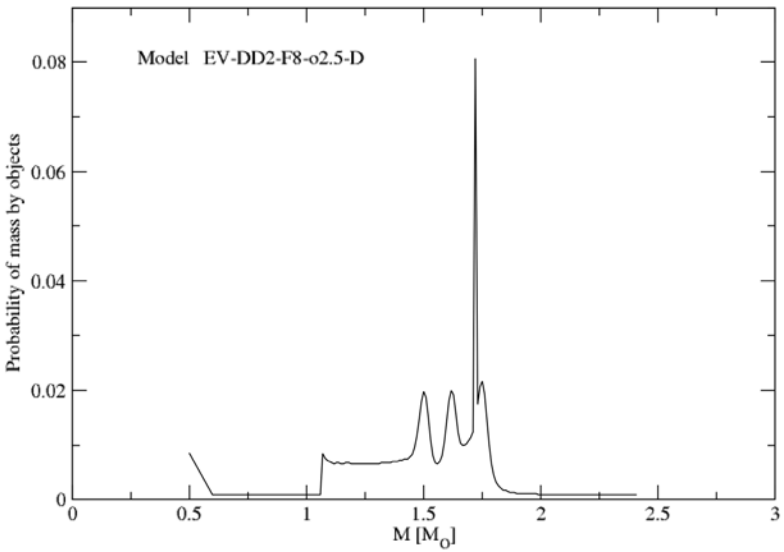
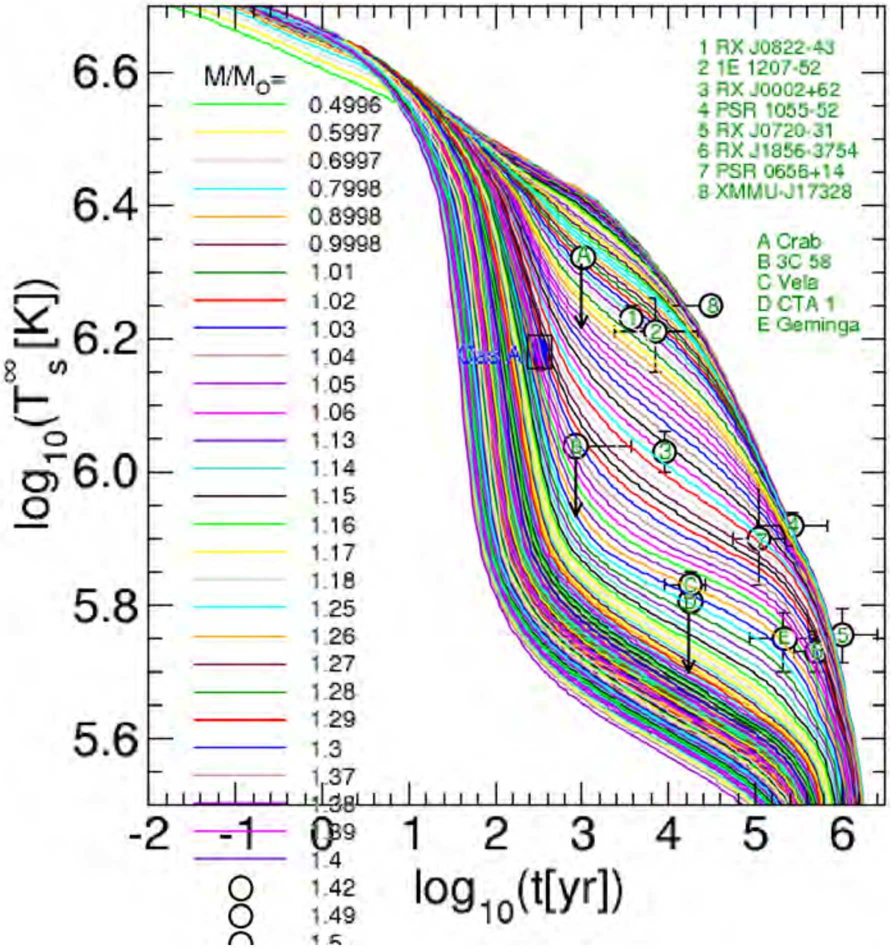




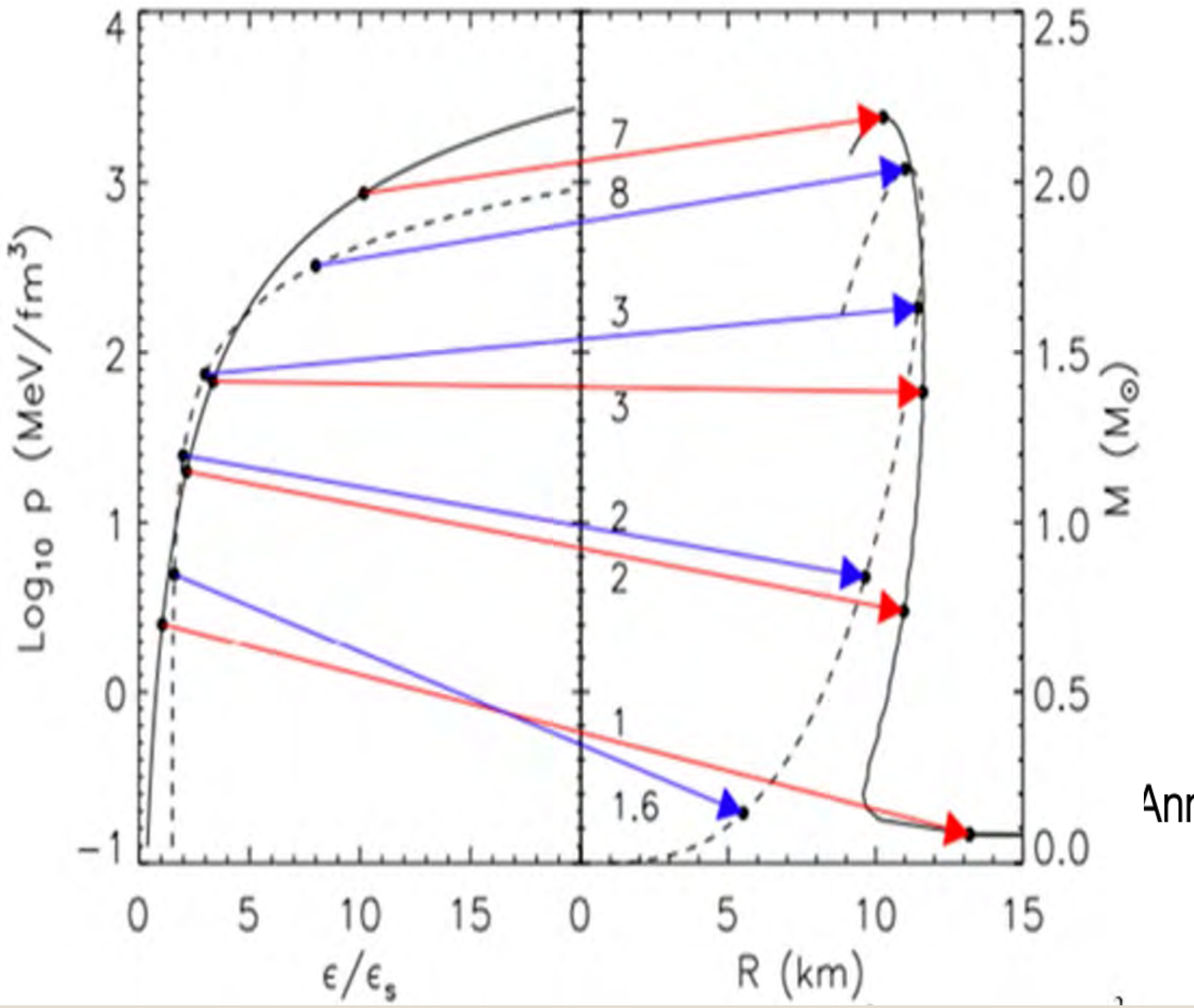


Conclusions

- All known cooling data including the Cas A rapid cooling consistently described by the "nuclear medium cooling" scenario
- Influence of stiffness on EoS and cooling can be balanced by the choice of corresponding gap model.
- Parallelization allowed to make the calculations for statistical analyses of models in reasonable time
- For each choice of different gap models and the influence of the medium effects it is possible to extract the masses of observed objects and the mass spectrum of neutron stars.

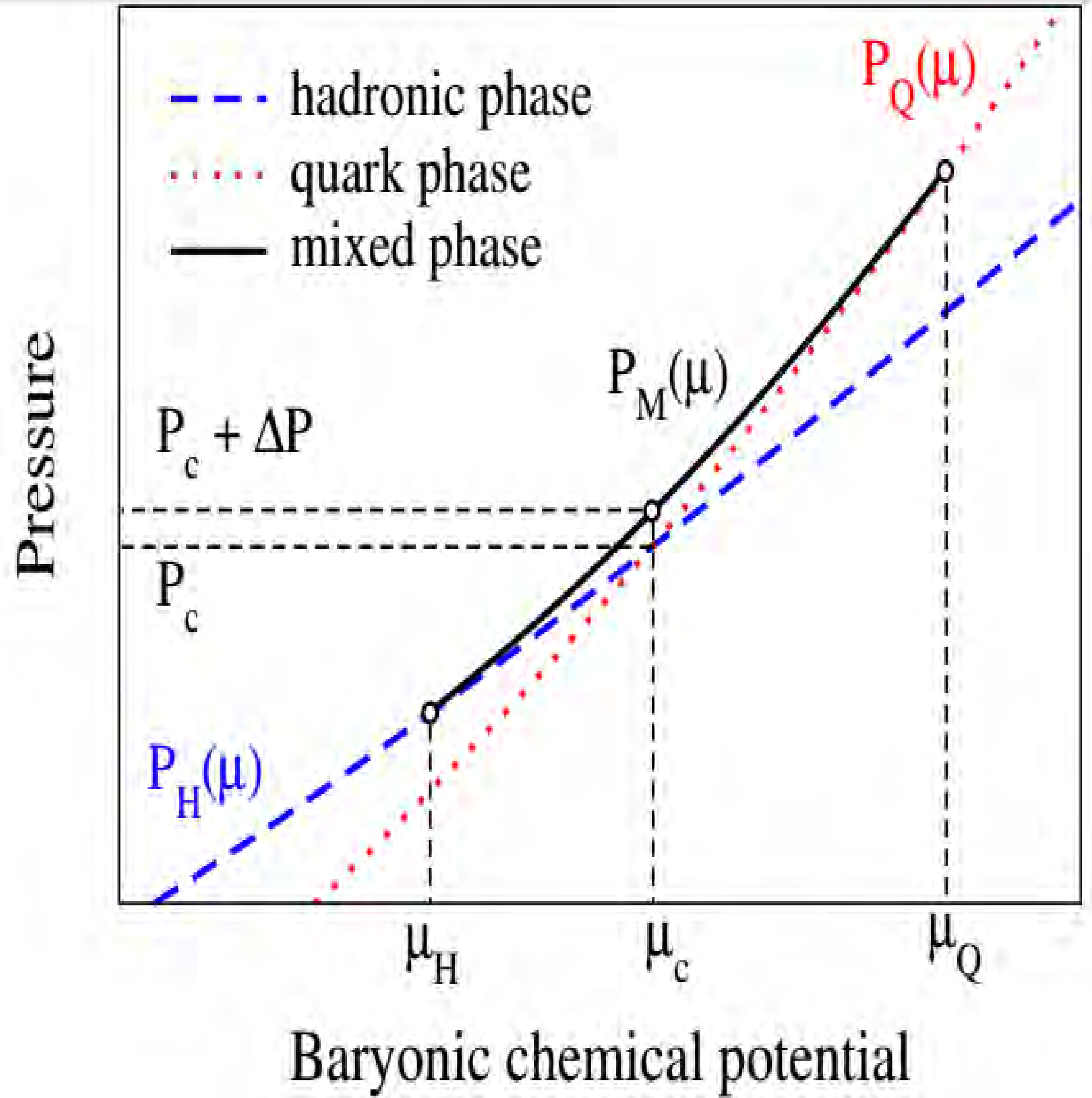
Thank YOU!!!!!

