On numerical singularities of computational scheme in the reconstruction problem

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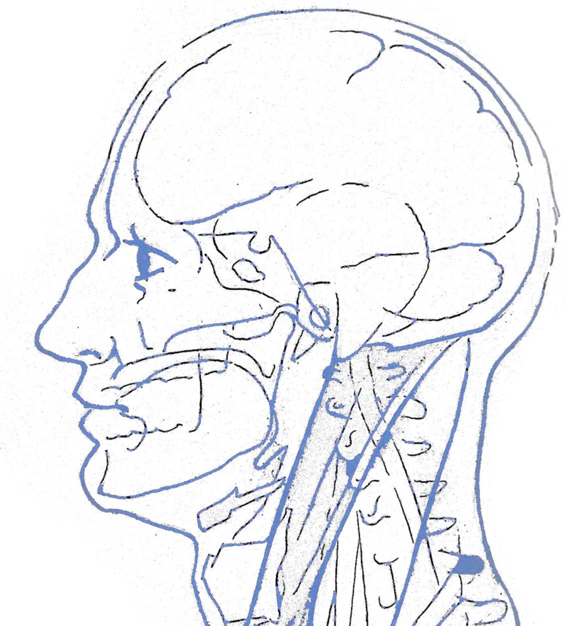

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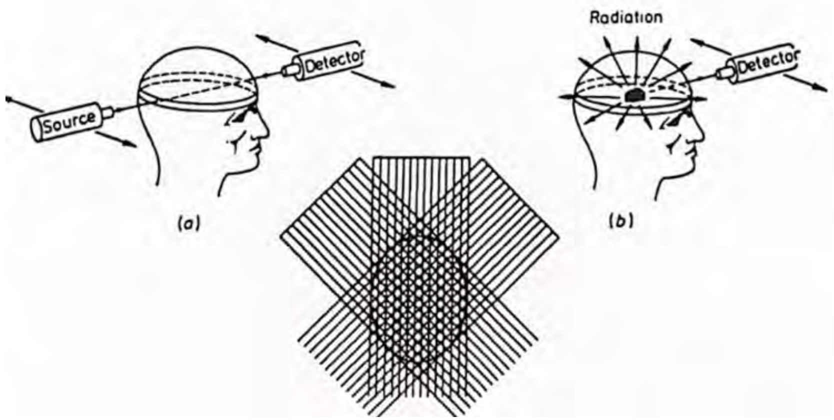
The task of image reconstruction

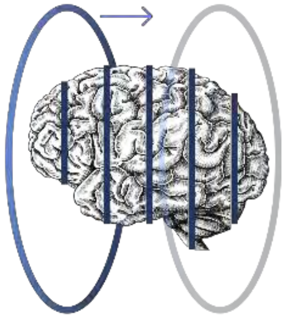
The task is to study the internal structure of subjects without breakings and cutting. The medical CT methods which exist nowaday suffer from many different difficulties (blurred zones on images). The practical application involves the analysis and visualization of CT data including **transmission (a)** and **emission (b)** processes.

The purpose of project is the development of CT data visualization and possible improvements of CT images which should eliminate the blurred zones.

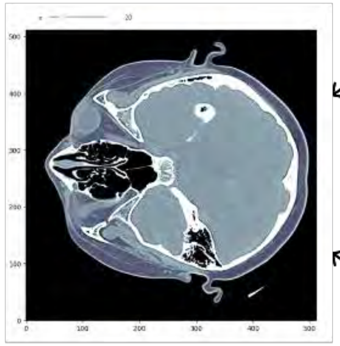


Scheme of reconstruction

CT machine → CT scan → Matrix of intensity ratio → Reconstruction of images



$$\begin{array}{c|ccc} & \tau_1 & \tau_2 & \tau_3 \\ \hline \phi_1 & * & * & * \\ \phi_2 & * & * & * \\ \phi_3 & * & * & * \end{array}$$



standard method leading the blurred zones

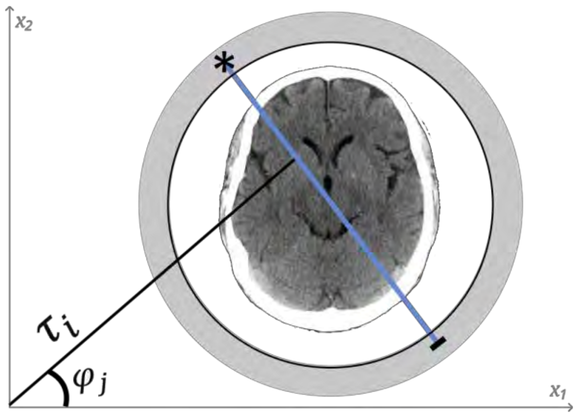
proposed method that should eliminate the blurred zones

