

# Size, Shape and Deformation of Nuclei

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# Content

- Motivation
- The model: Strongly Correlated Quark Model (SCQM)
  - Nucleon Structure
  - Nuclear Structure: SCQM+FCC
- Nuclear Properties and SCQM
  - Size
  - Shape
  - Deformation
- Summary

# Motivation

## Nuclear Size and Shape

### Experimental Observations

- Compactness of  $^4\text{He}$  and a hole inside it
- Halo nuclei: the radius of halo nucleus appreciably larger than that predicted by the liquid drop model
- Neutron skin
- Fluctuation of the central nuclear matter density distribution

# Motivation

## Nuclear Size and Shape

### Experimental Observations

- Compactness of and a hole inside  ${}^4\text{He}$

Point-nucleon charge distributions of  ${}^3\text{He}$  and  ${}^4\text{He}$   
Hole inside  ${}^3\text{He}$  and  ${}^4\text{He}$

*I. Sick, PRC, vol. 15, No.4; LNP, vol. 87, p.236*

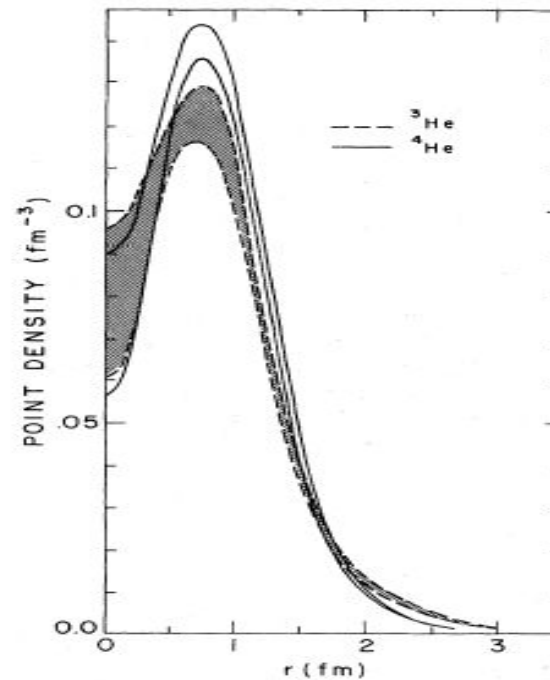


FIG. 15. Model-independent densities of pointlike protons in  ${}^3,{}^4\text{He}$ .

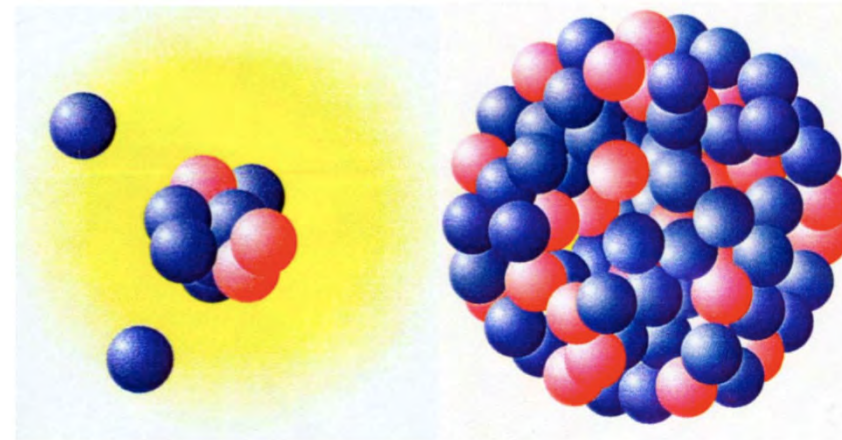
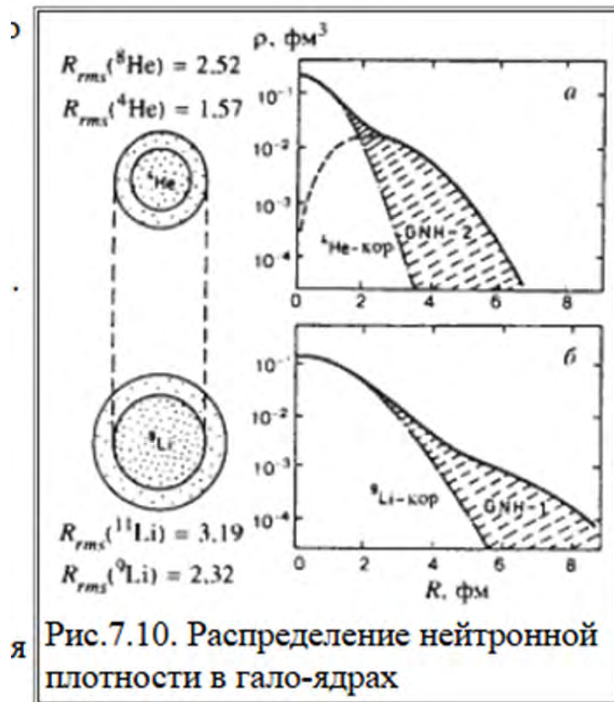
# Motivation

## Nuclear Size and Shape

### Experimental Observations

- Halo nuclei: the radius of halo nucleus  $R_{halo}$  appreciably larger than that predicted by the liquid drop model
- Halo nuclei:  ${}^6\text{He}$ ,  ${}^8\text{He}$ ,  ${}^{11}\text{Li}$ , ...

