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prove their worth  
by hitting back

*Piet Hein*

В задачах тех ищи удачи, где  
получить рискуешь сдачи.

*Перевод Я. А. Смородинского*

# TARGET DEPENDENCE OF THE ISOTOPE DISTRIBUTIONS IN HEAVY-ION REACTIONS AT FERMI ENERGIES

**T. I. Mikhailova<sup>1</sup> (MLIT)**

in collaboration with

***B. Erdemchimeg*<sup>1,2</sup>, *Yu Sereda*<sup>1</sup> (FLNR)**

<sup>1</sup>JINR, Dubna, Russia

<sup>2</sup>Mongolian National University, NRC,  
Ulaanbaatar, Mongolia

# Outline

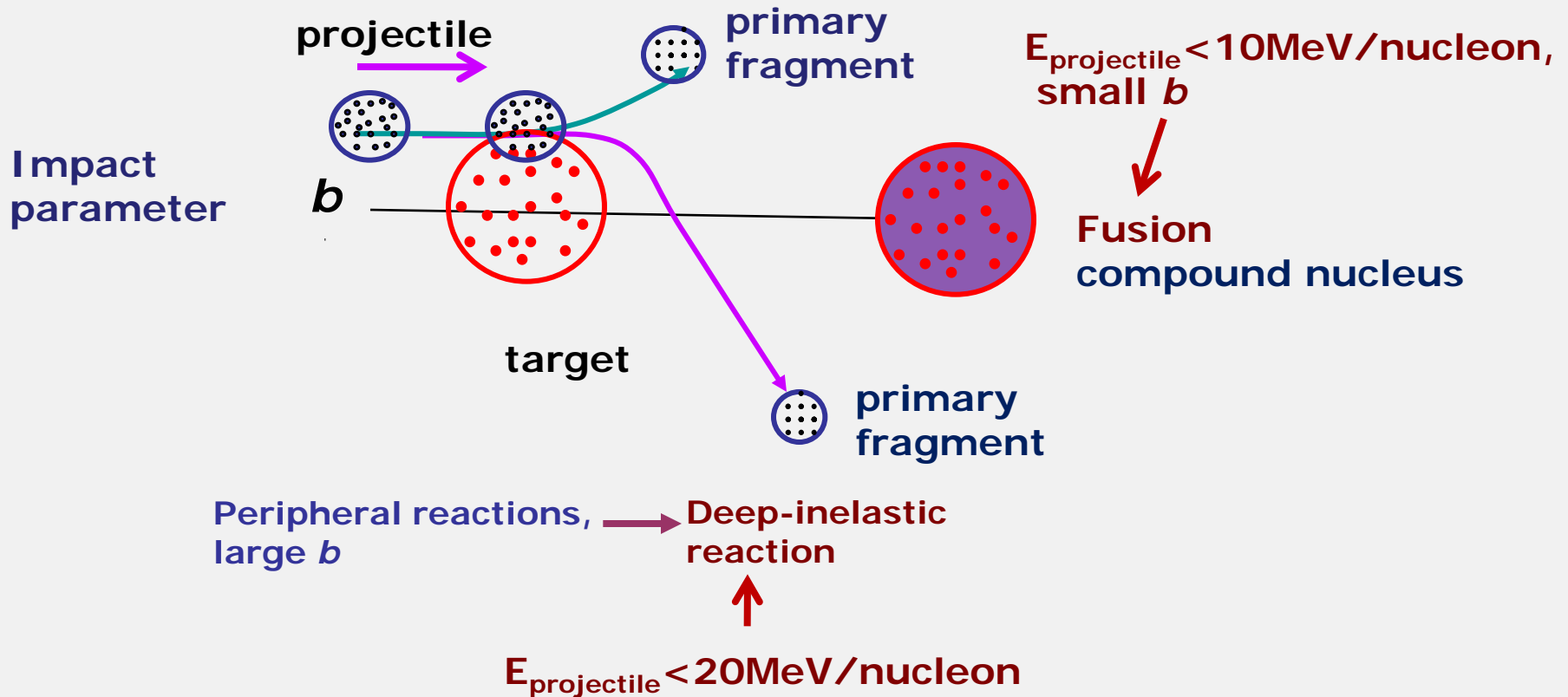
- **Motivation.**
- **Description of heavy-ion collisions with transport-statistical approach (BNV-SMM) and its numerical implementation.**
- **Comparison with experimental data and other model calculations**
- **Explanation of target dependence from the point of view of BNV-SMM calculations**
- **Conclusion**

# Schematic view of the collision of two heavy ions

$$E_{\text{projectile}} > 100 \text{ MeV/nucleon}$$



Direct reaction



How to describe the collision in the energy range  
 $10 \text{ MeV / nucleon} < E_{\text{projectile}} < 100 \text{ MeV / nucleon}$



To model heavy-ion collision microscopically we use kinetic theory:

**Transport theory: Boltzmann-Nordheim-Vlasov (BNV) approach**

time evolution of the one-body phase space density:  $f(\mathbf{r},\mathbf{p};t)$