EoS vs. Mass Radious of NS

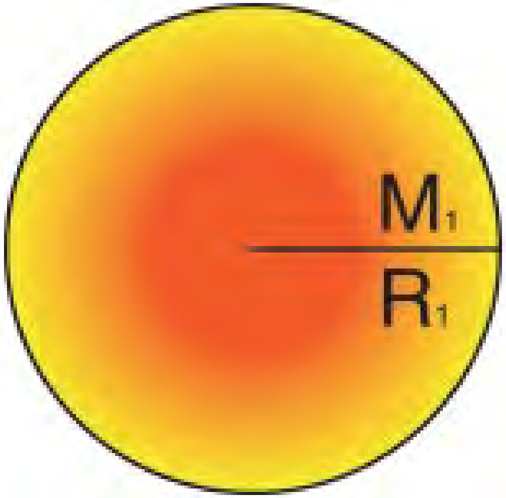


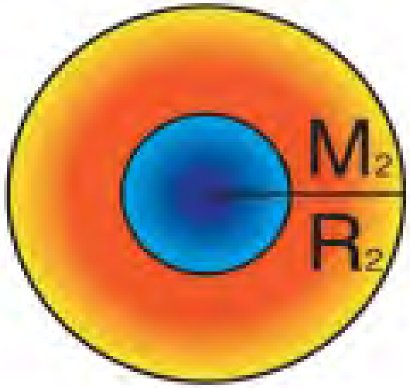
Lattimer,
Annu. Rev. Nucl. Part. Sci. 62, 485 (2012)
arXiv: 1305.3510

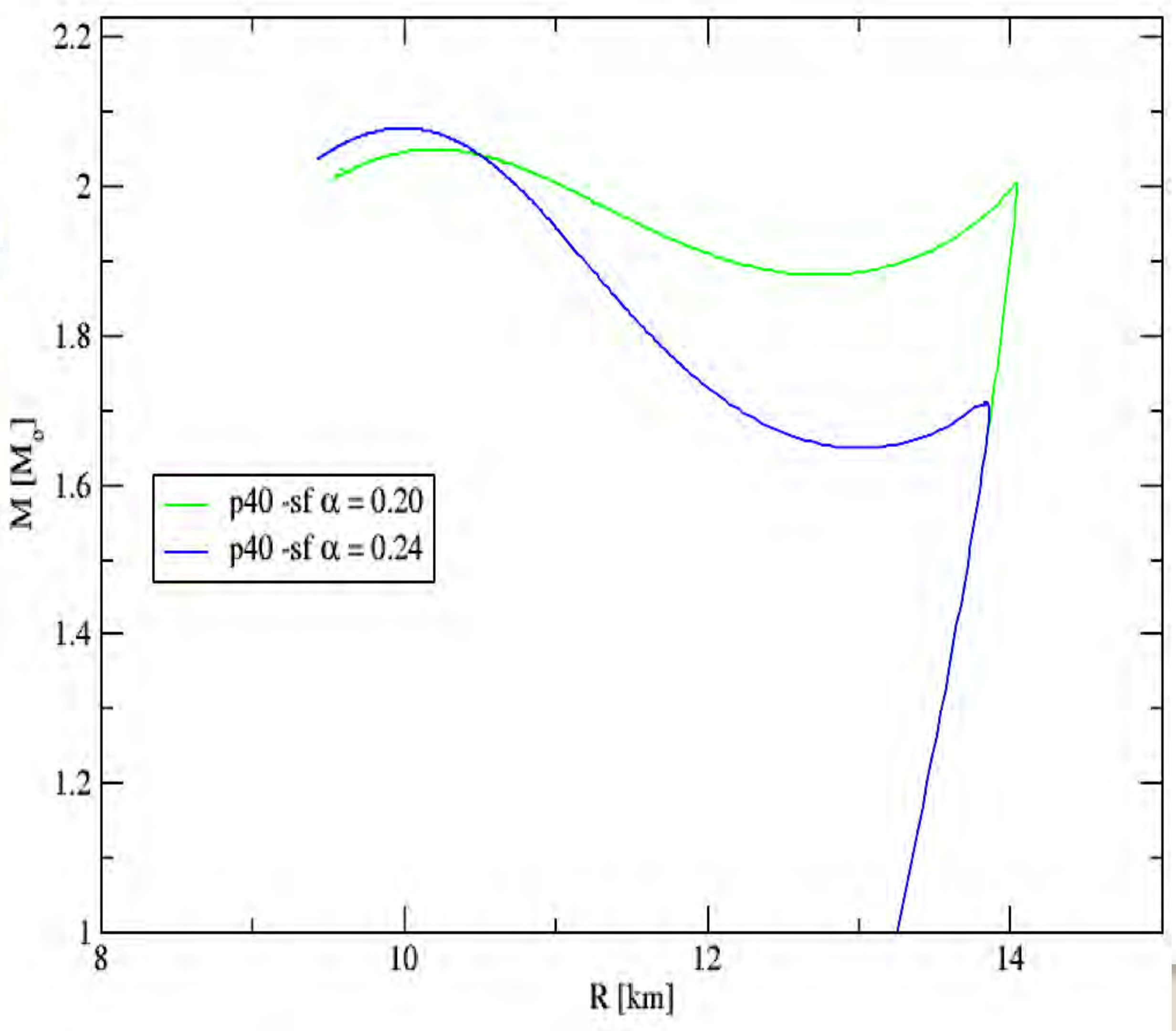
Mixed Phase in Quark-Hadron Phase Transition