$$R_{halo} \gg 1.3 A^{1/3}$$



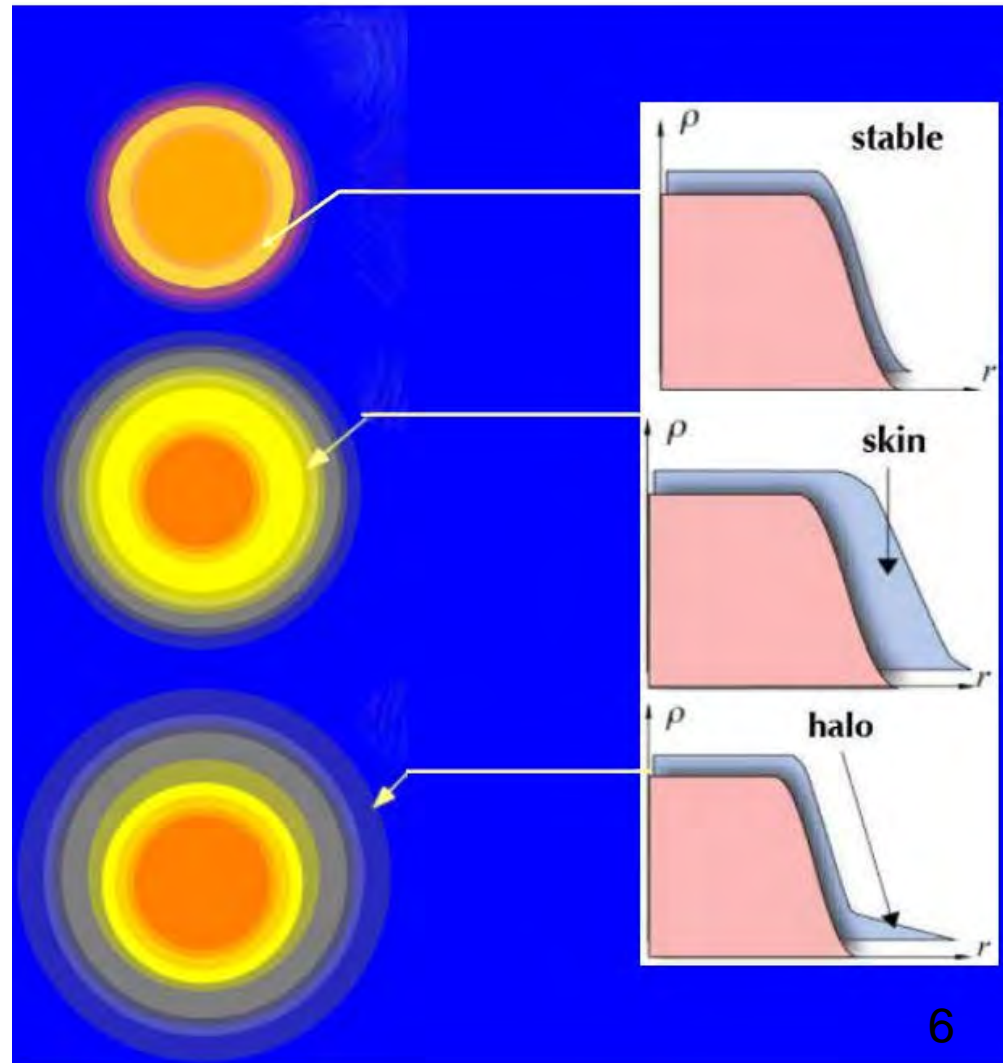
${}^{11}\text{Li}$

${}^{208}\text{Pb}$

# Motivation

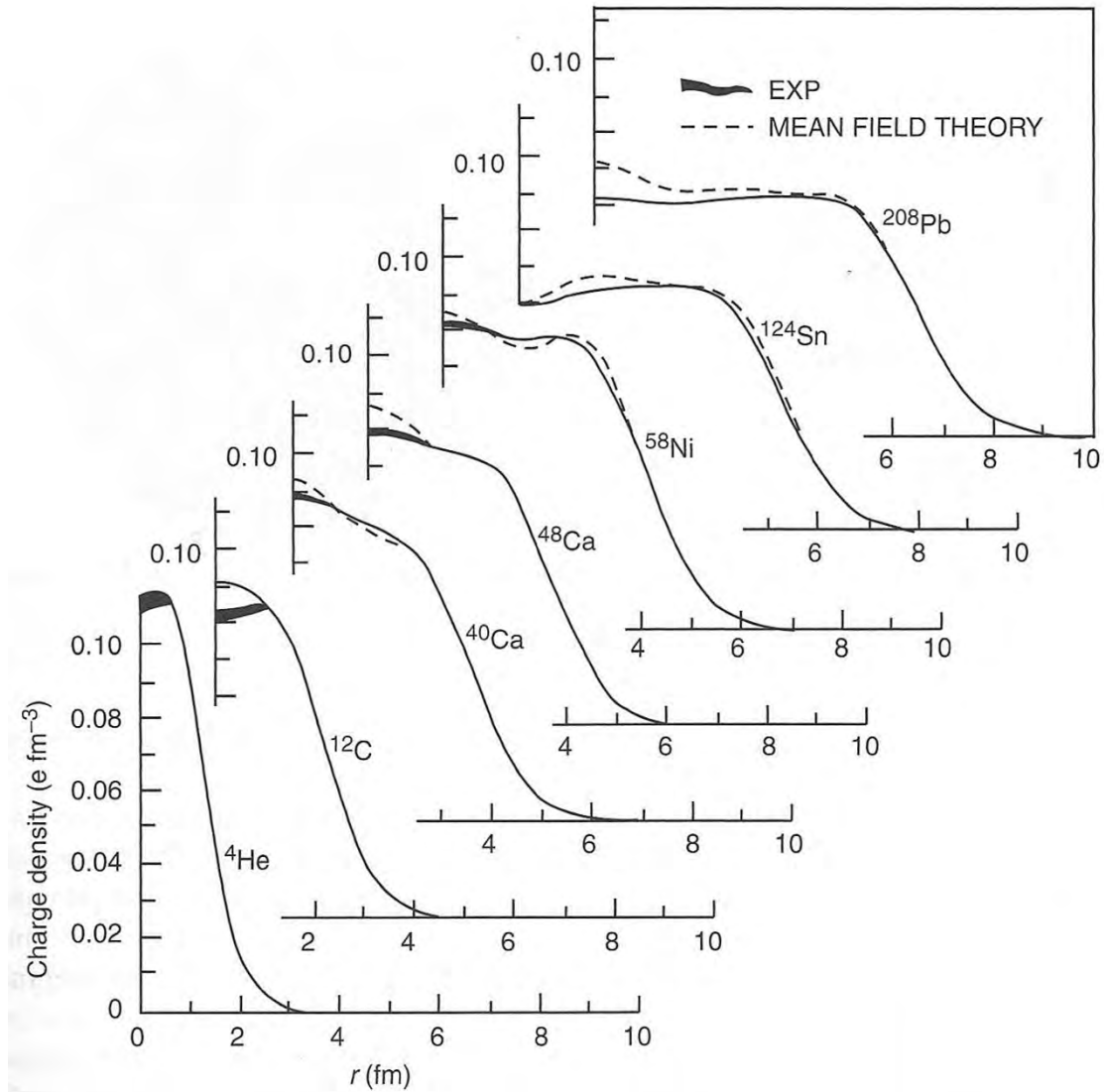
## Nuclear Size and Shape

### Neutron skin



# Motivation

## Fluctuation of central nuclear density



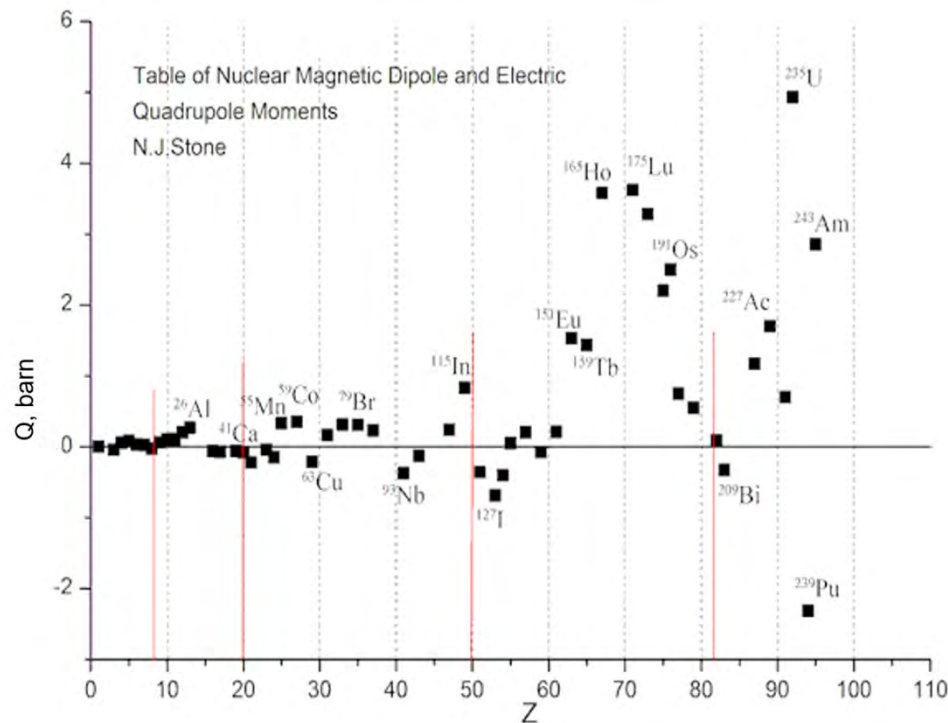
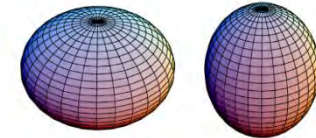
# Motivation

## Nuclear Deformation

### Experiment

All nuclei are deformed

- The simplest deformation: **electric** quadrupole deformation



- Nuclear deformation is much more complicated: **multipole** deformations
- Nothing is known about deformation of the **neutron matter**



# Diversity of models of Nuclear Structure

**Nuclear Models** in terms of nucleons and mesons

- Conventional models
  - Independent Particle Models (Shell Model, ...)
  - Collective models (Liquid Drop Model, ...)
  - Cluster models
  - Modifications of above models
- Non-conventional model
- There are more than 40 models ... (*W.Greiner et al.*)

**Effective Field Theories, EFT**

QCD  $\rightarrow$  CSB: quark, gluon fields  $\rightarrow$  meson fields

# Diversity of models of Nuclear Structure

Is it possible to build a model composing the features of all conventional models?

Do quarks manifest themselves explicitly in nuclear structure?

**Yes**  
**It is possible!**

Conventional models

- Shell Model.
- Liquid Drop Model
- Cluster models

**FCC** –Face-Centered Cubic Model  
based on  
**SCQM**

# SCQM – Strongly Correlated Quark Model of Nucleon Structure

*G. Musulmanbekov, “Quarks as Vortices in Vacuum”  
in book *Frontiers of Fundamental Physics*,  
Kluwer Acad./Plenum Pub., 2001, p. 109-120.*

*G. Musulmanbekov, “Hadron Modifications in a Dense Baryonic Matter”  
PEPAN Lett., Vol., № 5, p. 548-558*

# QCD – fundamental theory of strong interactions

- **Constituents of hadrons – quarks** of different flavors carrying spin, charge, color.
  - **flavors:** **u, d, s, c, b, t**
  - **spin:**  $1/2$
  - **charge:**  $1/3, 2/3$
  - **color:**  $SU(3)_{\text{Color}} - R, G, B, \bar{R}, \bar{G}, \bar{B}$
- **Fields – gluons** – perform interactions between quarks.
- **Nucleons** – 3–quark (**u/d**), color-singlet systems
- **Mesons** – quark-antiquark systems

# QCD (cont.)

QCD is non-abelian theory

## Hadronic processes with high $Q^2$

pQCD:  $\alpha_s < 1$ ,  $m_q \rightarrow 0$ , chiral symmetry

## Low energy hadron and nuclear physics

non-pQCD:  $\alpha_s > 1$ ,  $m_q \neq 0$ , chiral symmetry breaking

- Low energy approx. of QCD, effective theories., ...
- QCD–inspired phenomenology
  - NR constituent quark models
  - Bag models
  - Chiral quark models
  - Soliton models

# pQCD → Low energy physics

## Hard processes

- $m_q = 0$

- Chiral Symmetry

$SU(2)_L \times SU(2)_R$  for  $\psi_{L,R} = u, d$  – current quarks

## Low energy, hadron properties

**Chiral symmetry breaking**  $\equiv$  quark or *chiral* condensate:

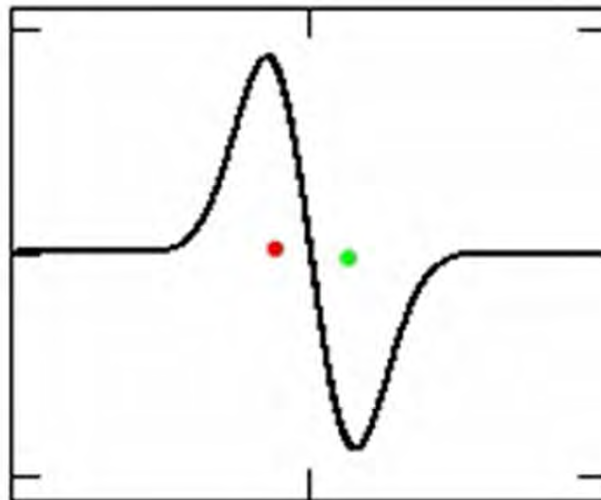
$$\langle \bar{\psi} \psi \rangle \simeq - (250 \text{ MeV})^3, \quad \psi = u, d$$