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla} f - \vec{\nabla} U \vec{\nabla}_p f = I_{coll} [f, \sigma]$$

Physical input:

mean field potential  $U$  (equation of state)

and in-medium elastic cross section

F. Bertsch, S. Das Gupta, Phys. Rep. **160** (1988) 189

V. Baran, M. Colonna, M. Di Toro, Phys. Rep., **410** (2005) 335

**Density functional**

$U(\rho(\mathbf{r})) =$  Nuclear Mean Field + Symmetry terms + Coulomb

$$U(\rho) = A \left[ \frac{\rho}{\rho_0} \right] + B \left[ \frac{\rho}{\rho_0} \right]^d + C (-1)^k (\rho_n - \rho_p) / (\rho_n + \rho_p) + U_{coul}$$

$$A = -356 \text{ MeV}, B = 303 \text{ MeV}, d = 7/6, k = 1(p), 2(n), C = 36 \text{ MeV}$$

# Collision term

$$I_{coll}[f_1, \sigma] = \frac{g}{h} \int d^3 p_2 d^3 p_3 d^3 p_4 \sigma(12, 34) \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 + \vec{p}_4) \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 + \varepsilon_4) \left[ \bar{f}_1 \bar{f}_2 f_3 f_4 - f_1 f_2 \bar{f}_3 \bar{f}_4 \right]$$

Pauli blocking factors for final state  
 $g$  degeneracy

$$(1 - f(r, v_i; t)) \equiv (1 - f_i) := \bar{f}_i$$

Collision term: treatment by stochastic simulation

1. Select in each time step  $dt$  TPs with distance  $d \leq \sqrt{\sigma / \pi}$
2. Collide with probability  $P = \sigma_{el} / \sigma_{max}$  with random scattering angle
3. Check Pauli blocking of final state in phase space

Computationally most expensive part of calculation

# Solution of transport equation

Partial integro-differential equation for  $f(r,p;t)$  is solved by simulation with the **test particle** method:

$N$  - finite element test particles (TP) per nucleon.

Each TP carries charge and isospin number.

$A$  – number of nucleons in the system

$g$  – the shape of the TP in space

$\rho$  – the density

$$f(\vec{r}, \vec{p}, t) = \frac{1}{N} \sum_i^{NA} g(\vec{r} - \vec{r}_i(t)) \bar{g}(\vec{p} - \vec{p}_i(t))$$

$$g = e^{-(\vec{r} - \vec{r}_i(t))^2 / L^2} \dots; \bar{g} = e^{-(\vec{p} - \vec{p}_i(t))^2 / l^2}$$

$$\rho(r;t) = \int d\vec{p} f(\vec{r}, \vec{p}; t)$$

Equations of motion of TP (Hamiltonian EoM's):

$$\frac{\partial \vec{p}_i(t)}{\partial t} = -\vec{\nabla}_r U(r_i, t) \quad \frac{\partial \vec{r}_i(t)}{\partial t} = \frac{\vec{p}_i(t)}{m}$$

Velocity Verlet ( or leapfrog) algorithm, accuracy  $(dt)^2$  :

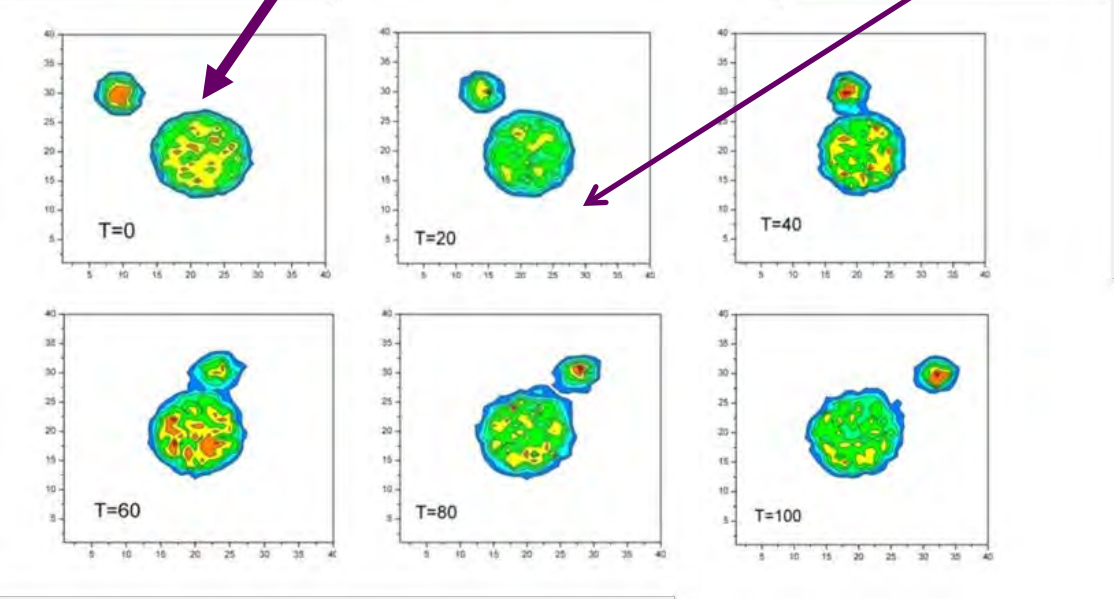
$$\vec{p}_i(t + \frac{1}{2}\Delta t) = \vec{p}_i(t) - \frac{1}{2}\Delta t \vec{\nabla}_r U(r_i(t))$$