In practice, the original slice function differs from the reconstructed forms. Our principal goal is to minimize the mentioned difference using a new mathematical method. To this aim, as a preliminary stage the corresponding polygon of CT-data analysis is planned to be created.

The standard process of reconstruction

Let $f(\vec{x}, x_{3_k})$ be the original outset function that describes the given slice of the scanned object.

$f_R^{st}(\vec{x}, x_{3_k}) = \sum_{i=1}^{N} \sum_{j=1}^{M} \Delta\tau_i \Delta\varphi_i W_S(\tau_i, \varphi_i)[ln\frac{I}{I_0}(\tau, \varphi; \vec{x}, x_{3_k})]_{k;i,j}$, where



- $f_R^{st}(\vec{x}, x_{3_k})$ is the reconstructed function;
- $ln\frac{I}{I_0}(\tau, \varphi; \vec{x}, x_{3_k})$ gives the intensity ratio of incoming and outgoing X-rays;
- $W_S(\tau_i, \varphi_i)$ is the weight function of the standard procedure.

The problem of reconstruction: $f_R^{st}(\vec{x}, x_{3_k}) \neq f(\vec{x}, x_{3_k})$

The proposed approach

$$f_R^{prop}(\vec{x}, x_{3_k}) = \sum_{i=1}^{N} \sum_{j=1}^{M} \Delta\tau_i \Delta\varphi_i \{W_S(\tau_i, \varphi_i) + W_A(\tau_i, \varphi_i)\}[ln\frac{I}{I_0}(\tau, \varphi; \vec{x}, x_{3_k})]_{k;i,j},$$

where $W_A(\tau_i, \varphi_i)$ is the weight function which reflects the extended and improved method of reconstruction.

The principal aim is to obtain that

$$f_R^{prop}(\vec{x}, x_{3_k}) \sim f(\vec{x}, x_{3_k})$$

I. V. Anikin, “Universal inverse Radon transforms: Complexity of Radon transforms and the hybrid functions,” [arXiv:2506.18911 [math.FA]];
I. V. Anikin, “Universal inverse Radon transforms: Inhomogeneity, angular restrictions and boundary,” Mod. Phys. Lett. A (in press) [arXiv:2504.01744 [math.CA]]
I. V. Anikin and X. Chen, “Inverse Radon transforms: Analytical and Tikhonov-like regularizations of inversion,” Mod. Phys. Lett. A 39, no.38, 2450181 (2024) [arXiv:2405.14897 [physics.comp-ph]].

Polygon for visualization of brain CT data

Component for educational purposes
JupyterLab Server [https://studhub1.jinr.ru] [https://studhub2.jinr.ru]
- 2x Intel Xeon E5-2680 v3 (12 Cores @ 2.5 GHz)
- 256 GB RAM

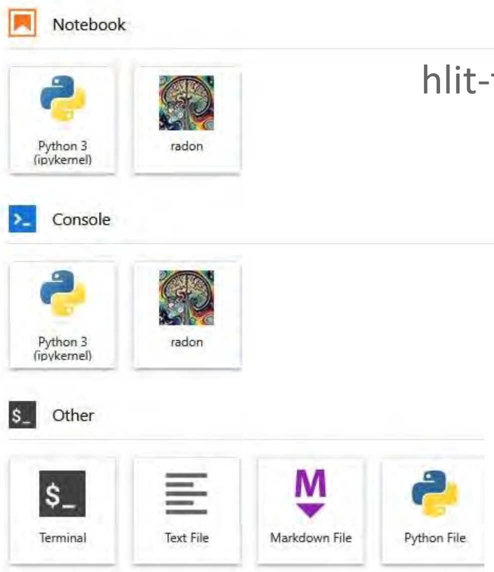
Component for computational tasks
Server with NVIDIA Volta [https://jhub1.jinr.ru] [https://jhub2.jinr.ru]
- 2x Intel Xeon Gold 6148 (20 Cores @ 2.4 GHz)
- 4x NVIDIA Volta V100 SXM2 32 GB HBM2
- 512 GB RAM