As a consequence massless valence quarks (u, d) acquire dynamical masses which we call **constituent quarks**

$$M_C \approx 350 - 400 \text{ MeV}$$

# quark – antiquark pair

$\varphi(x,t)$





# Quarks – Solitons

SCQM  $\equiv$  Breather Solution of Sine-Gordon equation

$$\partial_{\mu} \partial^{\mu} \phi(x, t) + \sin \phi(x, t) = 0$$

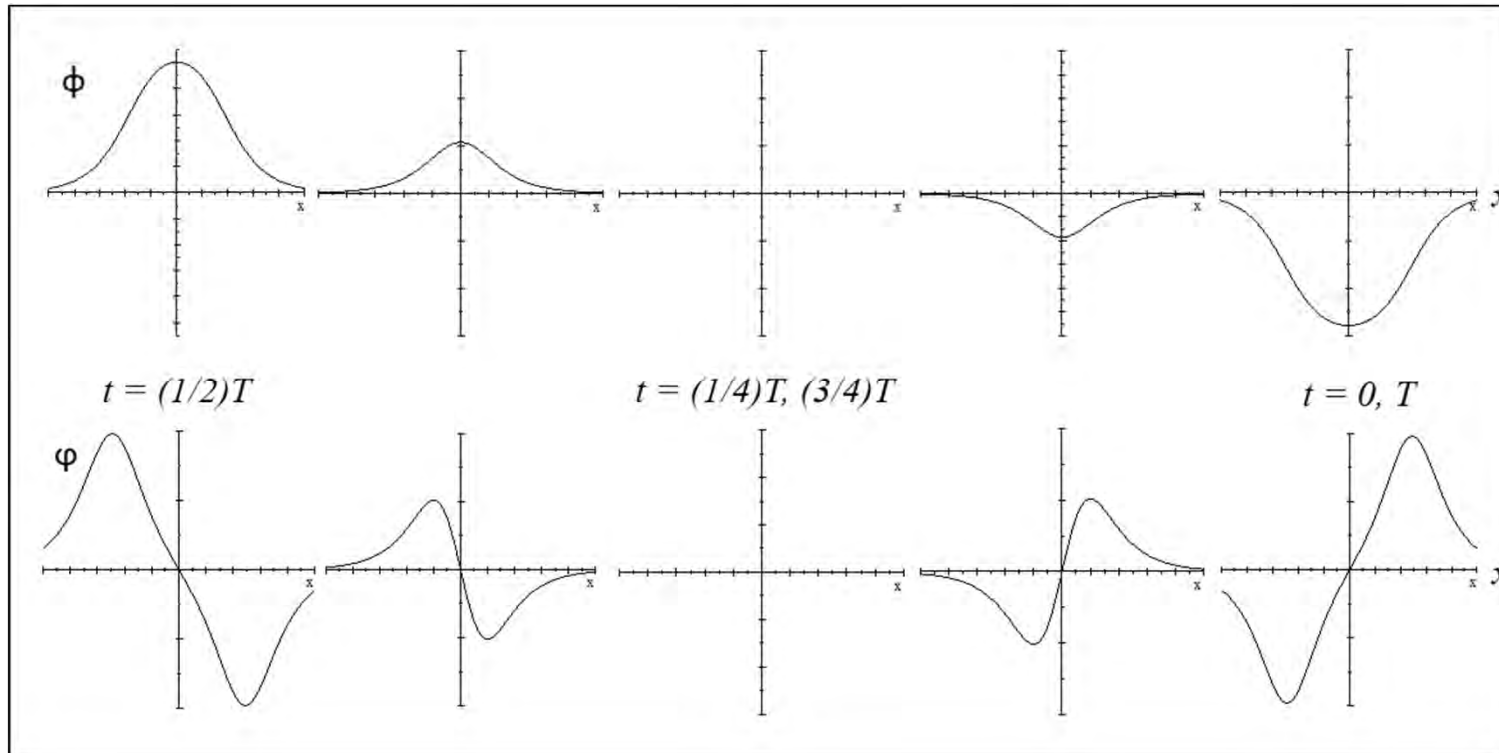
Breather – oscillating soliton-antisoliton pair, the periodic solution of SG:

$$\phi(x, t)_{s-as} = 4 \tan^{-1} \left[ \frac{\sinh\left(ut / \sqrt{1-u^2}\right)}{u \cosh\left(x / \sqrt{1-u^2}\right)} \right]$$

$$\varphi(x, t)_{s-as} = \frac{\partial \phi(x, t)_{s-as}}{\partial x}$$

is **identical** to our quark-antiquark system.

# Breather, $\phi(x, t)$ non-linear “Standing wave”



# The SCQM

Hamiltonian of the quark – antiquark system

$$H = \frac{m_q^-}{(1 - \beta_q^{-2})^{1/2}} + \frac{m_q}{(1 - \beta_q^2)^{1/2}} + V_{qq}^-(2x)$$

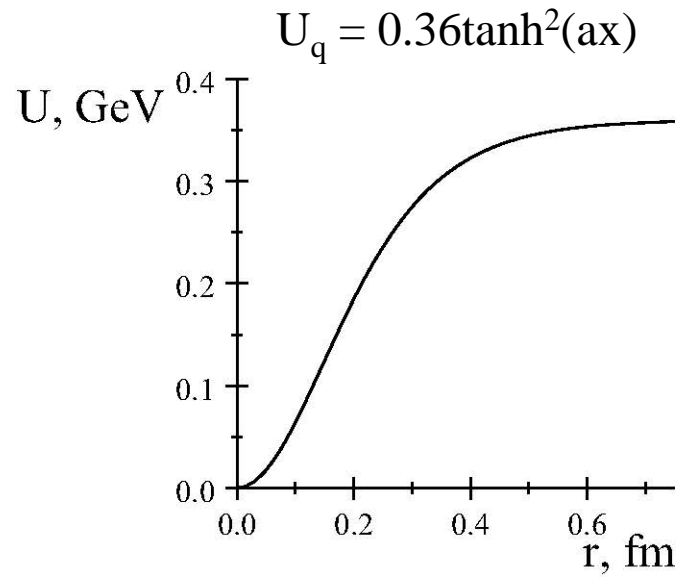
$m_q^-, m_q$  - current masses of quarks,  
 $\beta = \beta(\mathbf{x})$  - velocity of the quark (antiquark),  
 $V_{qq}^-$  - quark–antiquark potential.

$$H = \left[ \frac{m_q^-}{(1 - \beta_q^{-2})^{1/2}} + U(x) \right] + \left[ \frac{m_q}{(1 - \beta_q^2)^{1/2}} + U(x) \right] = H_q^- + H_q$$

$U(x) = \frac{1}{2} V_{qq}^-(2x)$  is the potential energy of a single quark/antiquark.

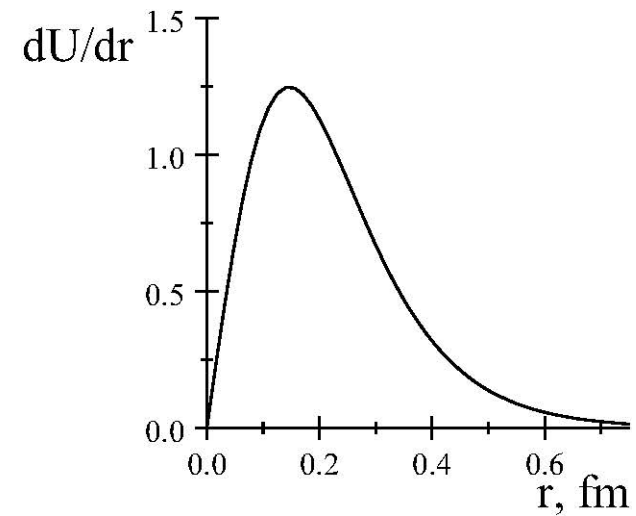
$$U(x) = \frac{1}{2} V_{qq}^-(2x) = m \tanh^2(ax)$$

## Quark Potential

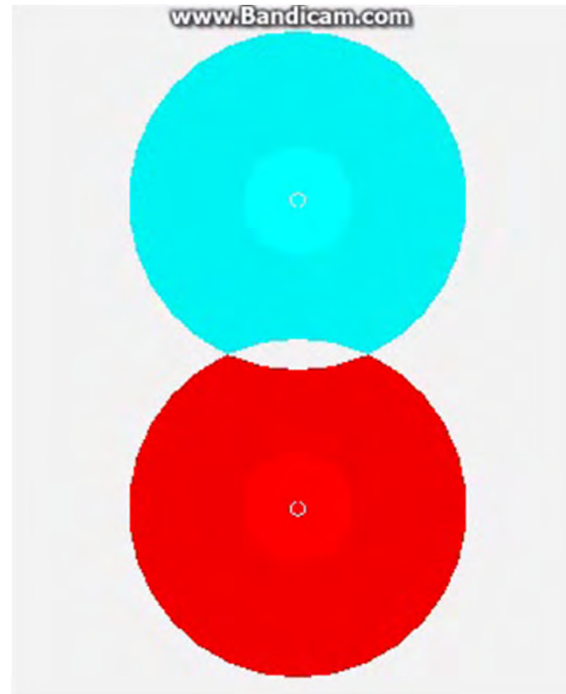


## Force

of quark-antiquark interaction



# quark–antiquark pair meson



**QCD:** Exchange by gluons  $\sqrt{\frac{1}{2}}(R\bar{R} + B\bar{B})$

**SCQM:** Overlap of color fields

# Generalization to the 3 – quark system (baryons)

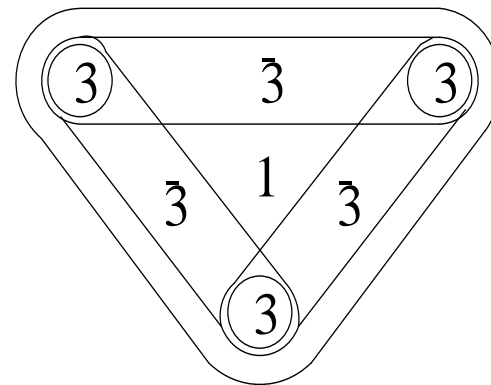
$SU(3)_{Color}$

$$q \Rightarrow SU(3) \Leftrightarrow RGB \quad q \Rightarrow SU(3) \Leftrightarrow CMY$$