$$\vec{r}_i(t + \Delta t) = \vec{r}_i(t) + \Delta t \vec{p}_i(t + \frac{1}{2}\Delta t) / m$$

$$\vec{p}_i(t + \Delta t) = \vec{p}_i(t + \frac{1}{2}\Delta t) - \frac{1}{2}\Delta t \vec{\nabla}_r U(r_i(t + \Delta t))$$

**STEP 1:** stochastic modeling of the system of two approaching ions. Minimization of energy in Woods-Saxon potential, taking into account Coulomb and Symmetry energy. The initial values of coordinates  $\mathbf{R}$  and momenta  $\mathbf{P}$  in the center of mass system are added

**STEP 2:** Evolution in time in the mean field until freeze-out time: only Coulomb potential, nuclear forces between fragments are negligible



Identify final fragments by coalescence method  
Here:  
Cut-off criterion in density

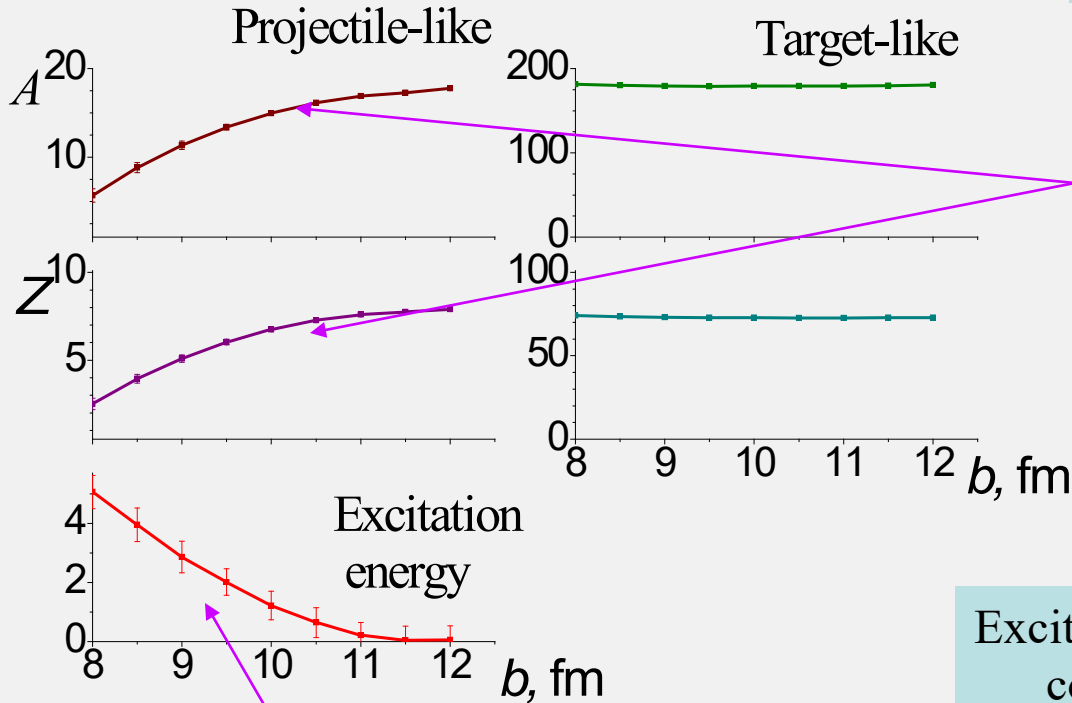
$$(\rho(r, t_{\text{freeze-out}}) < 0.02)$$

Density contour plots in the reaction  $^{18}\text{O}(35 \text{ A MeV}) + ^{181}\text{Ta}$  at  $b = 9 \text{ fm}$   
( $t=0, 20, 40, 60, 80, 100 \text{ fm} / c$  ( $10 \text{ fm}/c=3.3 \cdot 10^{-23} \text{ c}$ ))

Fragments characteristics:  
mass  $A$ , charge  $Z$ ,  
intrinsic energy  $E_{\text{int}}$ ,  
momentum  $\mathbf{P}$ , coordinates  $\mathbf{R}$

Calculations with 200 TP for nucleon

$^{18}\text{O}(35 \text{ AMev})+^{181}\text{Ta}$



The results smoothly depends on the value of impact parameter  $b$

Heavier ion stays practically the unchanged!