Component for scientific projects
Server with NVIDIA Ampere [https://ampere03.jinr.ru] [https://ampere05.jinr.ru]
- 2x AMD EPYC 7763 (64 Cores @ 2.4 GHz)
- 8x NVIDIA Ampere A100 SXM4 80 GB HBM2
- 2 TB RAM

Work tools for visualization

- **PyDicom** is a Python package used to read, modify, and write DICOM (Digital Imaging and Communications in Medicine) files, which are commonly used in medical imaging and related information. It provides a way to interact with DICOM datasets in a Pythonic way, allowing users to easily access and manipulate DICOM metadata and pixel data.

- **Nipype** (Neuroimaging in Python - Pipelines and Interfaces) is a Python-based open-source software package that offer users an incredible opportunity to analyze data using a variety of different algorithms and provides a uniform interface to neuroimaging software.
- **Pysurfer** is a Python library for visualizing cortical surface representations of neuroimaging data.



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The main goals of the polygon are:

I. developing of mathematical approach
II. developing of software environment for data analysis and for test new mathematical methods

Libraries used for calculations (SciPy)

- Analysis of the integral function of the standard contribution with angular dependence: analysis of a singularity, the **scipy.integrals.quad()** function.

1. Choosing a strategy (algorithm)

- QAGS (General-purpose)
- QAGI (Infinite intervals)
- QAGP (Points)
- QAWO / QAWF

→

2. The Gauss-Kronrod rule

Calculation of the integral

$$\int_a^b f(x)dx \approx \sum_{i=1}^n w_i f(x_i)$$

↙

3. Adaptive partitioning

If the error in the entire integration interval exceeds the value of **epsabs** or **epsrel**, the algorithm switches to iterative mode.

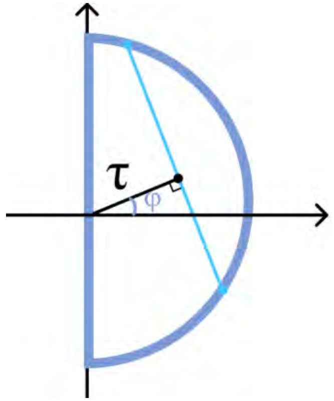
SciPy

Modeling the reconstruction procedure

Sum of contributions from the radial and angular dependence of the standard approach

$$\mathcal{R}[f](\tau,\varphi) = M\sqrt{1-\tau^2}\left\{\Theta(\tau\in[\sin\varphi,1])+\Theta(\tau\in[0,1])+\Theta(\tau\in[\cos\varphi,1])\right\}$$
$$+M\,\tau\cot\varphi\,\Theta(\tau\in[0,\sin\varphi])-M\,\tau\tan\varphi\,\Theta(\tau\in[\cos\varphi,1]),$$

$$f_\varepsilon(\vec{\mathbf{x}}) = f_S(\vec{\mathbf{x}})+f_A(\vec{\mathbf{x}})$$



where

$$f_S^{(\tau)}(\vec{\mathbf{x}}) = -M\int_0^{\pi/2}(d\varphi)\left\{\int_{-A_1(\varphi)}^{A_4(\varphi)}(d\eta)\frac{\sqrt{1-(\eta+A_1(\varphi))^2}}{\eta^2}\right.$$
$$\left.+\int_{A_2(\varphi)}^{A_4(\varphi)}(d\eta)\frac{\sqrt{1-(\eta+A_1(\varphi))^2}}{\eta^2}+\int_{A_2^\star(\varphi)}^{A_4(\varphi)}(d\eta)\frac{\sqrt{1-(\eta+A_1(\varphi))^2}}{\eta^2}\right\}$$

$$A_1(\varphi)\equiv\vec{\mathbf{n}}_\varphi\vec{\mathbf{x}} = \sin(\varphi)x_2+\cos(\varphi)x_1$$
$$A_2(\varphi) = \sin(\varphi)(1-x_2)-\cos(\varphi)x_1$$
$$A_4(\varphi) = 1-A_1(\varphi)$$
$$A_2^\star(\varphi) = (1-x_1)\cos\varphi-x_2\sin\varphi$$