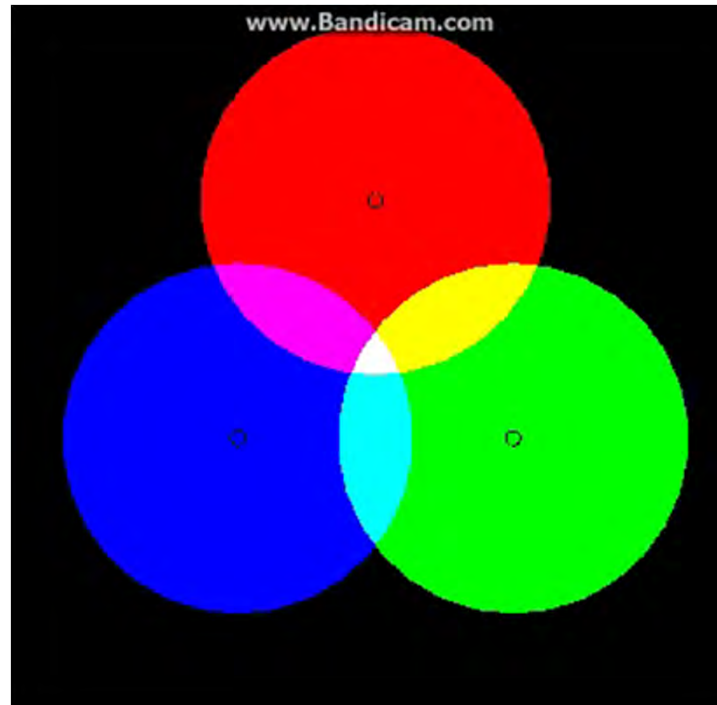
$$\bar{q}q \Rightarrow \left( \begin{array}{ccc} \bar{3} & 1 & 3 \end{array} \right)$$

$$qq \rightarrow 3 \times 3 = 6 \oplus \bar{3} \quad \Rightarrow \quad \bar{q} \rightarrow qq$$

$$qqq \Rightarrow$$



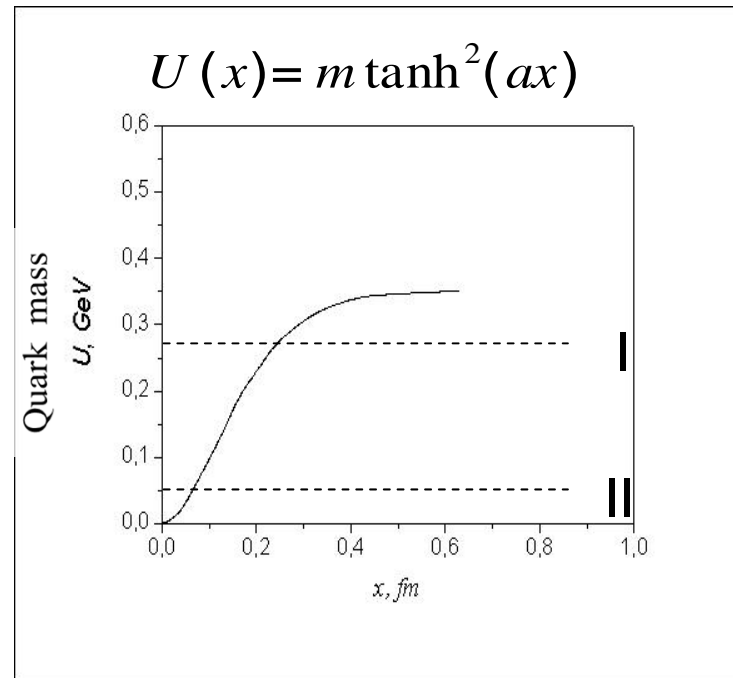
# Nucleon as 3 oscillating color quarks



“The wave packet solution of time-dependent Schrodinger equation for harmonic oscillator moves in exactly the same way as corresponding classical oscillator”

*E. Schrodinger, 1926*

# Dynamic Breaking-Restoration of Chiral Symmetry



$U(x) > I$  – constituent quarks

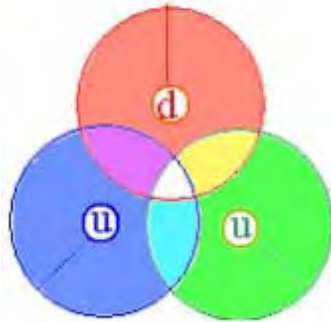
$U(x) < II$  – current (relativistic) quarks



# Interplay between constituent and current quark states

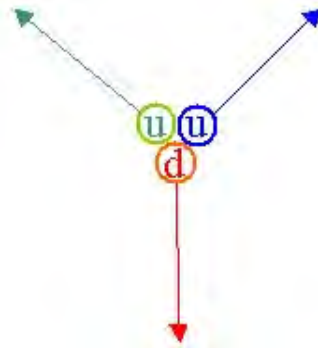
Chiral Symmetry Breaking  $\longleftrightarrow$  Restoration

$t = 0$   
 $x = x_{max}$



Constituent quaks

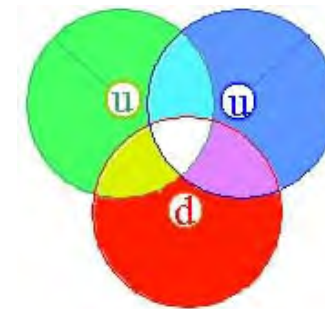
$t = T/4$   
 $x = 0$



current quarks

Asymptotic freedom

$t = T/2$   
 $x = -x_{max}$

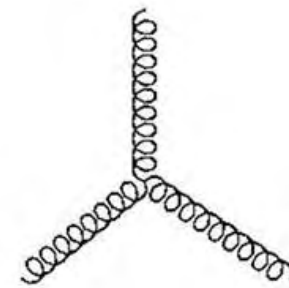
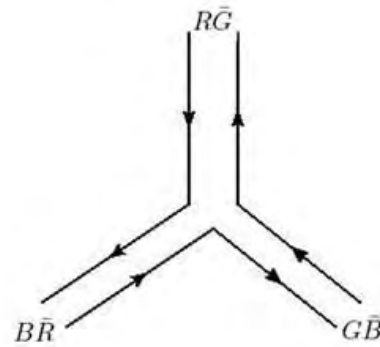
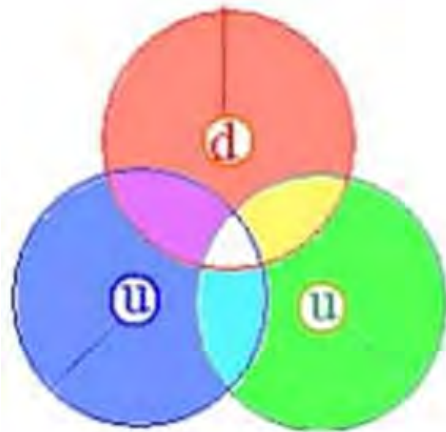


Constituent quarks

During the valence quarks oscillations:

$$|B\rangle = a_1 |q_1 q_2 q_3\rangle + a_2 |q_1 q_2 q_3 \bar{q} q\rangle + a_3 |q_1 q_2 q_3 q \bar{q}\rangle + \dots$$

# SCQM vs QCD



# Parameters of SCQM for the Nucleon

1. Mass of Constituent Quark

$$M_{Q(\bar{Q})}(x_{\max}) = \frac{1}{3} \left( \frac{m_{\Delta} + m_N}{2} \right) \approx 360 \text{ MeV},$$

2. Amplitude of VQs oscillations :  $x_{\max} = 0.64 \text{ fm}$ ,

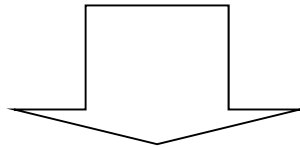
3. Constituent quark dimensions (parameters of gaussian distribution):  $\sigma_{x,y} = 0.24 \text{ fm}$ ,  $\sigma_z = 0.12 \text{ fm}$

Parameters 2 and 3 are derived from comparison of **Inelastic Overlap Function (IOF)** and  $\sigma_{tot}$  in  $p p$  and  $pp$  – collisions.

**Nucleons are nonspherical, triangular shaped!**  
**They are three-colored objects!**

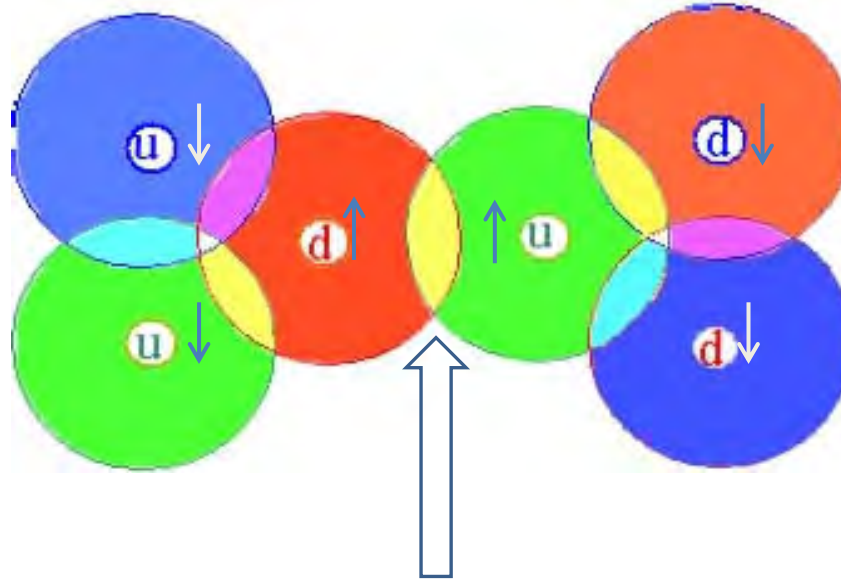
# Quark Arrangements inside Nuclei

Strongly Correlated Quark Model

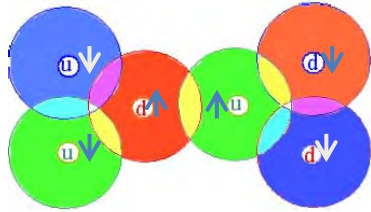


Lattice-like arrangement of Nuclear  
Structure

## Two Nucleon System in SCQM



Interaction between nucleons is due to **overlap** of their quark color fields



# Antisymmetrization

We need to define isospins, spins and colors at junctions

**${}^4\text{He}$ : 4 nucleons = 12 quarks in s-state**

## Antisymmetrization

$$\text{SU}(12) \longrightarrow \text{SU}(2)_{\text{isospin}} \otimes \text{SU}(2)_{\text{spin}} \otimes \text{SU}(3)_{\text{color}}$$

But  $\sim 90\%$  of 3-quark clusters are colored states (*Matveev, Sorba, 1978*)