Excitation energy is calculated in self-consistent way with the same potentials as dynamical calculations are fulfilled

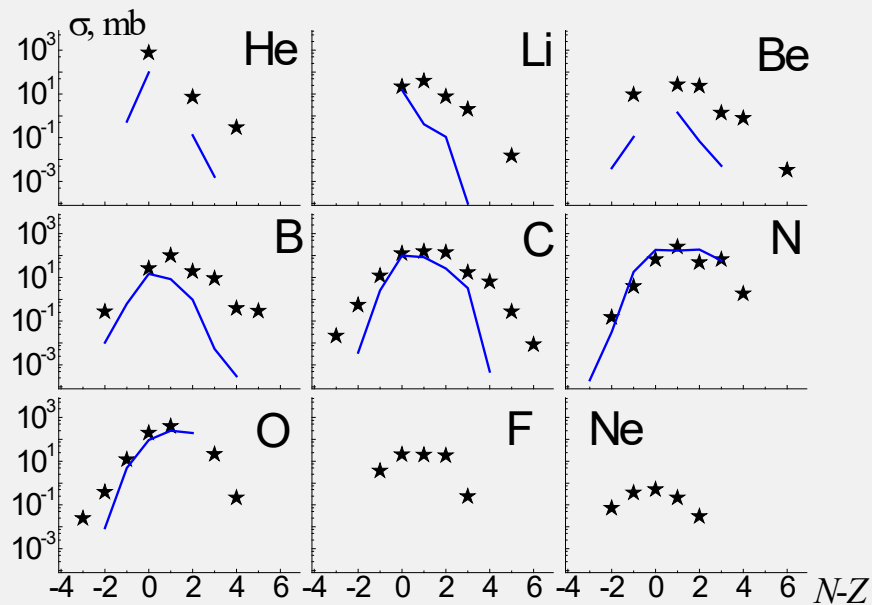
Fragments are excited!  
Transport approach is semiclassical approach, it can't describe quantum effects.

De-excite the fragments with statistical code

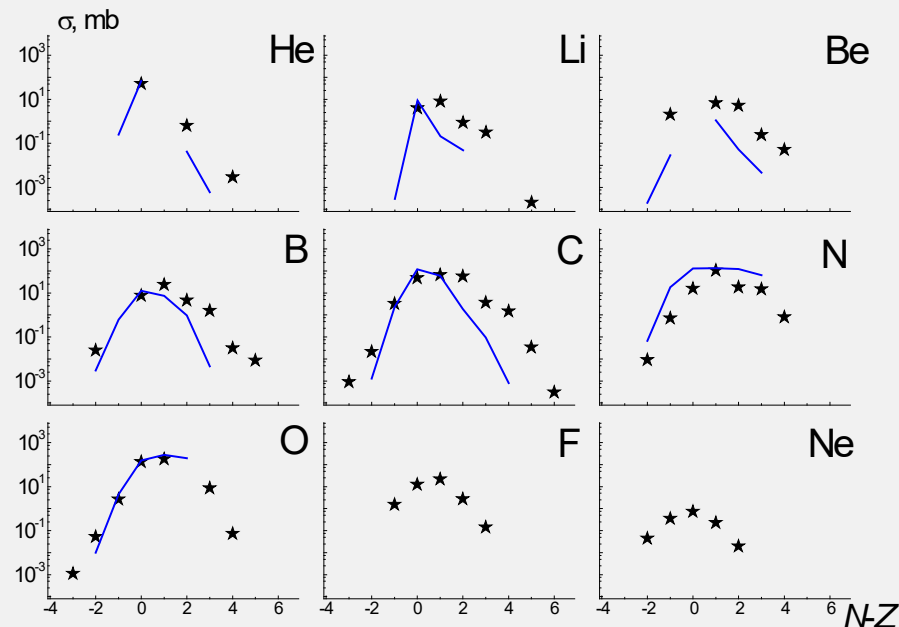


STEP 3: Calculating cold evaporation residues:  
 SMM code, P. Bondorf, et al., Phys. Rep. 257, 133 (1995)  
 Input parameters:  $A_{fr}$ ,  $Z_{fr}$ ,  $E_{exc}$ , R, P from BNV calculation