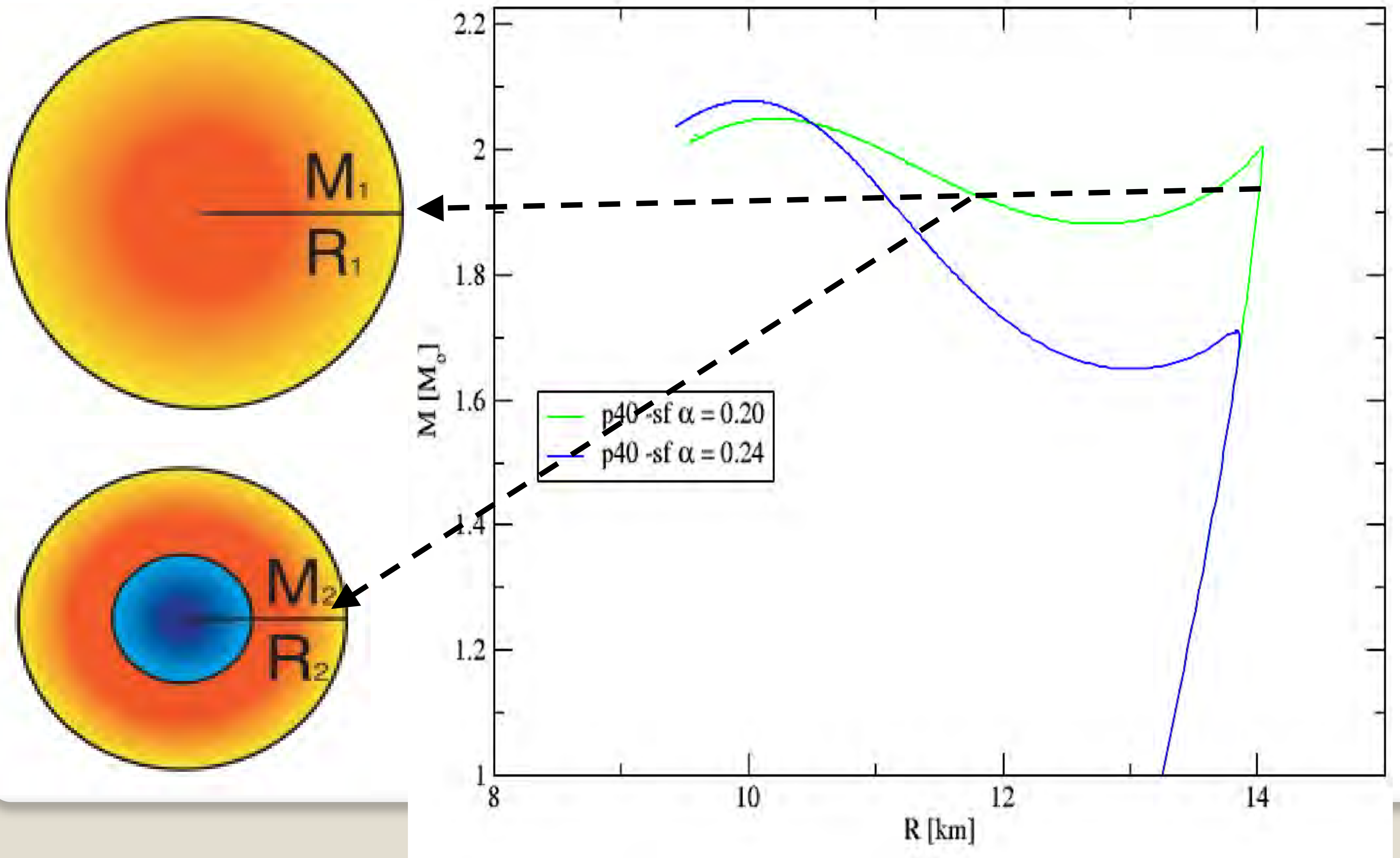
High Mass Twin CS



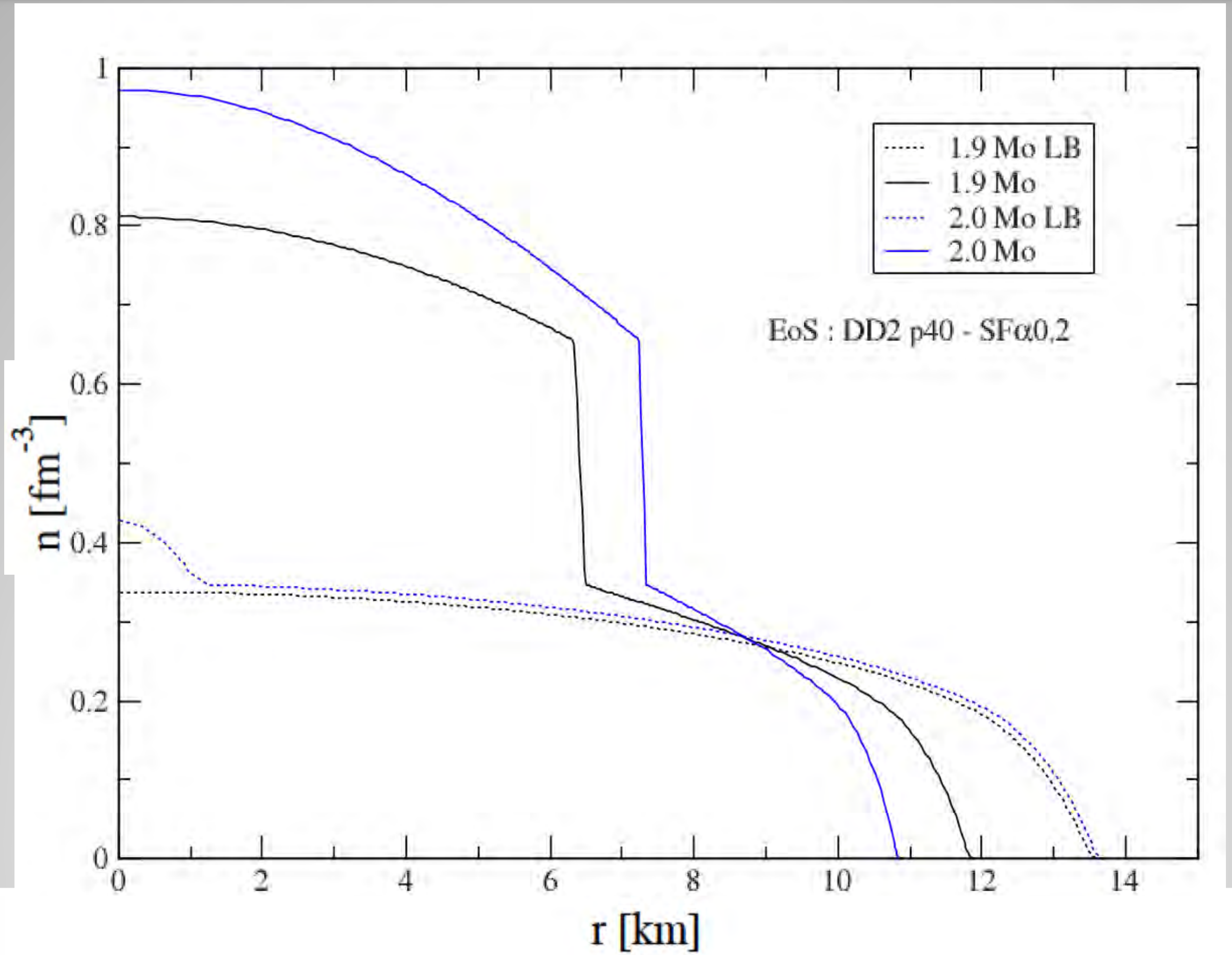




High Mass Twin CS



Different Configurations with the same NS mass

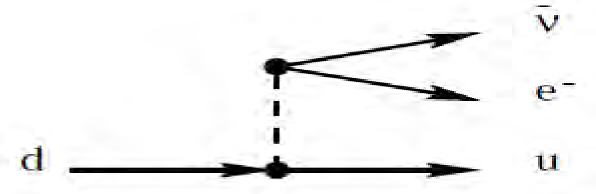


Neutrino emissivities in quark matter:

- Quark direct Urca (QDU) the most efficient processes

$d \to u + e + \bar{\nu}$ and $u + e \to d + \nu$

$\epsilon_\nu^{\rm QDU} \simeq 9.4 \times 10^{26} \alpha_s u Y_e^{1/3} \zeta_{\rm QDU}\, T_9^6 \ {\rm erg\ cm^{-3}\ s^{-1}}$,

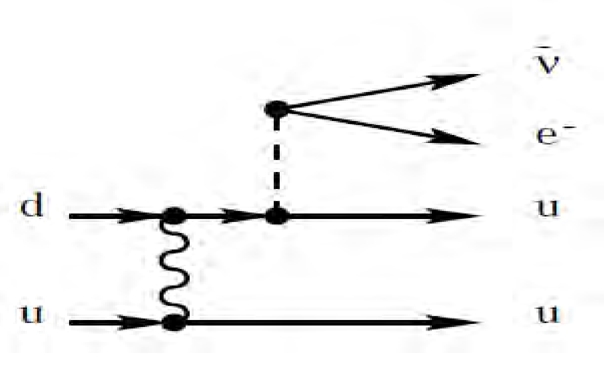


Compression n/no ≃ 2 , strong coupling αs ≈ 1

- Quark Modified Urca (QMU) and Quark Bremsstrahlung

$d + q \to u + q + e + \bar{\nu}$ and $q_1 + q_2 \to q_1 + q_2 + \nu + \bar{\nu}$

$\epsilon_\nu^{\rm QMU} \sim \epsilon_\nu^{\rm QB} \simeq 9.0 \times 10^{19} \zeta_{\rm QMU}\, T_9^8 \ {\rm erg\ cm^{-3}\ s^{-1}}$.



- Suppression due to the pairing

QDU : $\zeta_{\rm QDU} \sim \exp(-\Delta_q/T)$

QMU and **QB** : $\zeta_{\rm QMU} \sim \exp(-2\Delta_q/T)$ for $T < T_{{\rm crit},q} \simeq 0.57\, \Delta_q$

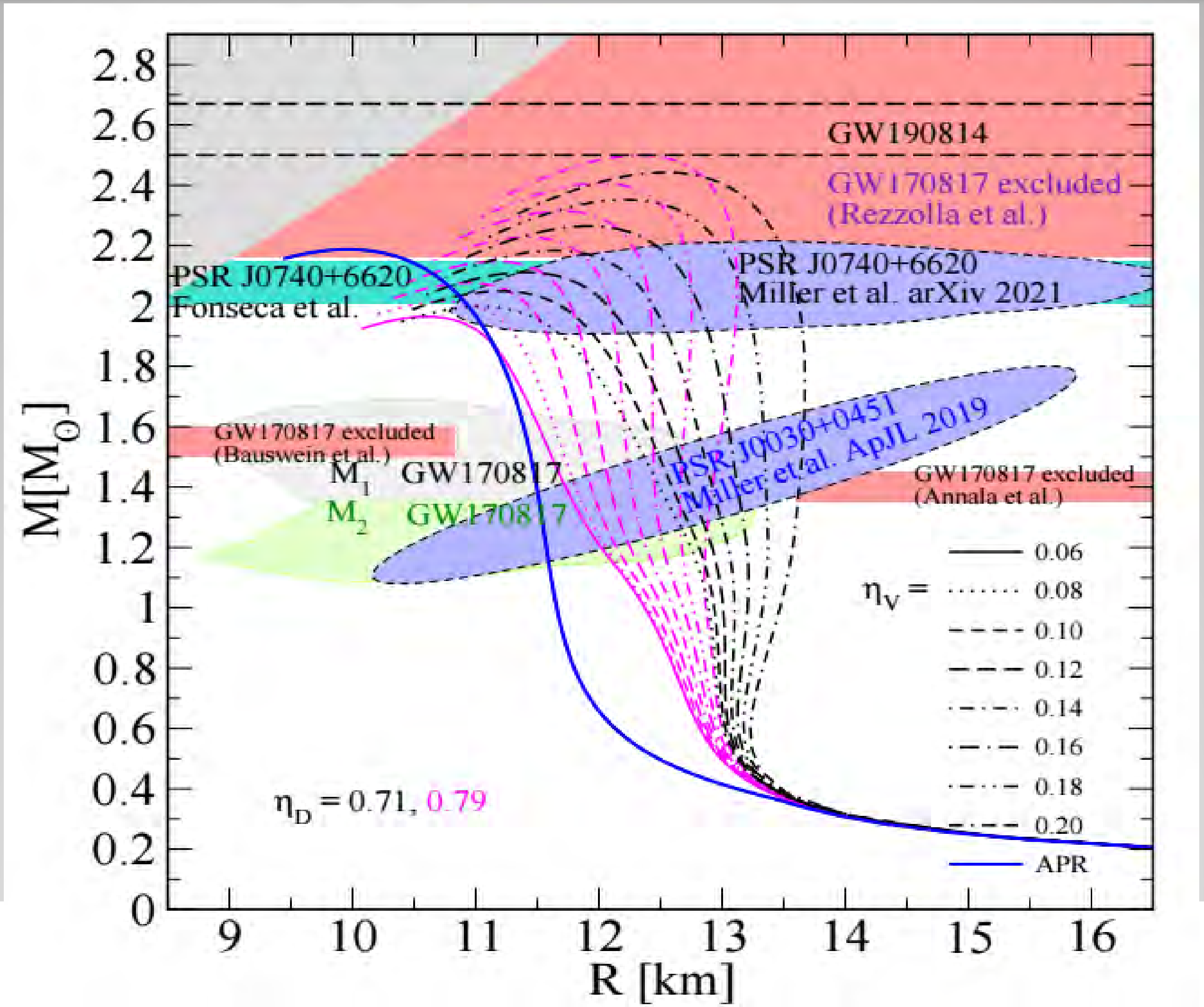
- Enhanced cooling due to the pairing

- $e + e \to e + e + \nu + \bar{\nu}$ (becomes important for $\Delta_q/T >> 1$)

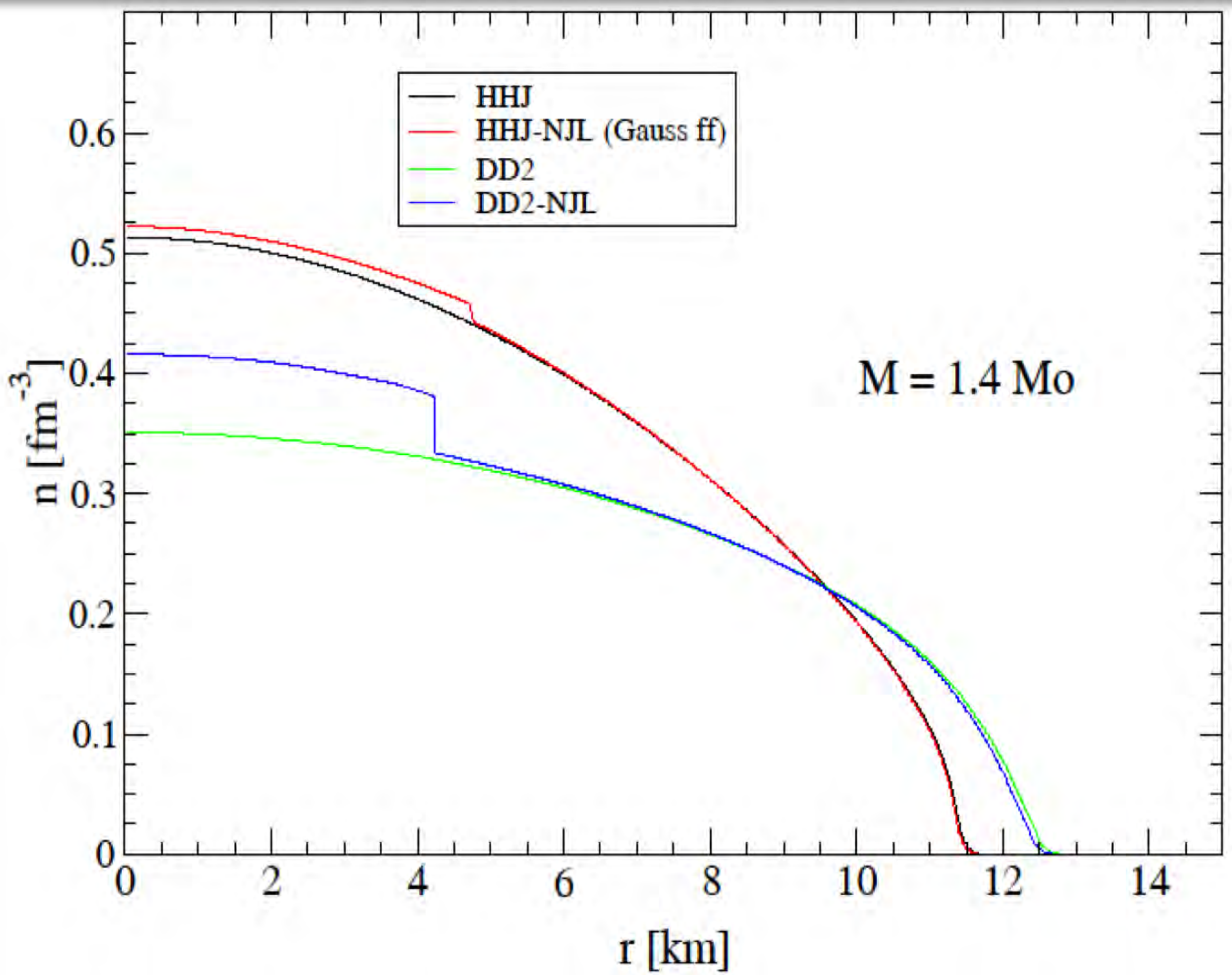
$\epsilon_\nu^{ee} = 2.8 \times 10^{12}\, Y_e^{1/3} u^{1/3} T_9^8 \ {\rm erg\ cm^{-3}\ s^{-1}}$,