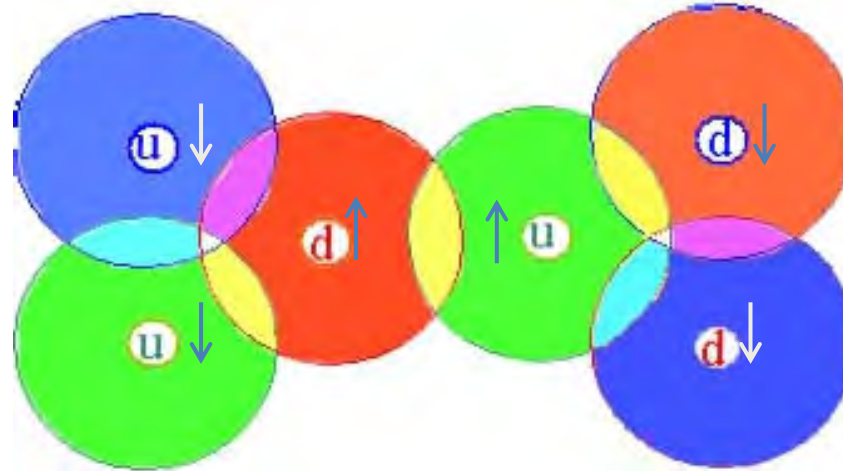
We select colorless 3-quark clusters by combinatorics imposing the following requirements to isospins, spins and colors at junctions:

$\text{SU}(2)_{\text{isospin}}$  – of different flavors (assumed)

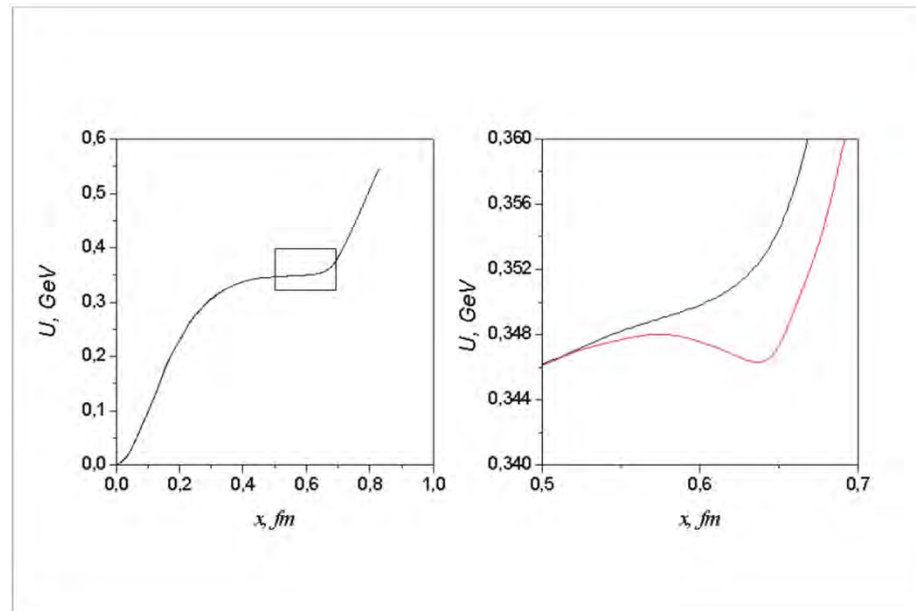
$\text{SU}(2)_{\text{spin}}$  – of parallel spins (calculated)

$\text{SU}(3)_{\text{color}}$  – of different colors (assumed)

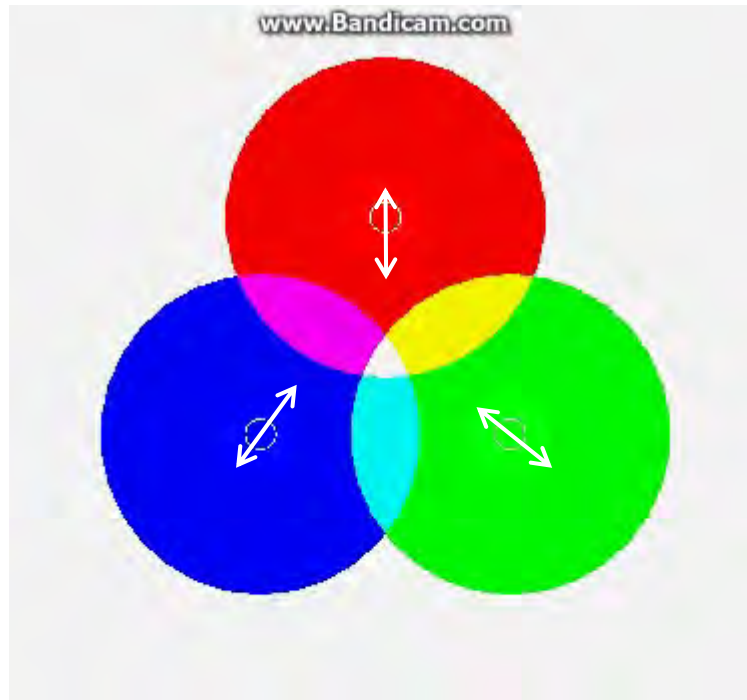
# Two Nucleon System in SCQM



## Quark Potential Inside Nuclei



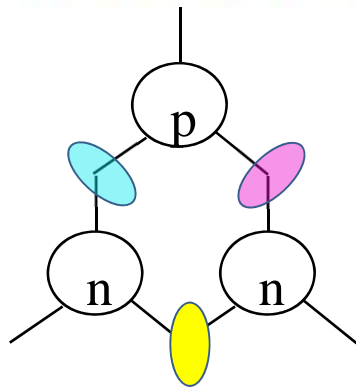
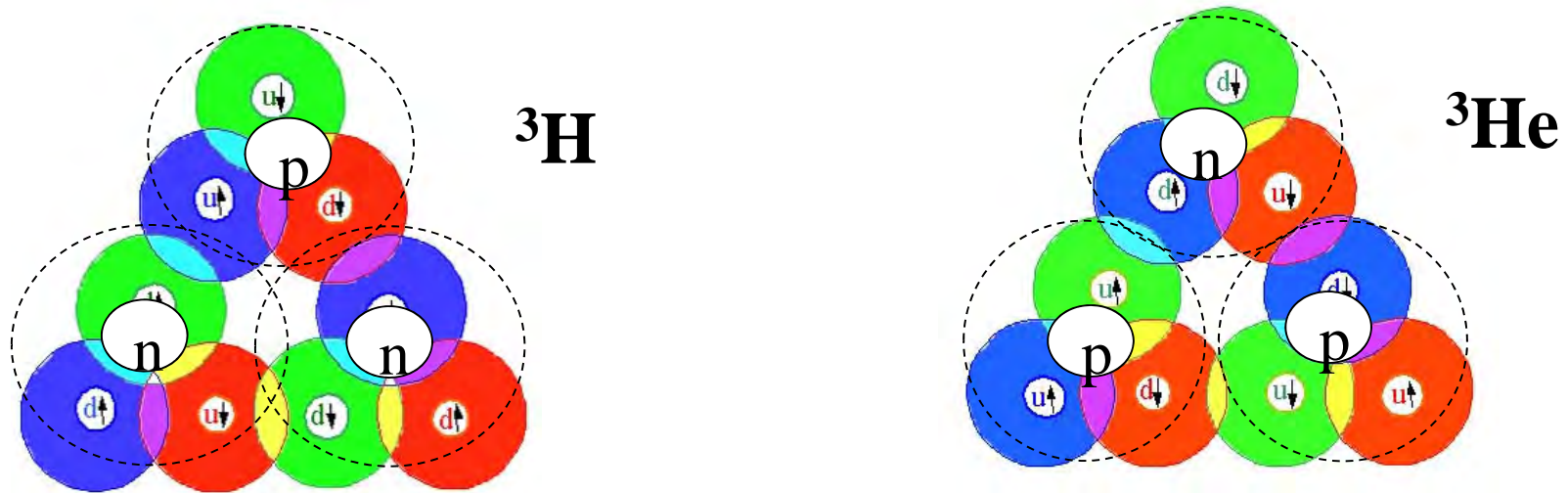
# Quarks inside nucleus



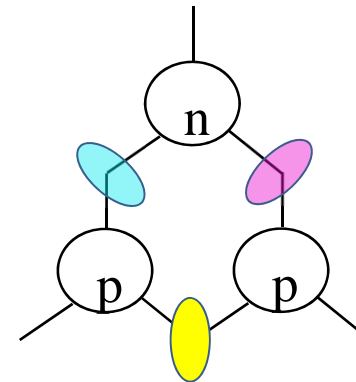
Quarks oscillate with small amplitudes  
near maximal displacements



# Three Nucleon Systems in SCQM



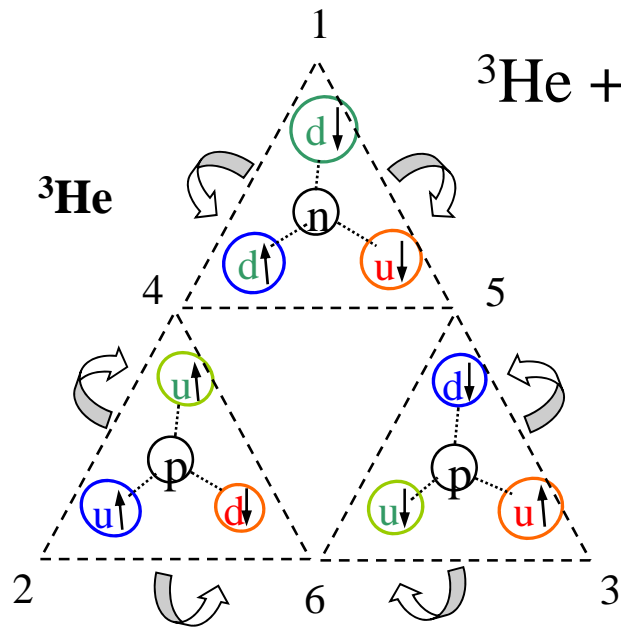
Summary color  
of 3 junctions is white,  
total color charge = zero!



$$qq \rightarrow 3 \times 3 = 6 \oplus \bar{3} \quad \Rightarrow \quad \bar{q} \rightarrow qq$$