$^{18}\text{O}(35 \text{ AMev}) + ^{181}\text{Ta}$



$^{18}\text{O}(35 \text{ AMev}) + ^9\text{Be}$



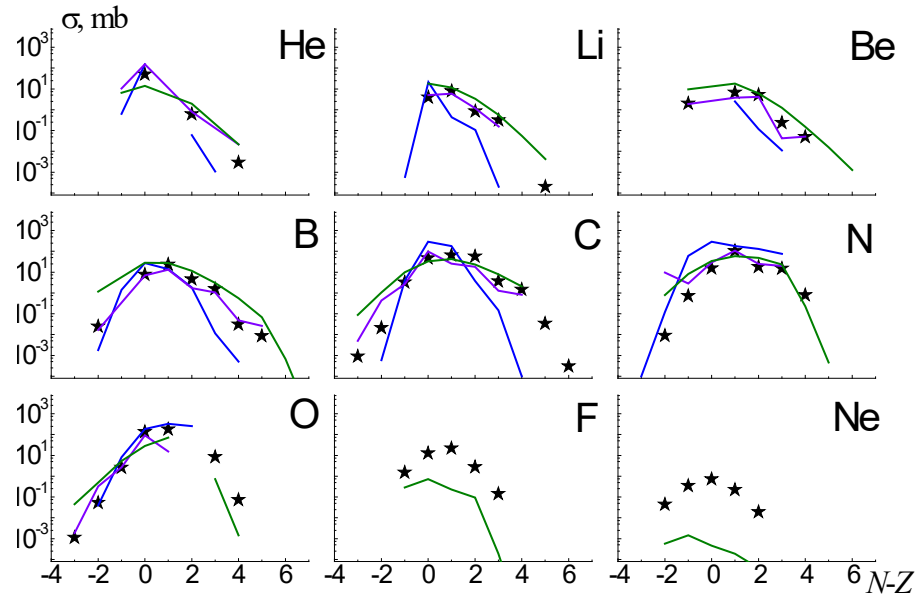
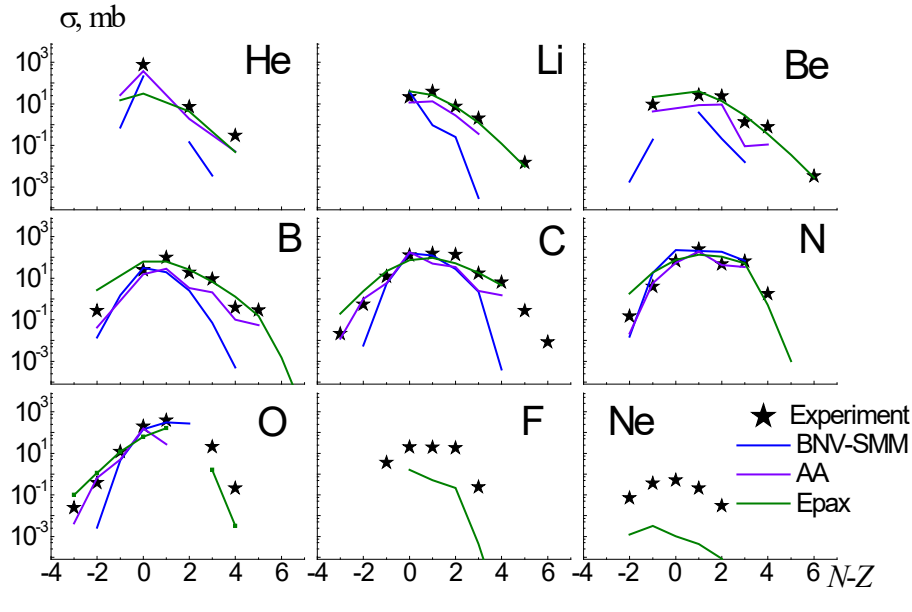
Experiment: measurement of projectile-like fragments emitted at forward angles with velocity close to the beam velocity.  
 Combas set-up, FLNR, JINR  
 Artukh, A.G, et al. Phys. Part. Nuclei Lett. 18, 19 (2021).

Calculations,  $1.25 E_{exc}$   
 from BNV calculations

# Calculations of isotope distributions with EPAX and Abrasion-Ablation models

$^{18}\text{O}(35 \text{ AMev})+^{181}\text{Ta}$

$^{18}\text{O}(35 \text{ AMev})+^9\text{Be}$



The best coincidence give EPAX calculations, BNV-SMM calculations give reasonable agreement for  $N-Z$  close to zero nuclides

**EPAX**-[K. Summerer and B. Blank, Phys. Rev. C. 61, 034607 (2000).]

**AA** - [Bowman J.D.// LBL Report. 1973. LBL-2908.]

## Ratio $\sigma(\text{Kr}+\text{Ta}) / \sigma(\text{Kr}+\text{Be})$ , 64 A MeV

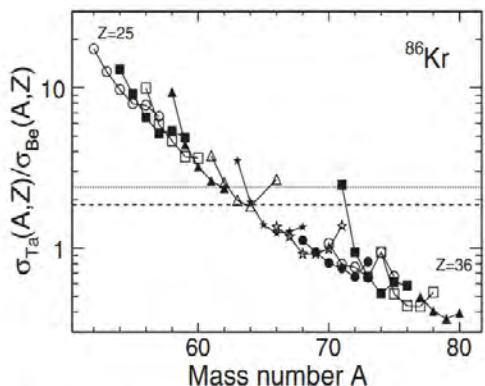
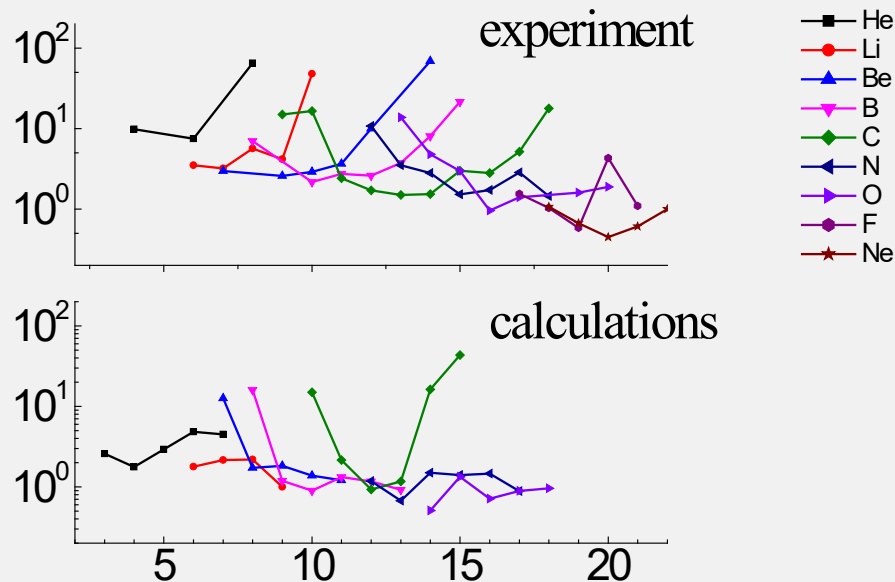