Quark PBF

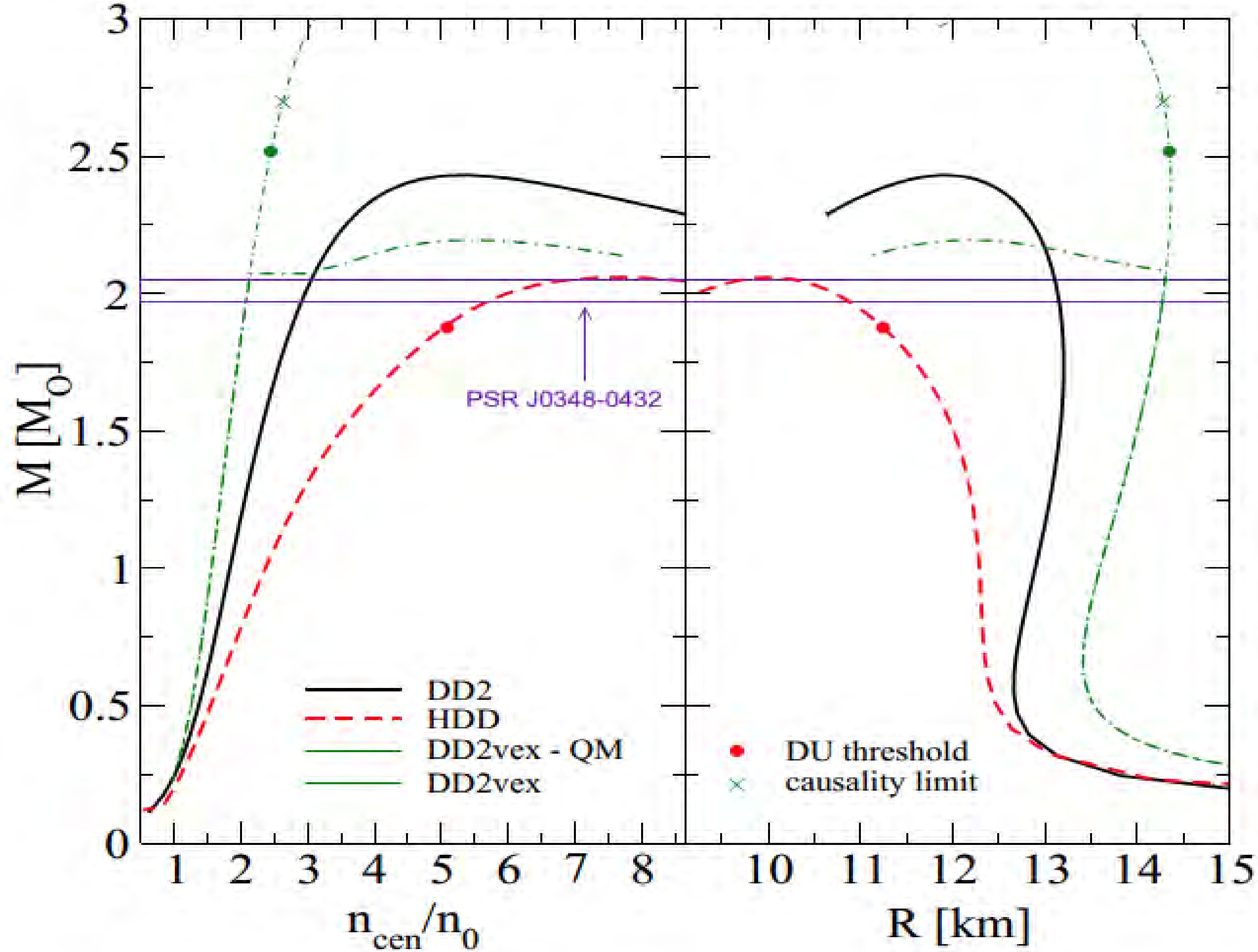
Modern MR Data and Models



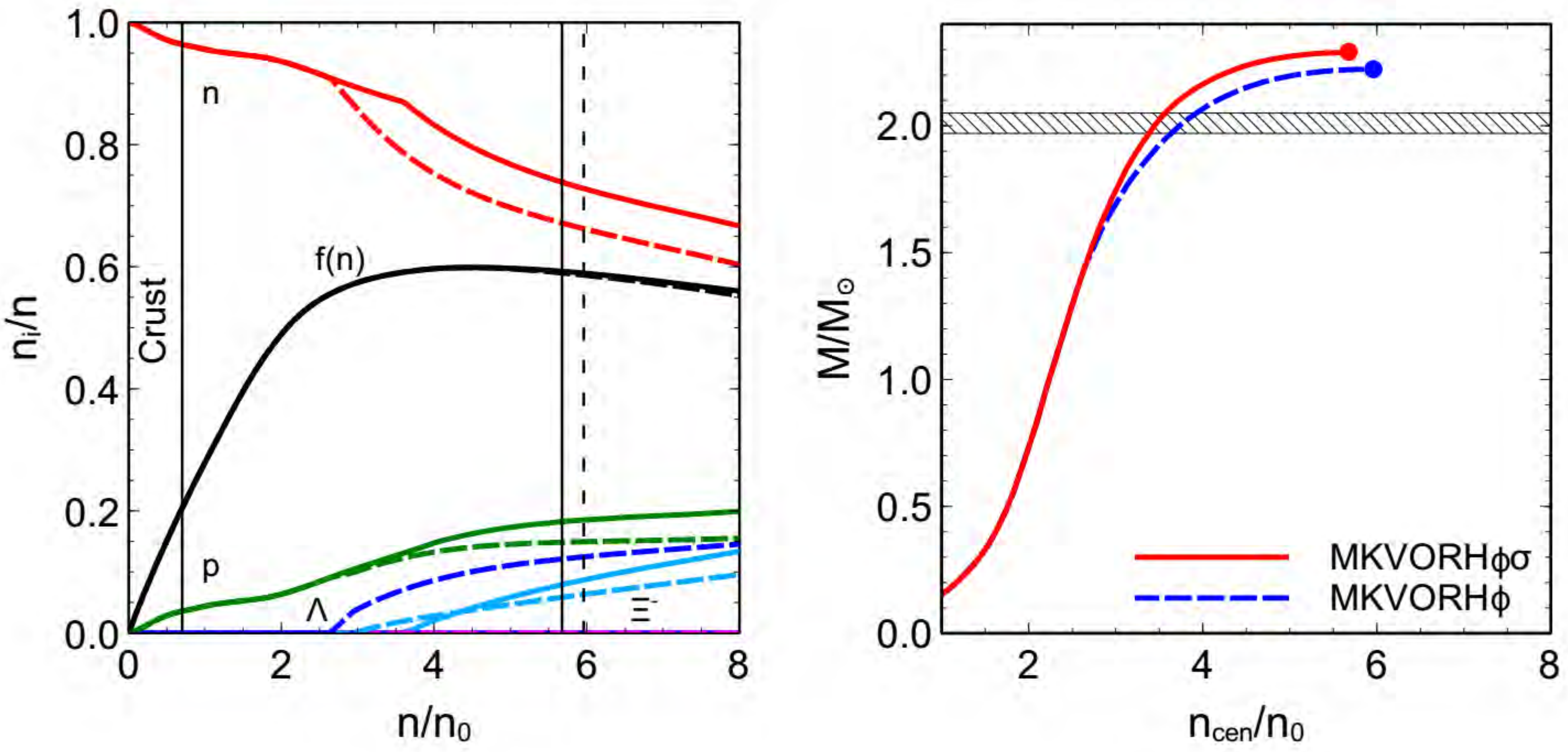
Different Configurations with the same NS mass



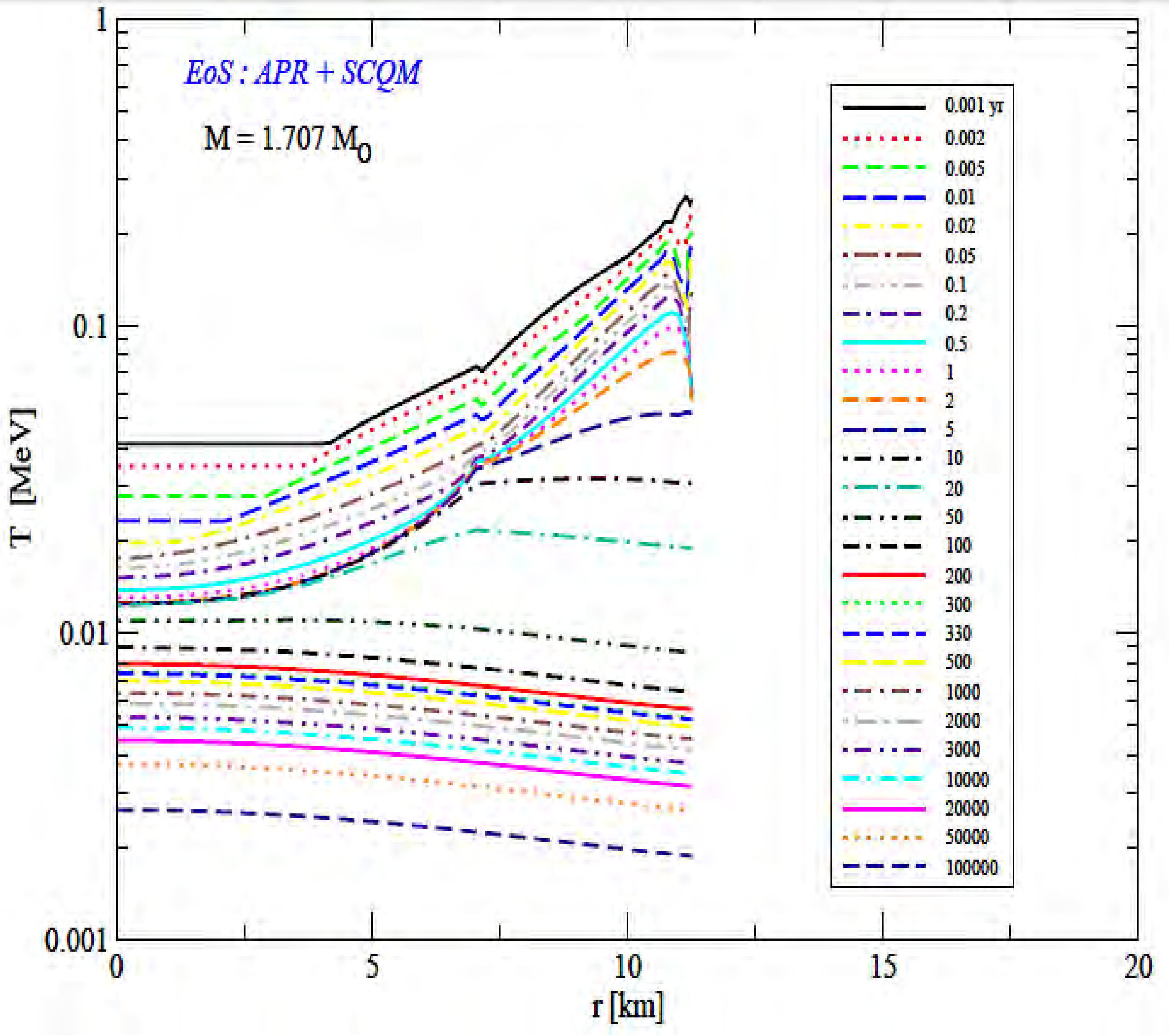
Stability of stars
HDD, DD2 & DDvex-NJL EoS model