Quark loop formed by 3 nucleons  $\rightarrow$  3-body force

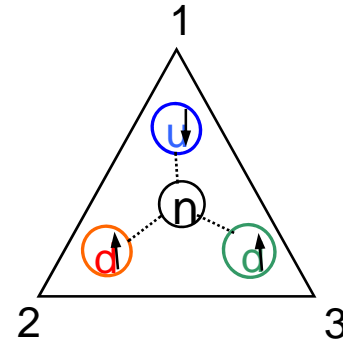
# 4-nucleon system: ${}^4\text{He}$



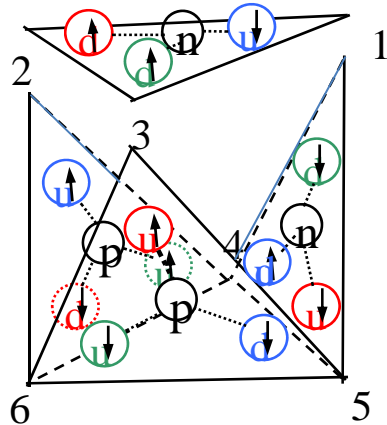
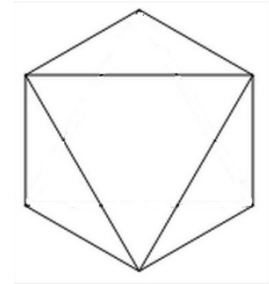
${}^3\text{He} + \text{neutron}$  or  ${}^3\text{H} + \text{proton}$

Junctures

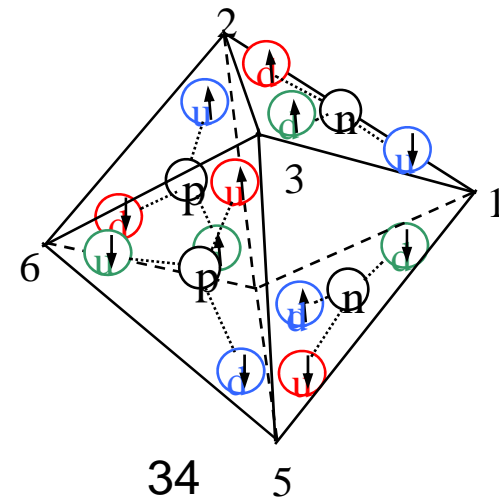
- 1  $\leftrightarrow$  1
- 2  $\leftrightarrow$  2
- 3  $\leftrightarrow$  3



Shell Closure



$\Rightarrow$  4 quark loops



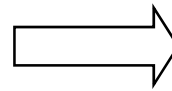
34

5

34

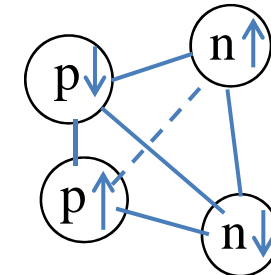
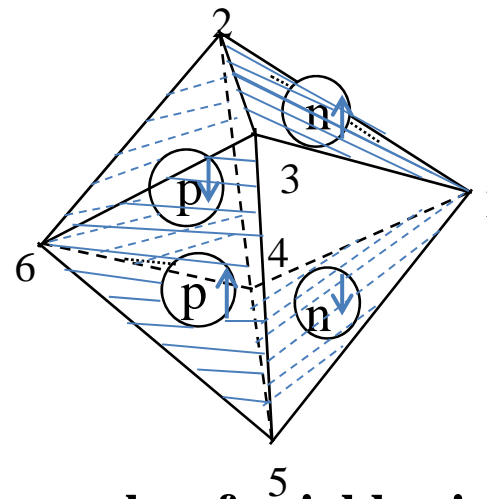
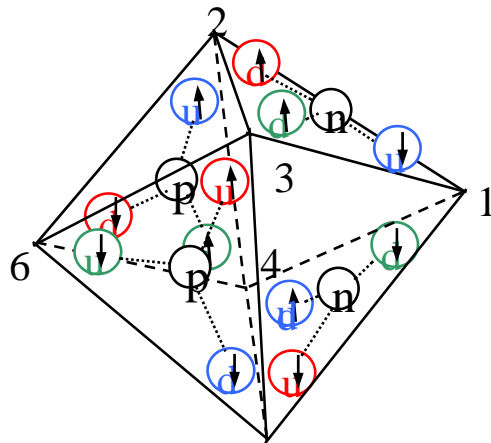
# The closed shell $n = 0$ , nucleus ${}^4\text{He}$

Antisymmetrisation of  
12 quarks in  $SU(12)$  state  
 $SU(2)_I \times SU(2)_S \times SU(3)_C$



Totally antisymmetrized  
4 nucleons in  $s$ -state

Shell Closure



**Selection<sup>5</sup> rules for binding two quarks of neighboring nucleons at a junction:**

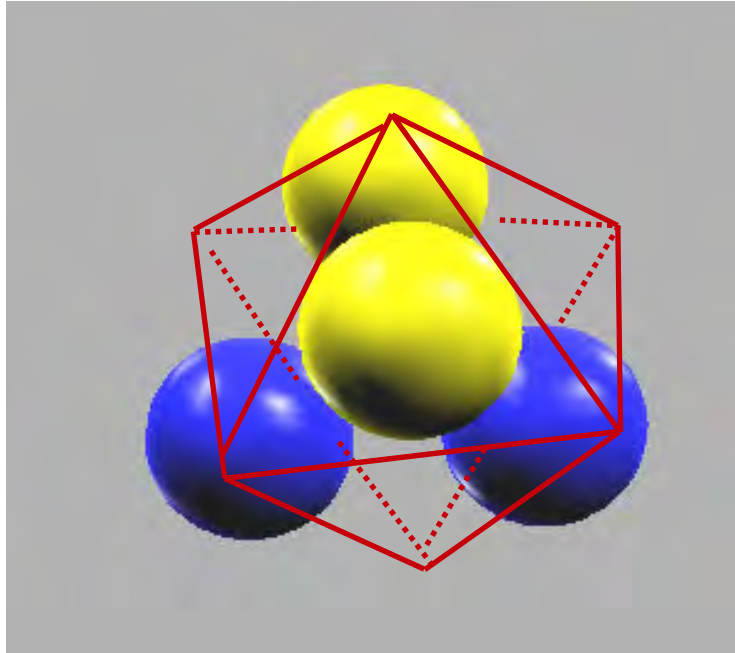
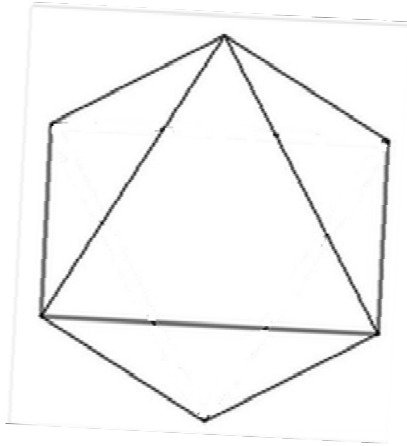
- $SU(2)_{\text{Isospin}}$  – of different flavors
- $SU(3)_{\text{Color}}$  – of different colors
- $SU(2)_{\text{Spin}}$  – of parallel spins

# Experimental Binding Energy of Stable Nuclei and Quark Loops in SCQM

Nucleus	$E_B$ MeV/junct. Exp.	Number of quark loops	Free quark ends	Nuclear forces
d	2.05	<b>0</b>	4	2-body
$^3\text{H}$	2.83	<b>1</b>	3	3-body
$^3\text{He}$	2.57	<b>1</b>	3	3-body
$^4\text{He}$	7.07	<b>4</b>	0	4-body

The more quark loops, the stronger the binding energy!

## The closed shell $n = 0$ , nucleus ${}^4\text{He}$

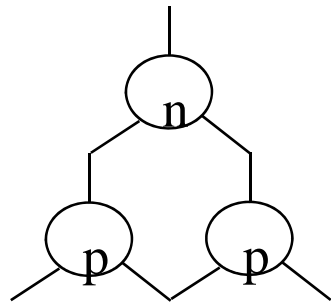
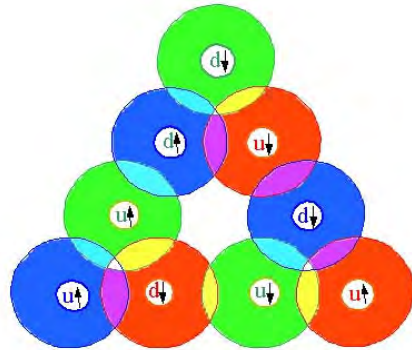


**Yellow** – protons are on opposite faces of upper pyramid

**Blue** – neutrons are on another faces of below lower pyramid

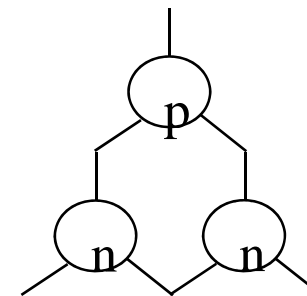
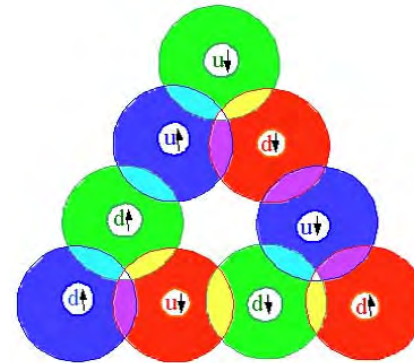
# Building blocks in Shell Structure