$\vec{\mathbf{n}}_\varphi = (\cos(\varphi),\sin(\varphi))$ and $\vec{\mathbf{n}}_\varphi^2 = 1$

$\vec{\mathrm{x}} = (x_1, x_2)$,
$\tau = \eta + A_1(\varphi)$,
$M = m_I + m_{IV}$

Modeling the reconstruction procedure

Sum of contributions from the radial and angular dependence of the standard approach

and

$$f_S^{(\varphi)}(\vec{\mathbf{x}})\Big|_{\text{cot-term}} = -M \int_0^{\pi/2} (d\varphi) \cot\varphi \times \left[\log \frac{x_1 \cos\varphi - (1-x_2)\sin\varphi}{x_1\cos\varphi + x_2 \sin\varphi} + \frac{x_1\cos\varphi + x_2\sin\varphi}{x_1\cos\varphi - (1-x_2)\sin\varphi} - 1 \right]$$

While the contributions which trace from the term with tan-functions have been omitted for the moment.

Result: `3379177106.430385`

Modeling the reconstruction procedure

- Analytical singularity: calculations of two-manifold integrations by **scipy.quad**

$$f_S^{(\tau)}(\vec{\mathbf{x}}) = M\int_0^{\pi/2}(d\varphi)\Big\{-\frac{1}{A_1(\varphi)}-\int_{A_2(\varphi)}^{A_4(\varphi)}(d\eta)\frac{1}{\sqrt{1-(\eta+\vec{\mathbf{n}}_\varphi\vec{\mathbf{x}})^2}}\frac{\eta+A_1(\varphi)}{\eta} - \int_{-A_1(\varphi)}^{A_4(\varphi)}(d\eta)\frac{1}{\sqrt{1-(\eta+\vec{\mathbf{n}}_\varphi\vec{\mathbf{x}})^2}}\frac{\eta+A_1(\varphi)}{\eta}\Big\}$$

$$f_S^{(\varphi)}(\vec{\mathbf{x}}) = M\int_0^{\pi/2}(d\varphi)\cot(\varphi)\log\left(\frac{\cos(\varphi)x_1-\sin(\varphi)(1-x_2)}{\cos(\varphi)x_1+\sin(\varphi)x_2}\right)$$

$$f_S^{(\tau)}(\vec{x}) = -M\int_0^{\pi/2}(d\varphi)\{\int_0^1 dt[\frac{1}{A_1(\varphi)}+(A_4-A_2)\cdot\frac{1}{\sqrt{1-(A_2+t(A_4-A_2)+A_1)^2}}\cdot\frac{A_2+t(A_4-A_2)+A_1}{A_2+t(A_4-A_2)}+ + (A_4+A_1)\cdot\frac{1}{\sqrt{1-(t(A_4+A_1))^2}}\cdot\frac{t(A_4+A_1)}{(-A_1)+t(A_4+A_1)}]\}$$

$$A_1(\varphi)\equiv\vec{\mathbf{n}}_\varphi\vec{\mathbf{x}} = \sin(\varphi)x_2+\cos(\varphi)x_1$$
$$A_2(\varphi) = \sin(\varphi)(1-x_2)-\cos(\varphi)x_1$$
$$A_4(\varphi) = 1-A_1(\varphi)$$

Result: `27.002214364042917`

Conclusions

- For the visualization of medical computed tomography data, the polygon based on heterogeneous platform "HybriLIT" has been developed.

The main purposes of polygon are:

1) CT and MRI/fMRI data storage;
2) The visualized work with data in web-browser thanks for the possibility to work within JupyterLab interactive environment;
3) The visualization of results using the special software;
4) The software development;
5) Mathematical modeling.

- Mathematical modeling for the study of possible calculations applied to Radon transformations with the help of Python libraries.

- For the study of analytical and numerical singularities, the corresponding of algorithms have been developed.

Thank you for attention!