FIG. 10. Ratios of the fragmentation cross sections on Ta and Be targets,  $\sigma_{\text{Ta}}(A, Z)/\sigma_{\text{Be}}(A, Z)$ , for fragments with  $25 \leq Z \leq 36$  for the  $^{86}\text{Kr}$  beam. Only ratios with relative errors smaller than 25% are shown. Open and solid symbols represent odd and even elements starting with  $Z = 25$ . The horizontal dashed and dotted lines indicate the ratio calculated by the EPAX formula and Eq. (4), respectively.

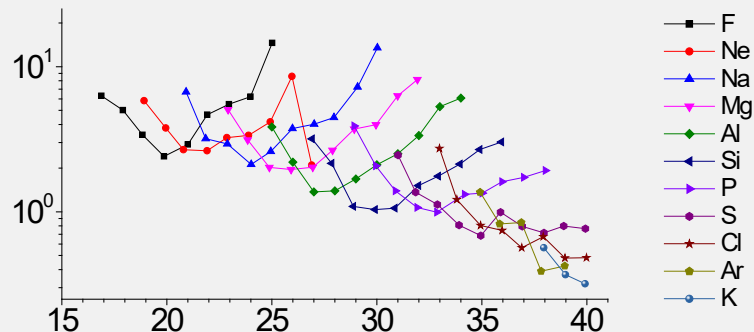
$$\frac{\sigma_{\text{Ta}}(A, Z)}{\sigma_{\text{Be}}(A, Z)} = \frac{(A_{\text{Kr}}^{1/3} + A_{\text{Ta}}^{1/3})^2}{(A_{\text{Kr}}^{1/3} + A_{\text{Be}}^{1/3})^2} = 2.4,$$

$$\frac{\sigma_{\text{Ta}}(A, Z)}{\sigma_{\text{Be}}(A, Z)} = \frac{(A_{\text{Kr}}^{1/3} + A_{\text{Ta}}^{1/3} - 2.38)}{(A_{\text{Kr}}^{1/3} + A_{\text{Be}}^{1/3} - 2.38)} = 1.9.$$