${}^3\text{He}$



${}^3\text{He}$  – block

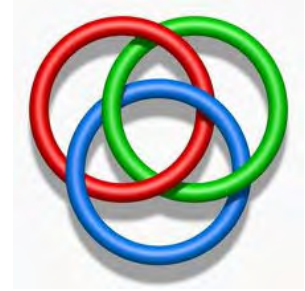
${}^3\text{H}$



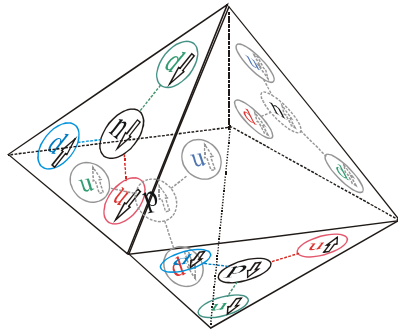
${}^3\text{H}$  – block

Forms Neutron Halo

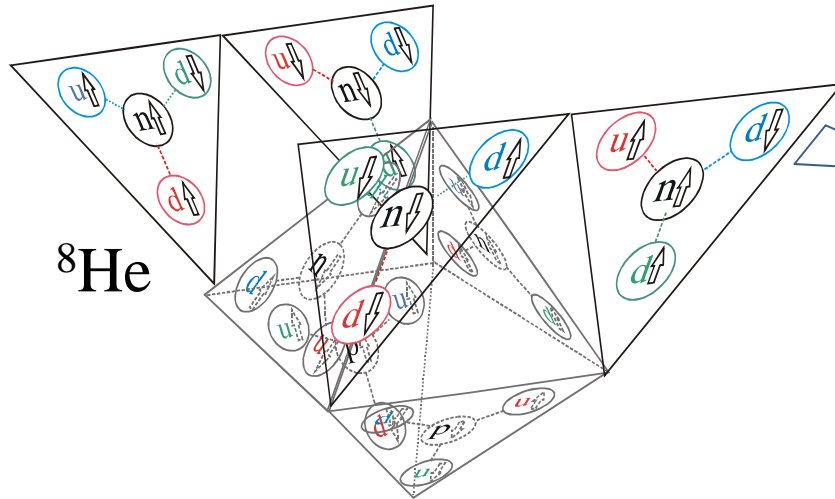
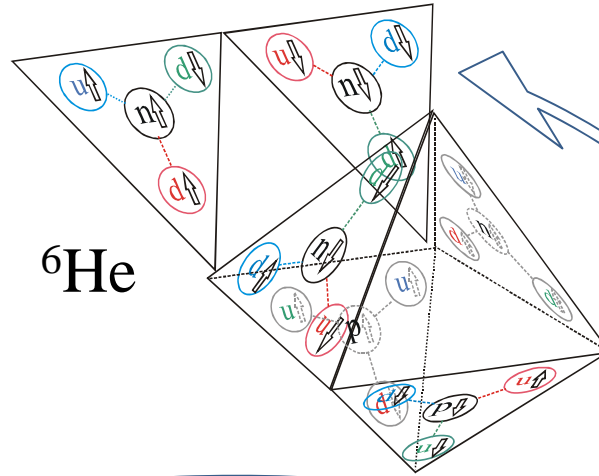
# Helium Isotopes Borromean Nuclei



${}^4\text{He}$   
Core

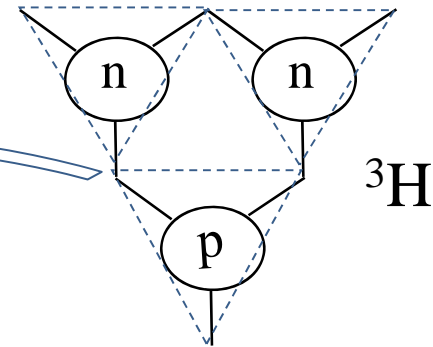


${}^6\text{He}$



${}^8\text{He}$

Quark loop



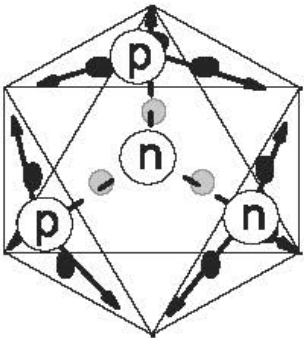
${}^3\text{H}$

# Helium Isotopes

## Borromean Nuclei

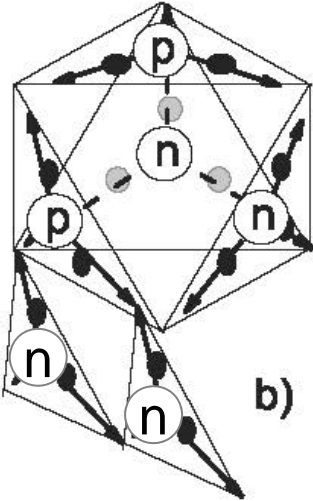


${}^4\text{He}$   
Core



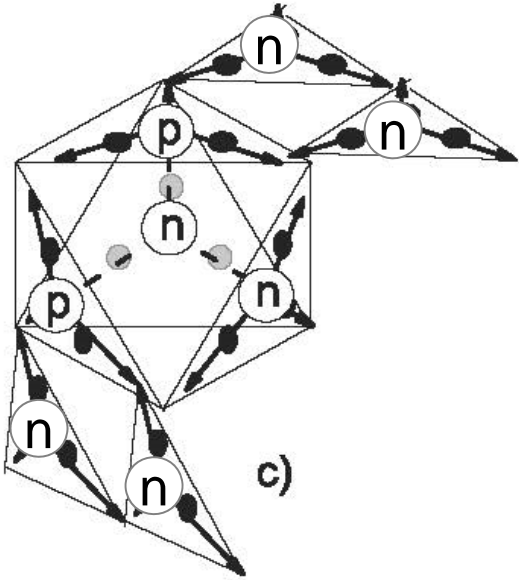
a)

${}^6\text{He}$



b)

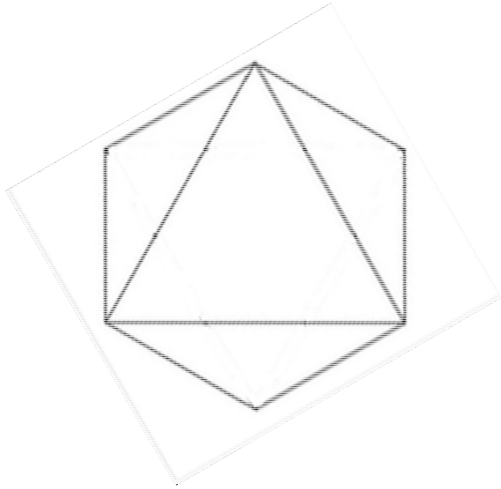
${}^8\text{He}$



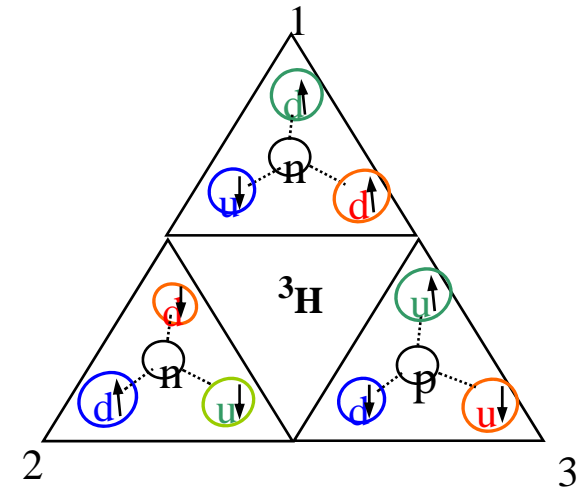
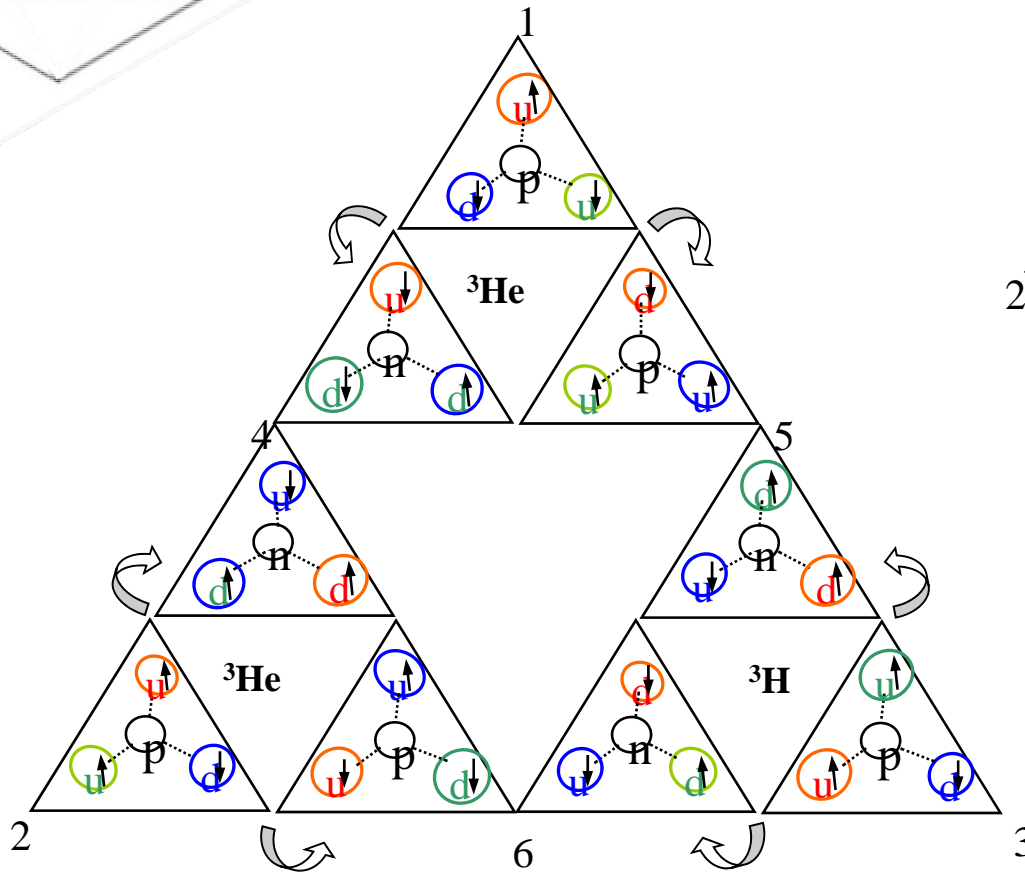
c)