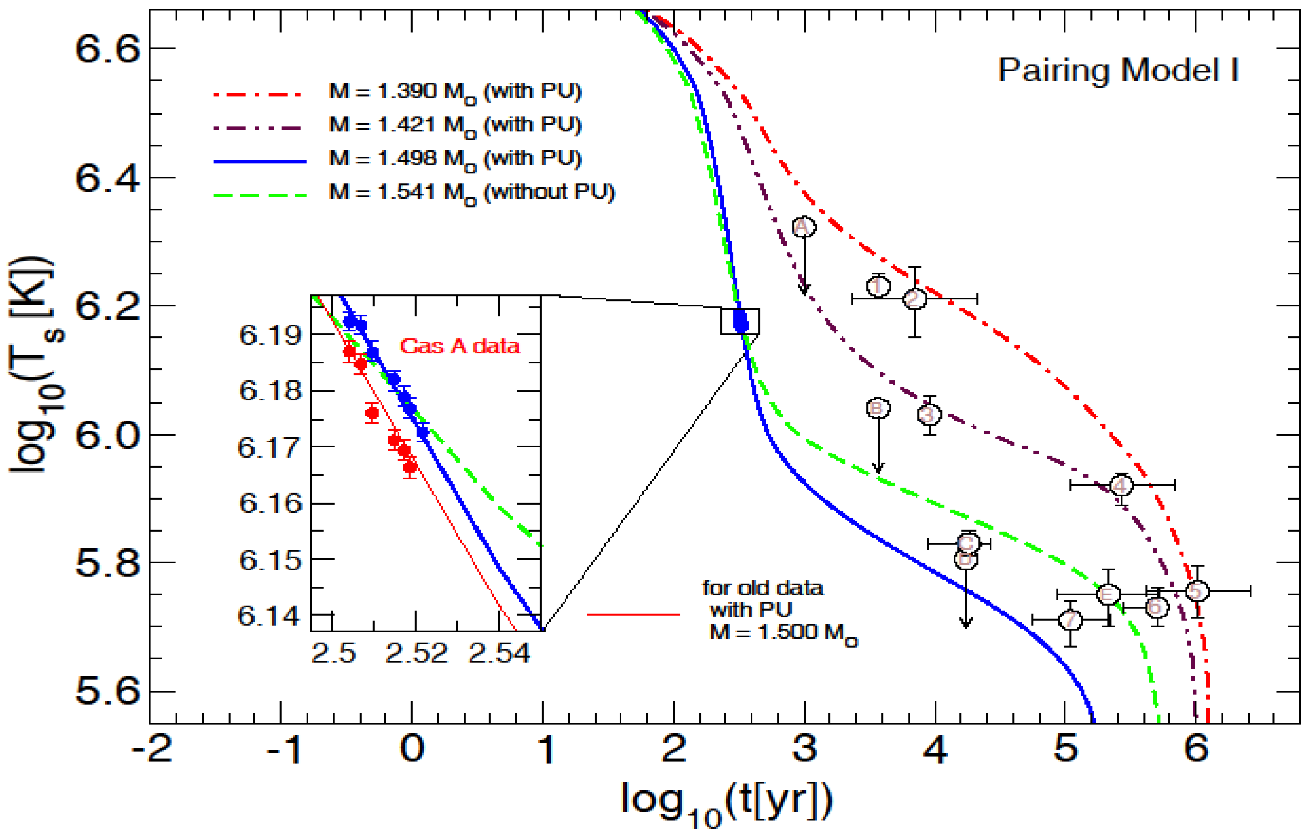
MKVOR – EoS model



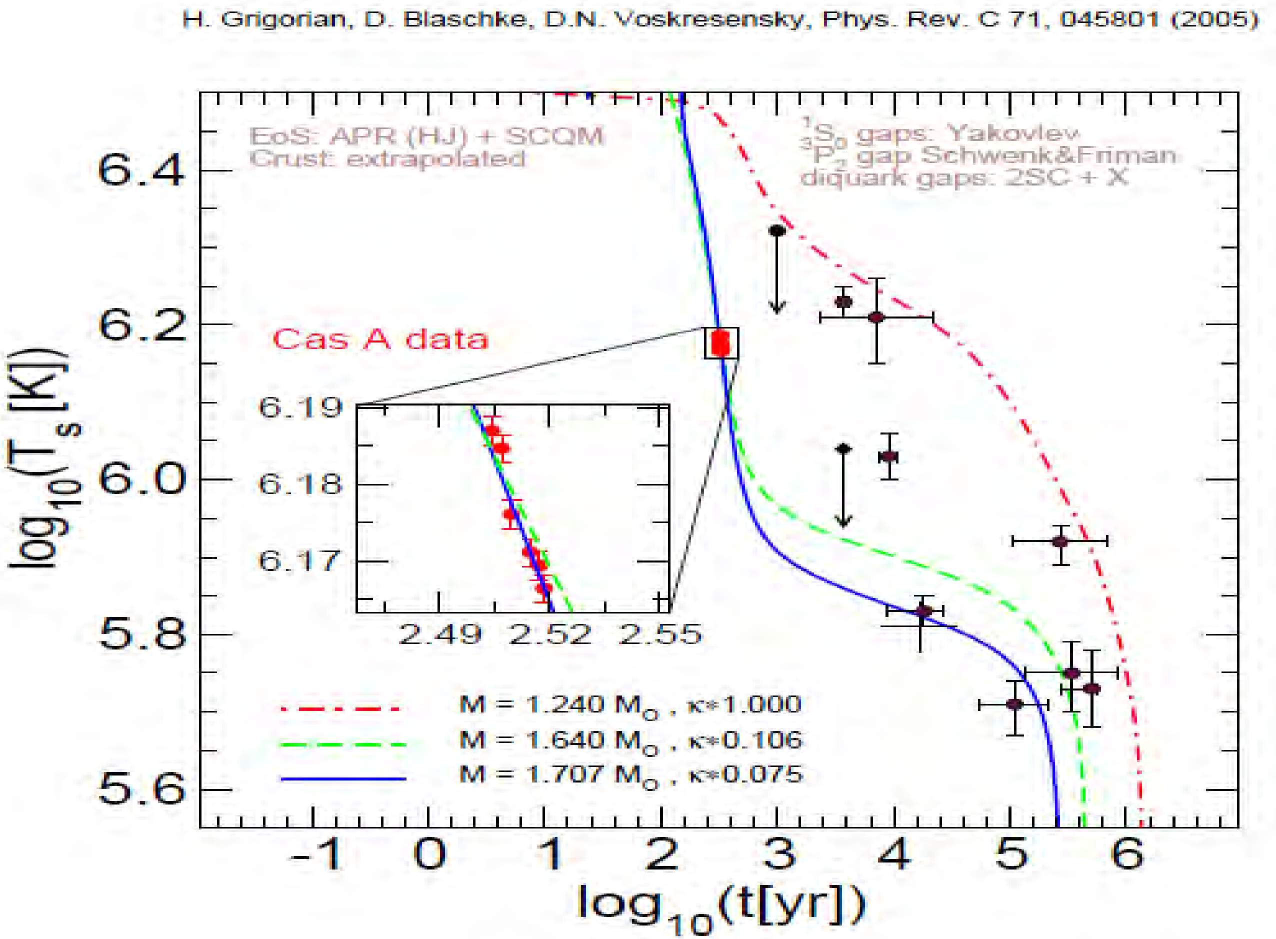
Temperature in the Hybrid Star Interior



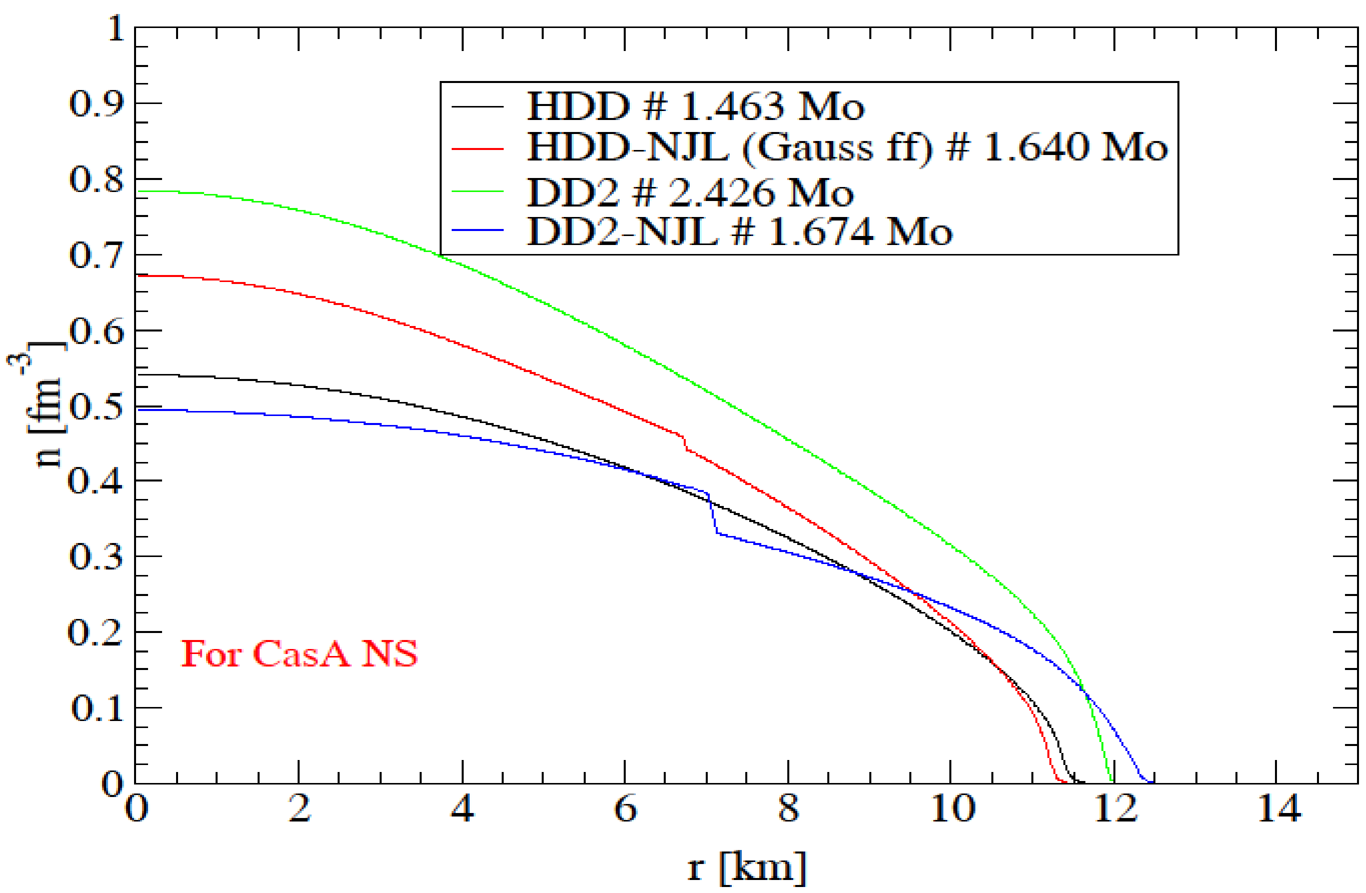
Cas A as an Hadronic Star



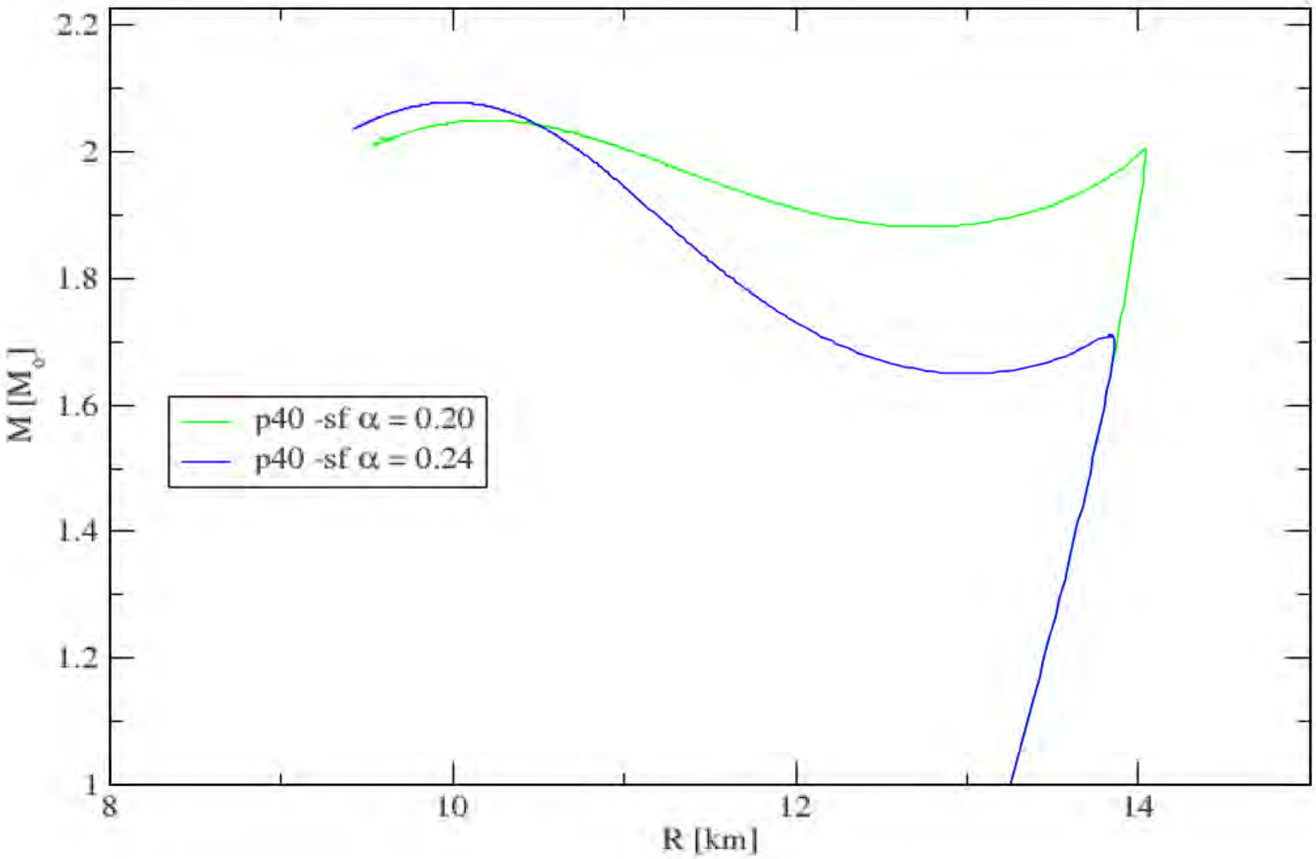
Cas A As An Hybrid Star



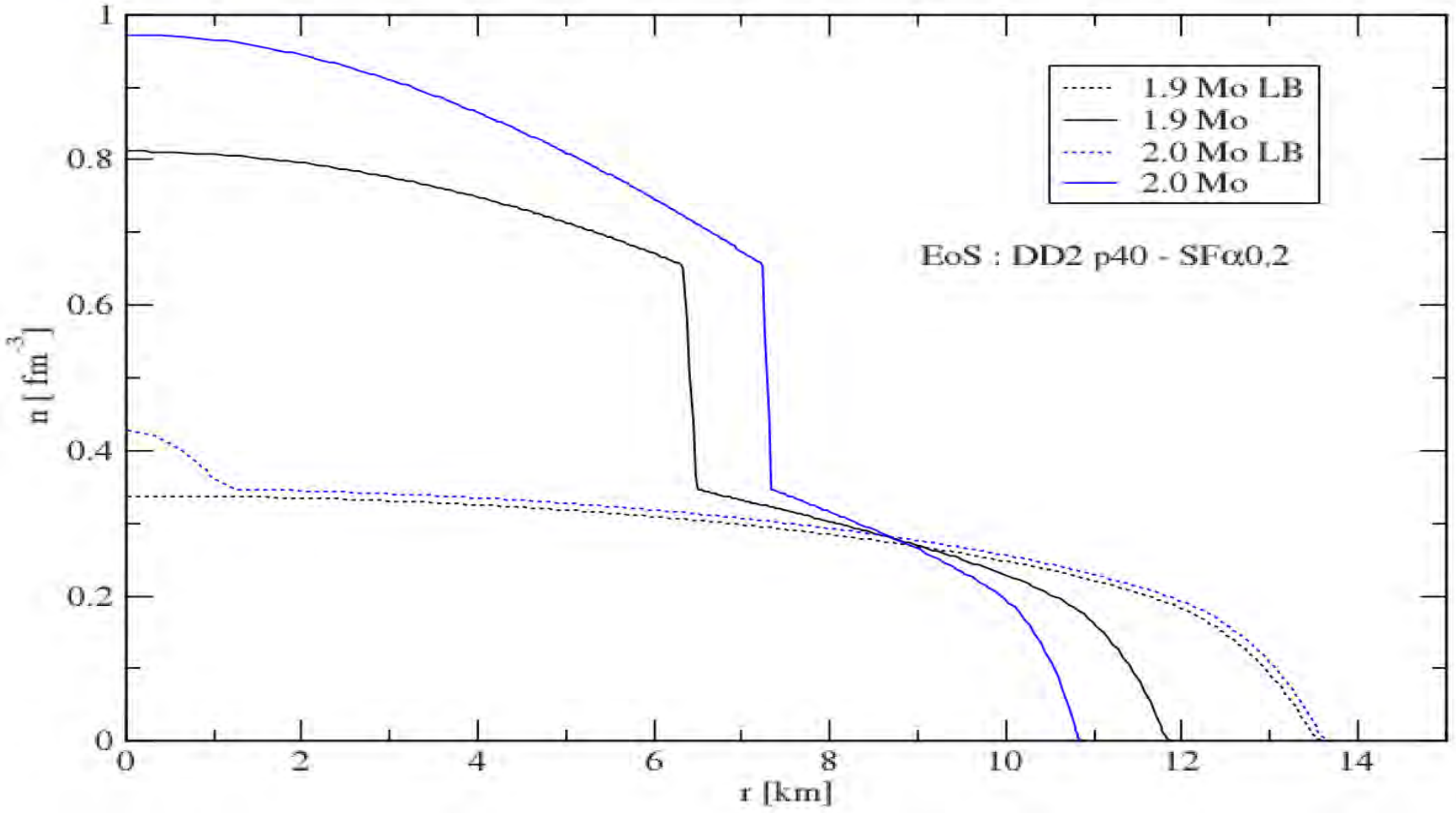
Possible internal structure of CasA



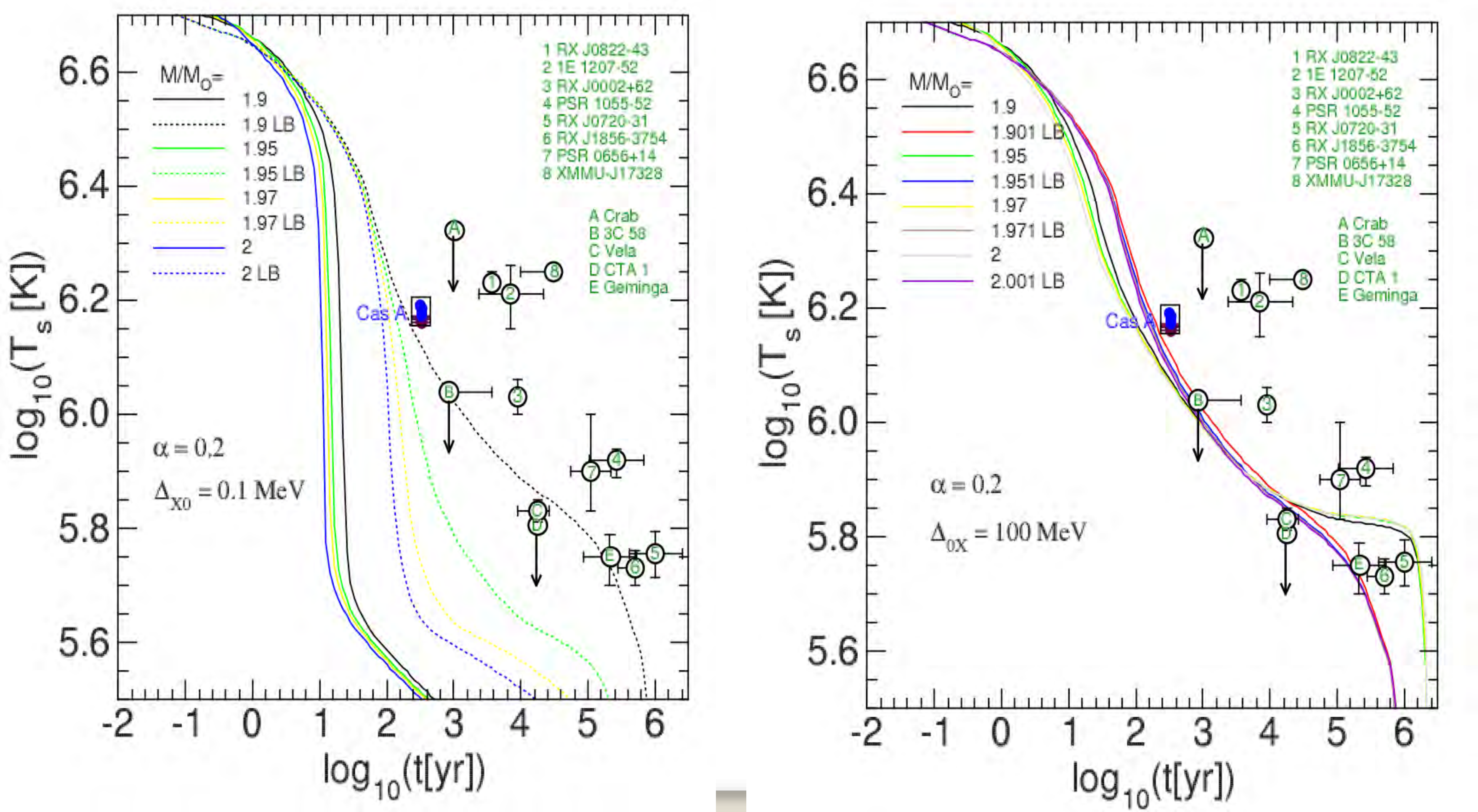
High Mass Twin CS

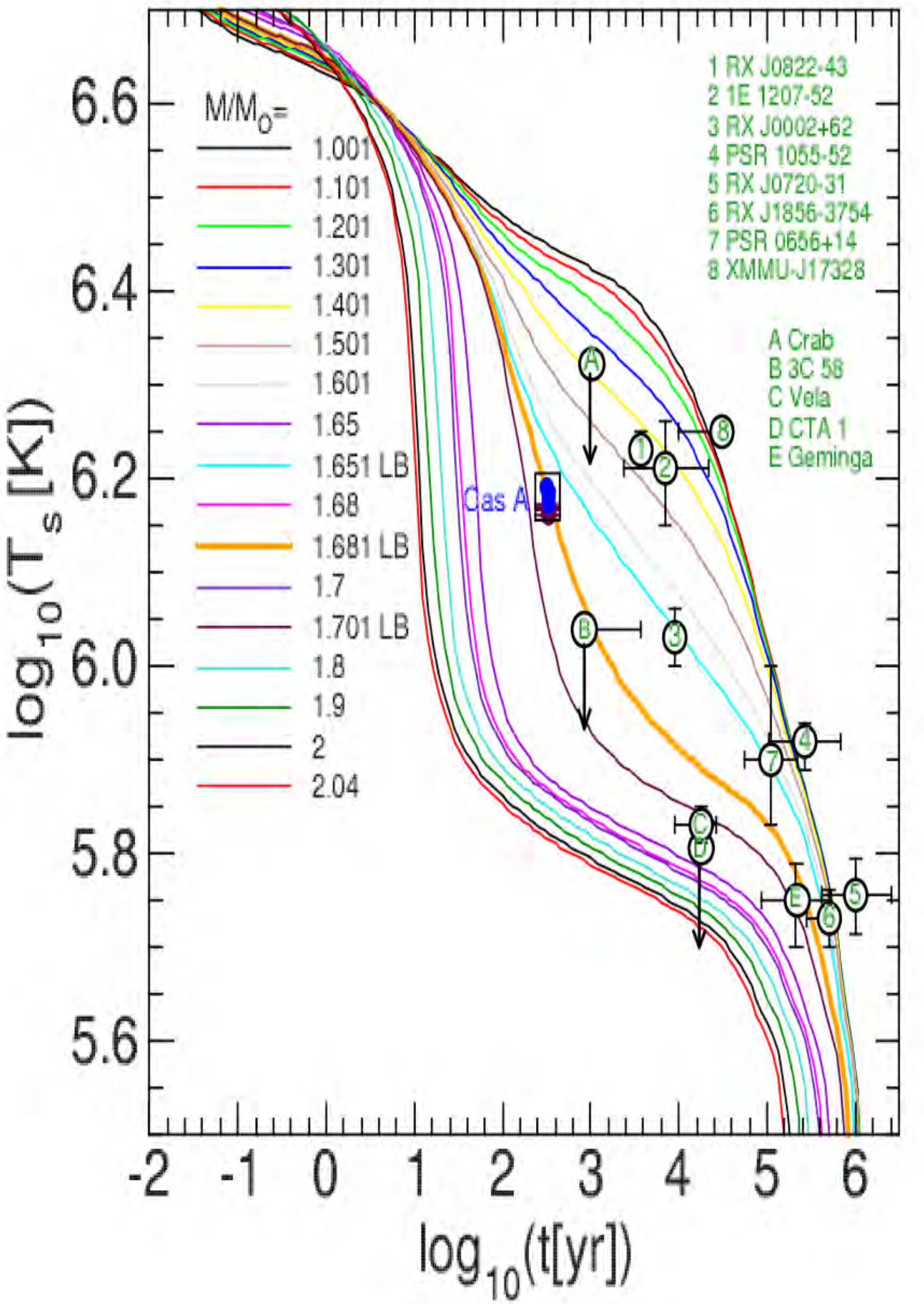