## Yield(Ar+Ta) / Yield(Ar+Be), 35 A MeV



## $\sigma(\text{Ar}+\text{Ta}) / \sigma(\text{Ar}+\text{Be})$ , 57 A MeV

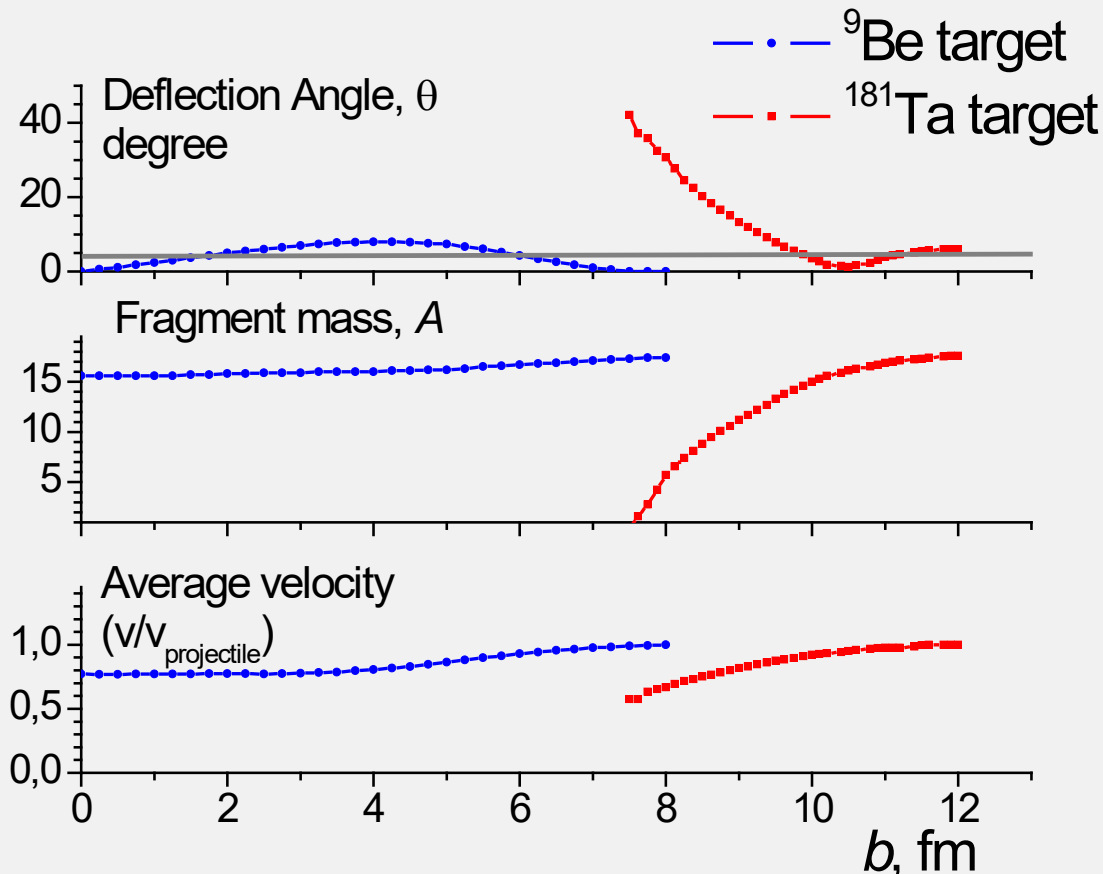


Data from

X. H. Zhang et.al Phys. Rev. C 85, 024621(2012)

# Explanation of the shape of the isotope cross-section ratio on heavy and light targets from the view of the BNV-SMM calculations

Results of BNV calculations for two reactions with projectile  $^{18}\text{O}$  at 35 MeV per nucleon on different targets



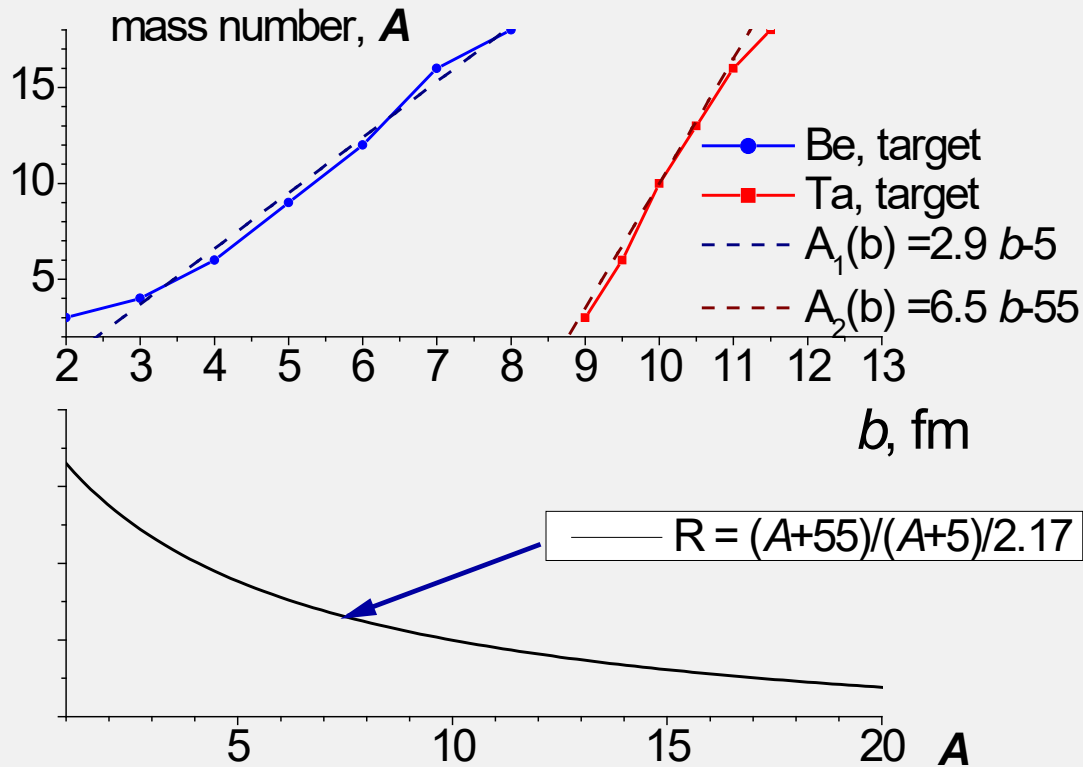
What can we learn from this figure:

1) Transport calculations describe only the dissipative contribution to the collision (see the velocity distributions)

2) The results of BNV calculations show that the lighter nucleus is more affected by the collision, the heavier one doesn't change much during the first stage of the reaction. But still it is excited (the velocity is decreasing as the collision becomes more central)!