Different Configurations with the same NS mass

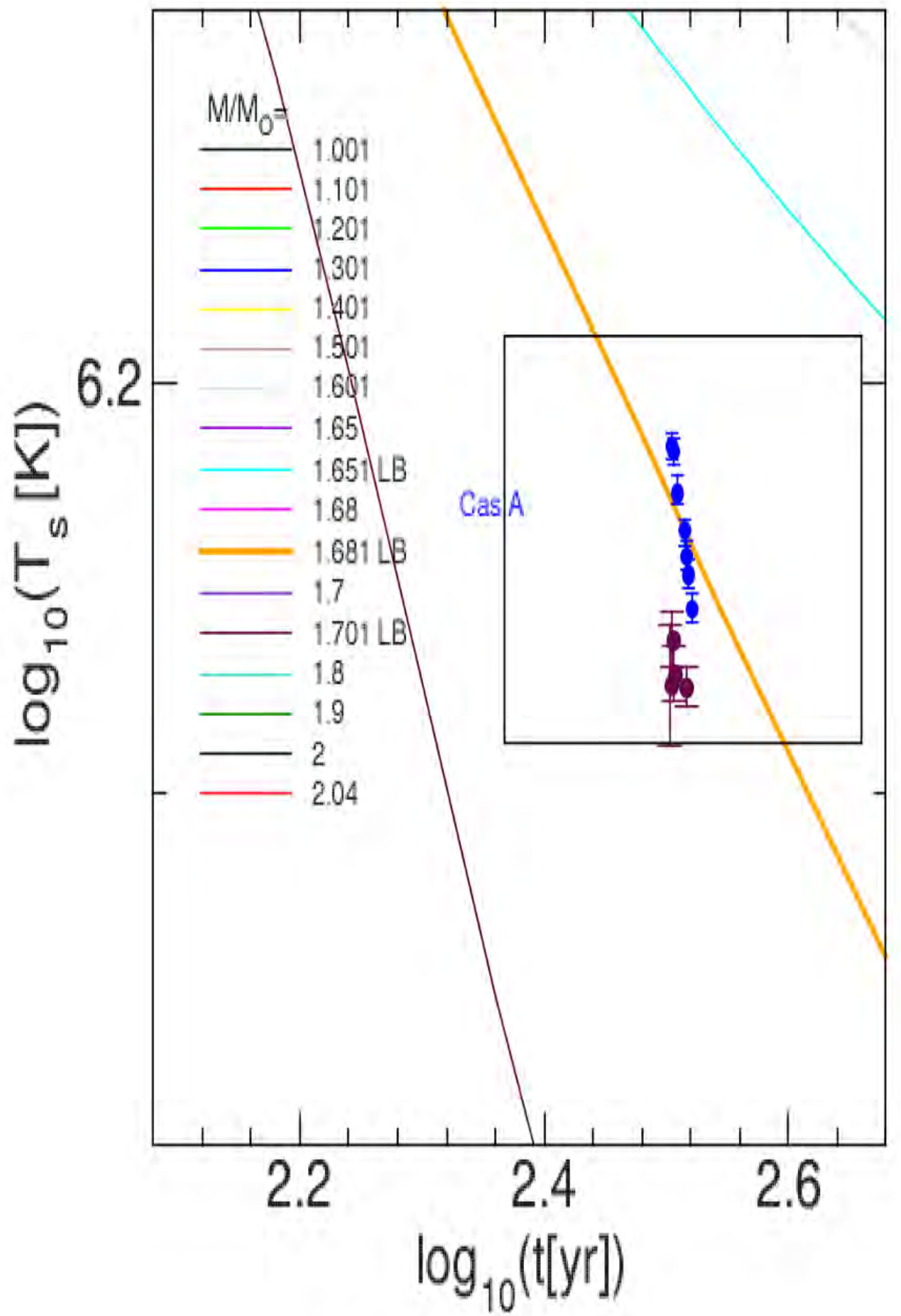


Highmass Twins: QM SC Effect





M/M$_O$=
1.001
1.101
1.201
1.301
1.401
1.501
1.601
1.65
1.651 LB
1.68
1.681 LB
1.7
1.701 LB
1.8
1.9
2
2.04
1 RX J0822-43
2 1E 1207-52
3 RX J0002+62
4 PSR 1055-52
5 RX J0720-31
6 RX J1856-3754
7 PSR 0656+14
8 XMMU-J17328
A Crab
B 3C 58
C Vela
D CTA 1
E Geminga
Cas A
log$_{10}$(T$_s$ [K])
log$_{10}$(t[yr])



M/M$_O$=
1.001
1.101
1.201
1.301
1.401
1.501
1.601
1.65
1.651 LB
1.68
1.681 LB
1.7
1.701 LB
1.8
1.9
2
2.04
Cas A
log$_{10}$(T$_s$ [K])
log$_{10}$(t[yr])

Cooling of Neutron Stars admixed with Light Dark Matter

$$\frac{e^{-\lambda-2\Phi}}{4\pi r^2}\frac{\partial}{\partial r}\left(e^{2\Phi}L\right) = -Q + Q_{\rm h} - \frac{c_V}{e^{\Phi}}\frac{\partial T}{\partial t},$$

$$\frac{L}{4\pi\kappa r^2} = e^{-\lambda-\Phi}\frac{\partial}{\partial r}\left(Te^{\Phi}\right)$$