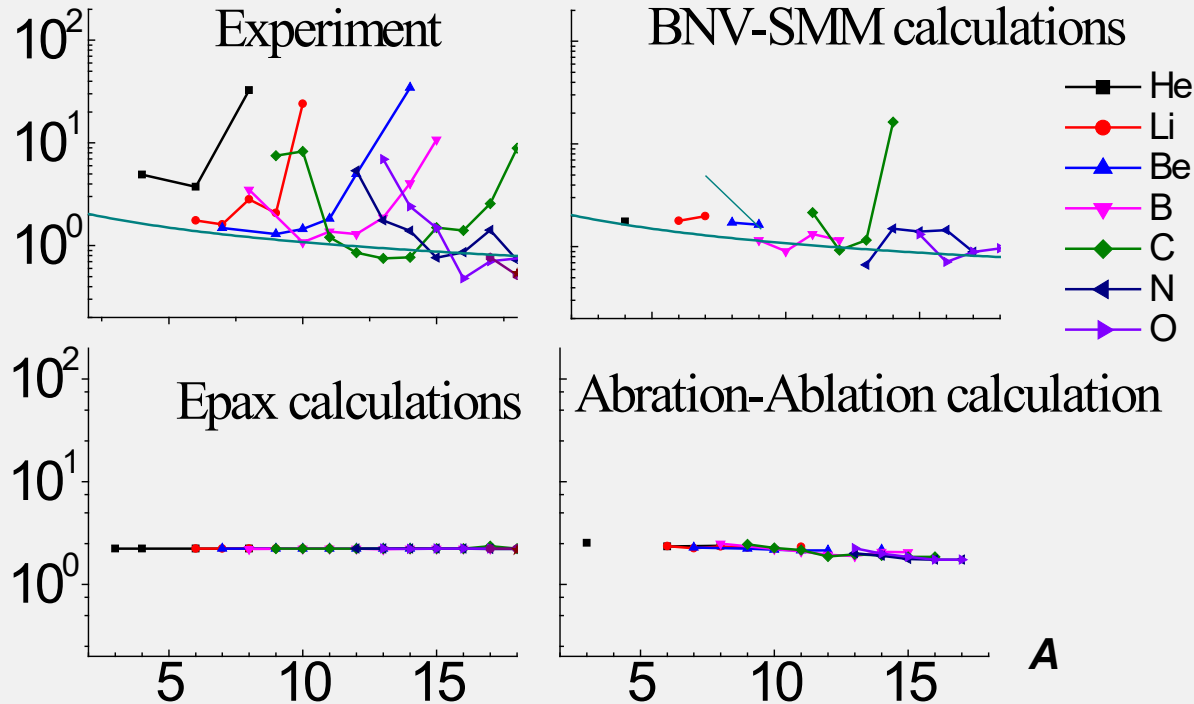
3) If the measurement is done in the restricted angular regions (forward angles), there is a big difference between direct and inverse reactions. In the case of the direct one only narrow set of impact parameters contribute to the reaction.

Results of SMM calculations for the reaction of  $^{18}\text{O}$  beam (35 A MeV),  
3 degrees acceptance angle for two targets  $^{181}\text{Ta}$  and  $^9\text{Be}$



- 1) The upper panel: dependence of the mass number of the final (cold) fragment on the value of initial impact parameter. One can see, that the range of impact parameters contributing to the final cross-section is much larger for light target  $^9\text{Be}$  than for heavy  $^{181}\text{Ta}$
- 2) Dotted lines - linear fits of the results
- 3)  $R = b_1(A)/b_2(A)$  – the ratio of the inverse functions of the fits from the upper panel

# Ratio Yield( $^{18}\text{O}+^{181}\text{Ta}$ ) / Yield( $^{18}\text{O}+^9\text{Be}$ ), 35 A MeV, experiment and three model simulations



The ratios shown in the figure point out on the importance of taking into account two crucial characteristics of the reaction:

- 1) target mass
- 2) impact parameter value

The excess of the neutron-rich nuclides in the reaction on heavy target can be explained by the fact that in the collision on heavy target even at the first stage of the reaction exotic combinations of  $N$  and  $Z$  can be obtained, while in the reactions on light target primary fragments are similar to the projectile

# Conclusions

Heavy-ion fragmentation reactions are interesting for physics and important for applications. A microscopic understanding is desirable.

In this work we use transport theory. The solution of the highly non-linear transport equation is mathematically challenging. To de-excite hot fragments produced in transport calculations we use SMM code. We compared the results with experimental data of COMBAS set-up (FLNR JINR ):  $^{18}\text{O}$  (35 A MeV) on targets  $^{181}\text{Ta}$  and on  $^9\text{Be}$

We also compared calculations in combined transport-statistical model with two well known and highly used models: EPAX and AA

We found that our calculations describe fragments close to  $N-Z = 0$  rather well, but for neutron rich isotopes our calculations give smaller values than the experiment

It is shown that our self-consistent calculations predicts smaller values of excitation energy that is necessary to describe the data, this may be connected with not self-consistent description of initial state.

The decrease of the ratio of cross-sections of fragments on heavy and light nuclei with increasing mass number of fragments  $A$  can be the manifestation of different dependence of deflection angle of primary fragments on impact parameter  $b$

The excess of neutron-rich nuclides in the reactions on heavy target may be explained by the fact that in collision on heavy target the exotic fragments could be produced at first stage of the reaction (before evaporation).



**Thank you  
for  
attention**