$$N_\chi(t) \simeq N_{\chi,0} + \frac{dN_\chi}{dt}(t-t_0),$$

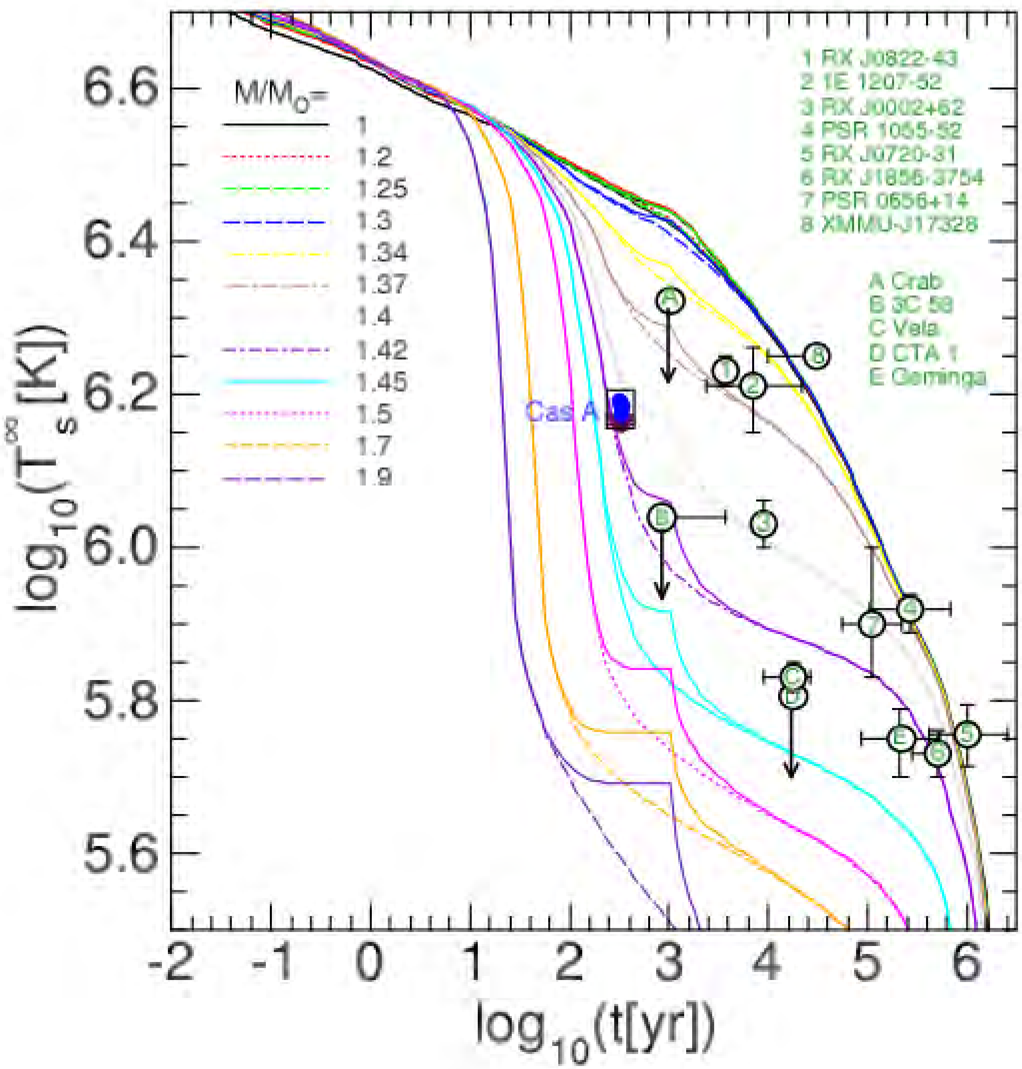
$$\frac{dN_\chi}{dt} = C_\chi - C_a N_\chi^2.$$

The DM capture rate can be approximated by

$$C_\chi \simeq 5.6\times10^{26}\left(\frac{M}{1.5M_\odot}\right)\left(\frac{R}{14\ \text{km}}\right)\left(\frac{0.1\text{GeV}}{m_\chi}\right)\left(\frac{\rho_\chi}{0.4\frac{\text{GeV}}{\text{cm}^3}}\right)\ \text{s}^{-1}$$

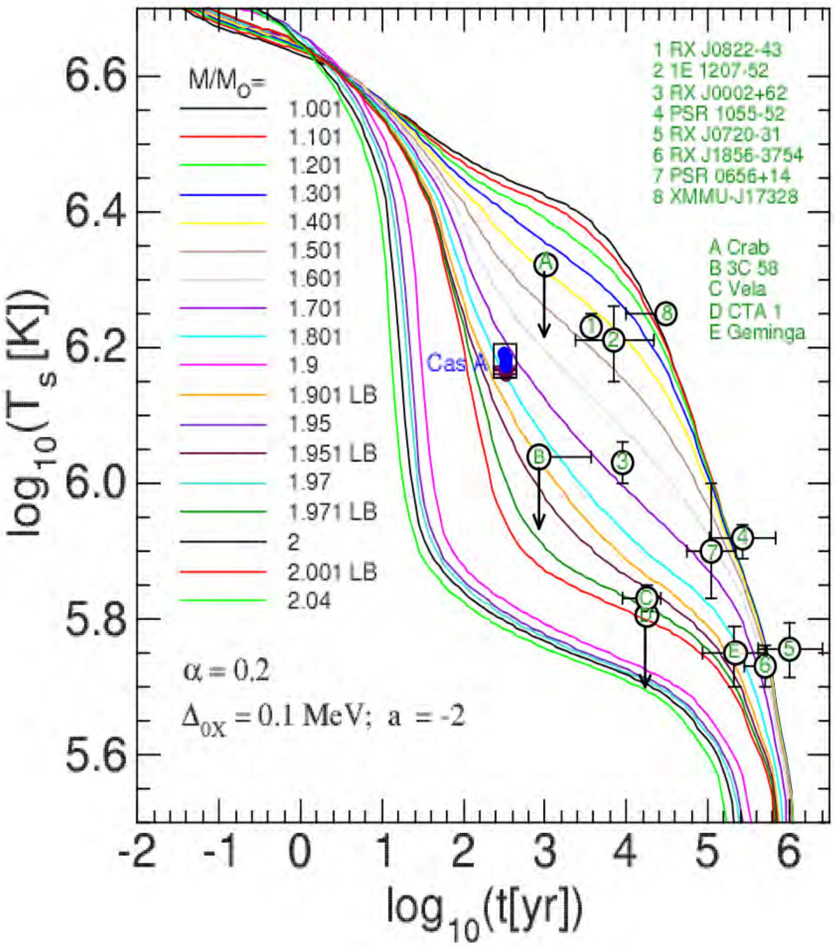
the thermally-averaged self-annihilation rate $\langle\sigma v\rangle \sim 3$ $10^{-26}\,\text{cm}^3\text{s}^{-1}$,

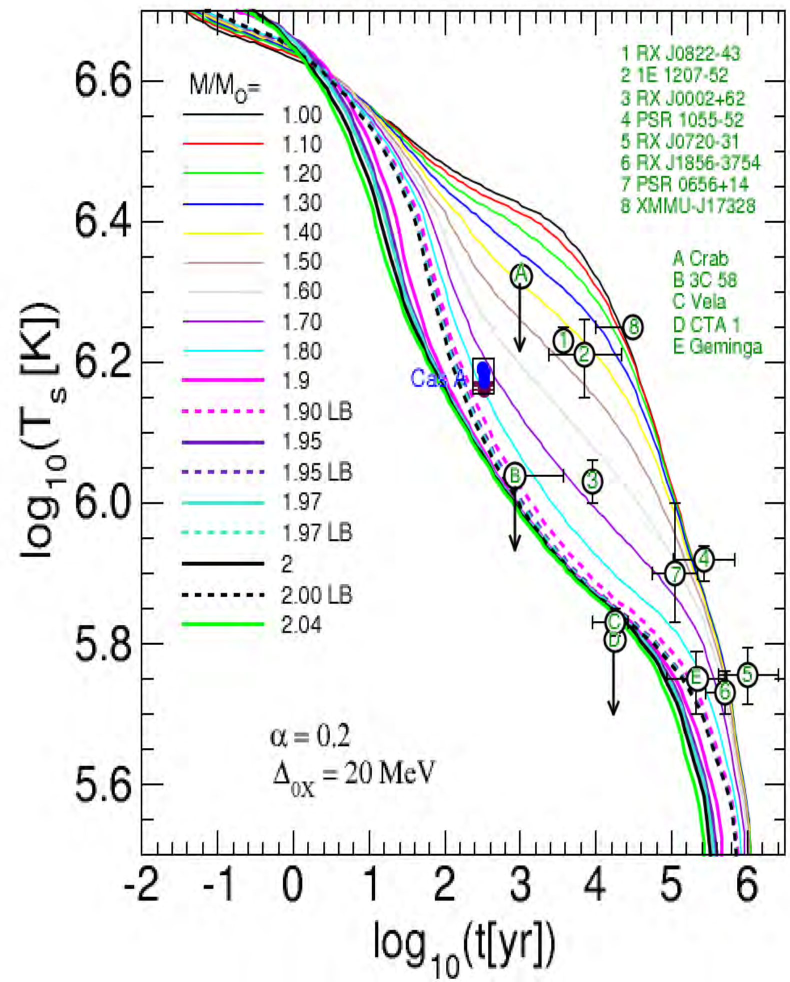
$$C_a \simeq 2\times10^{-42}\left(\frac{0.1\ \text{GeV}}{m_\chi}\frac{2\rho_0}{\rho_N}\frac{T}{0.5\,\text{MeV}}\right)^{-3/2}\text{s}^{-1}.$$



NSs with masses $M \in [1, 1.9]M_\odot$ with the effect of self-annihilating LDM ($m_\chi = 0.1$ GeV) originating a plateau or without LDM (continuous decline). Existing series of cooling

Cooling of Twin CS





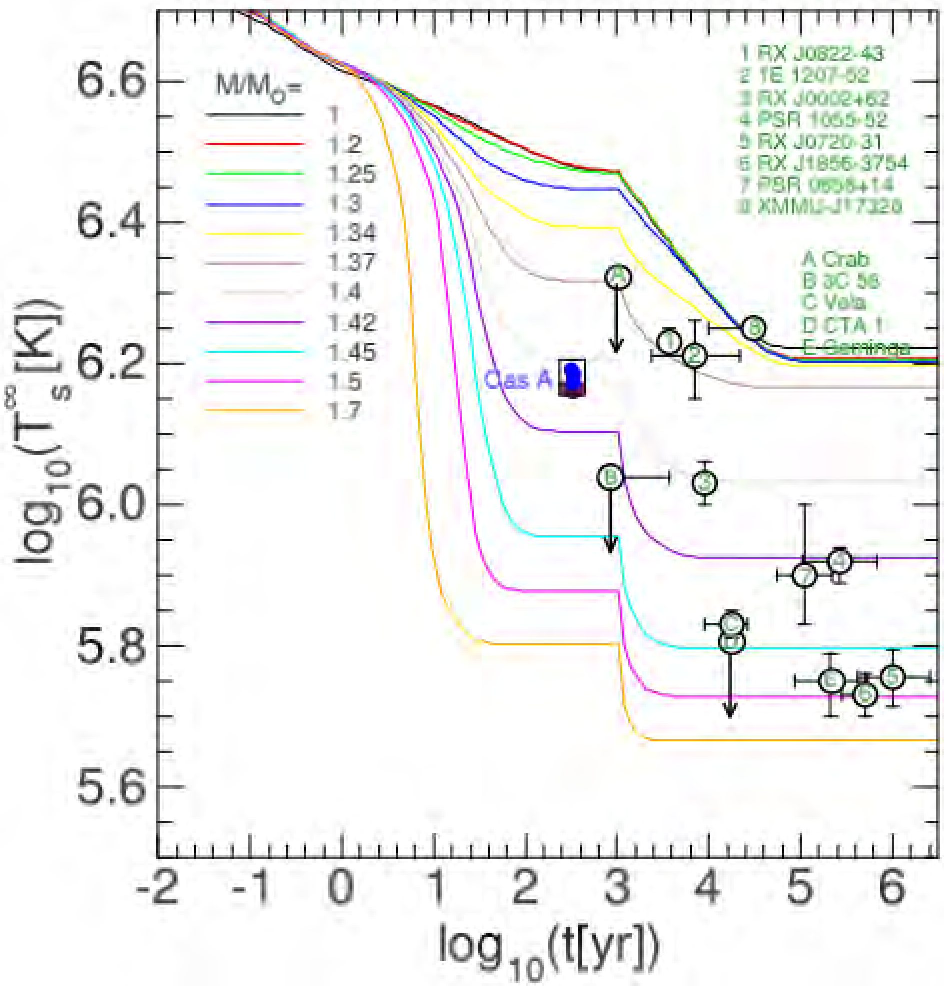


FIG. 2. Surface temperature as a function of NS age with masses $M \in [1, 1.7]M_\odot$ including self-annihilating conducting DM ($m_\chi = 0.1$ GeV). χ emissivity has been enhanced a factor 5 larger than in Figure 1. LDM enhanced processes are active up to $\tau \sim 10^3$ yr, followed by a period of decline, and again for $t \gtrsim 1.5 \times 10^3$ yr. See text for details.

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