# Next closed shell $n = 1, {}^{16}\text{O}$



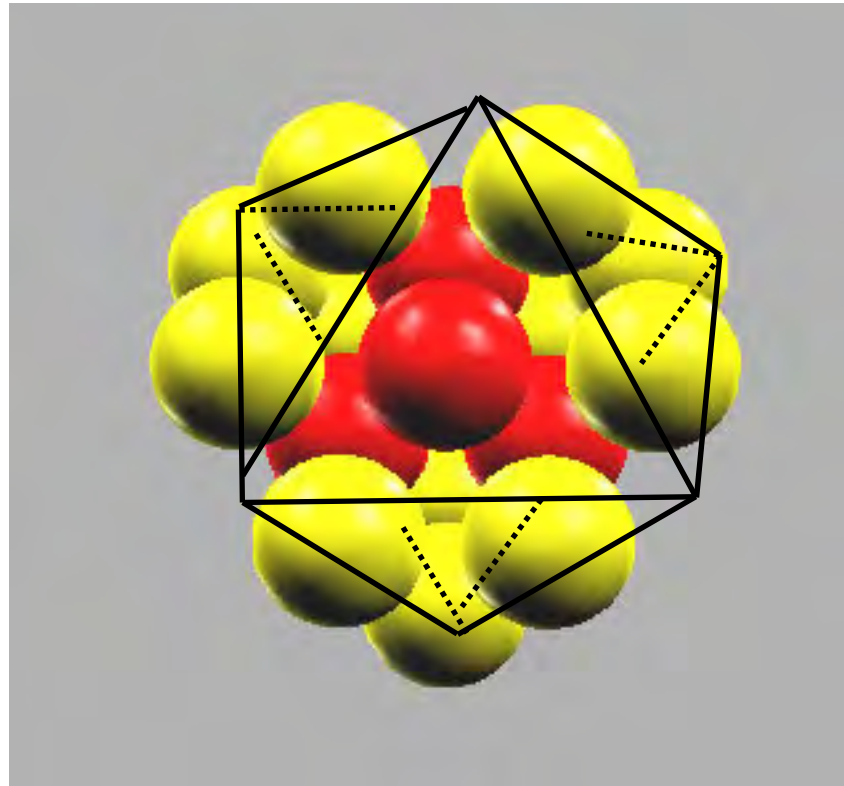
Face of  ${}^{16}\text{O}$  octahedron



In analogy with  ${}^4\text{He}$

${}^3\text{He}$  and  ${}^3\text{H}$  as  
proton and neutron  
in  ${}^4\text{He}$

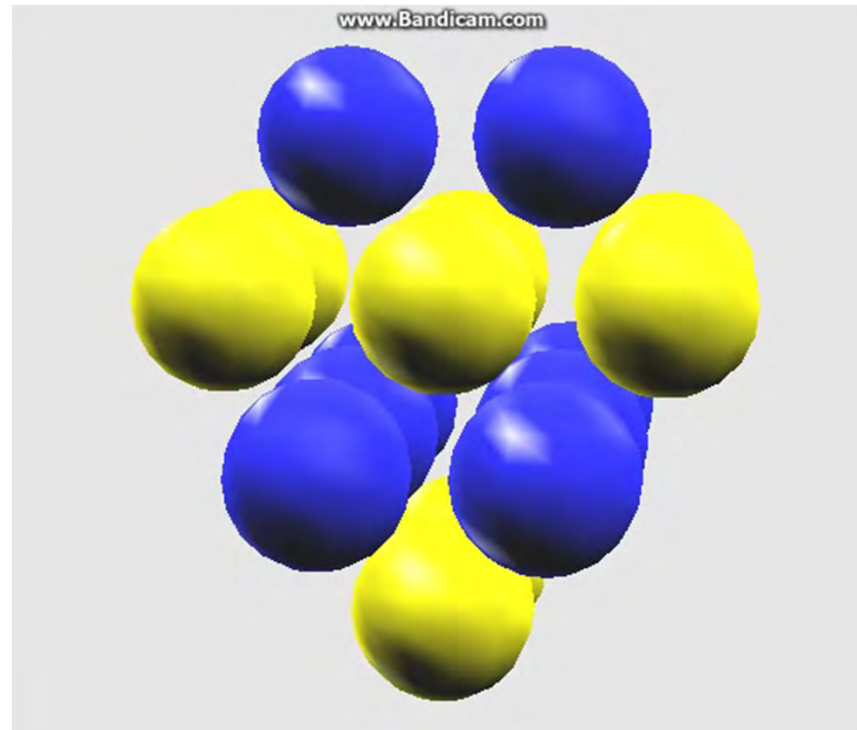
$^{16}\text{O}$



**RED** – s-shell

**YELLOW** – p-shell

$^{16}\text{O}$

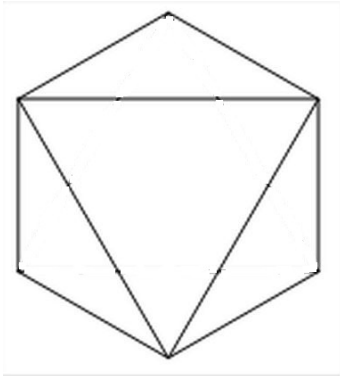


**Yellow** – protons

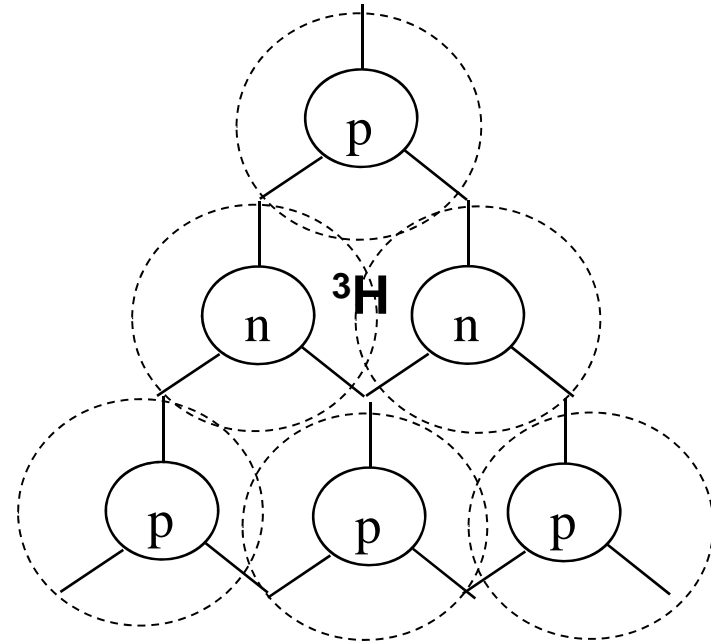
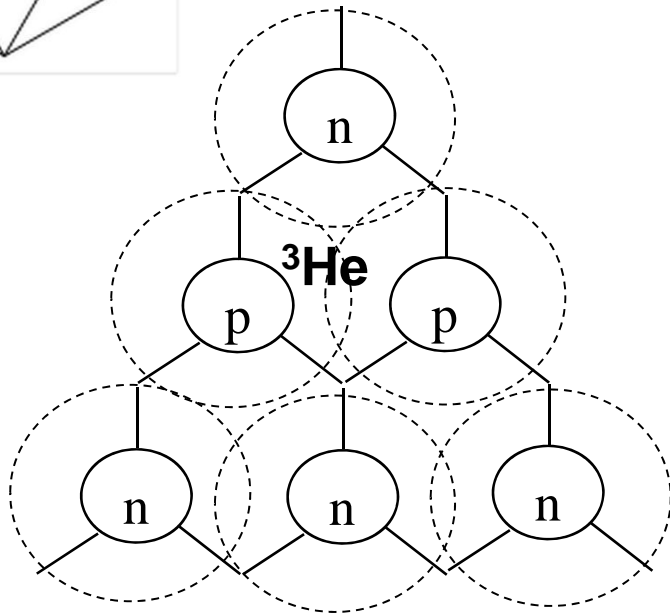
**Blue** – neutrons

# The closed shell $n = 2$ , $^{40}\text{Ca}$

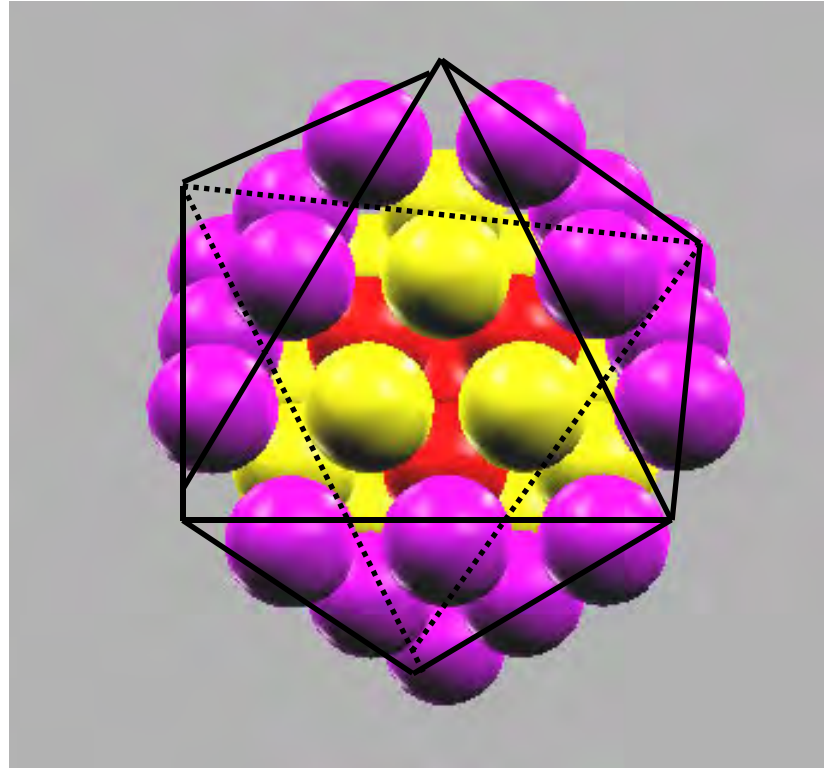
Shell Closure



Faces of  $^{40}\text{Ca}$  octahedron

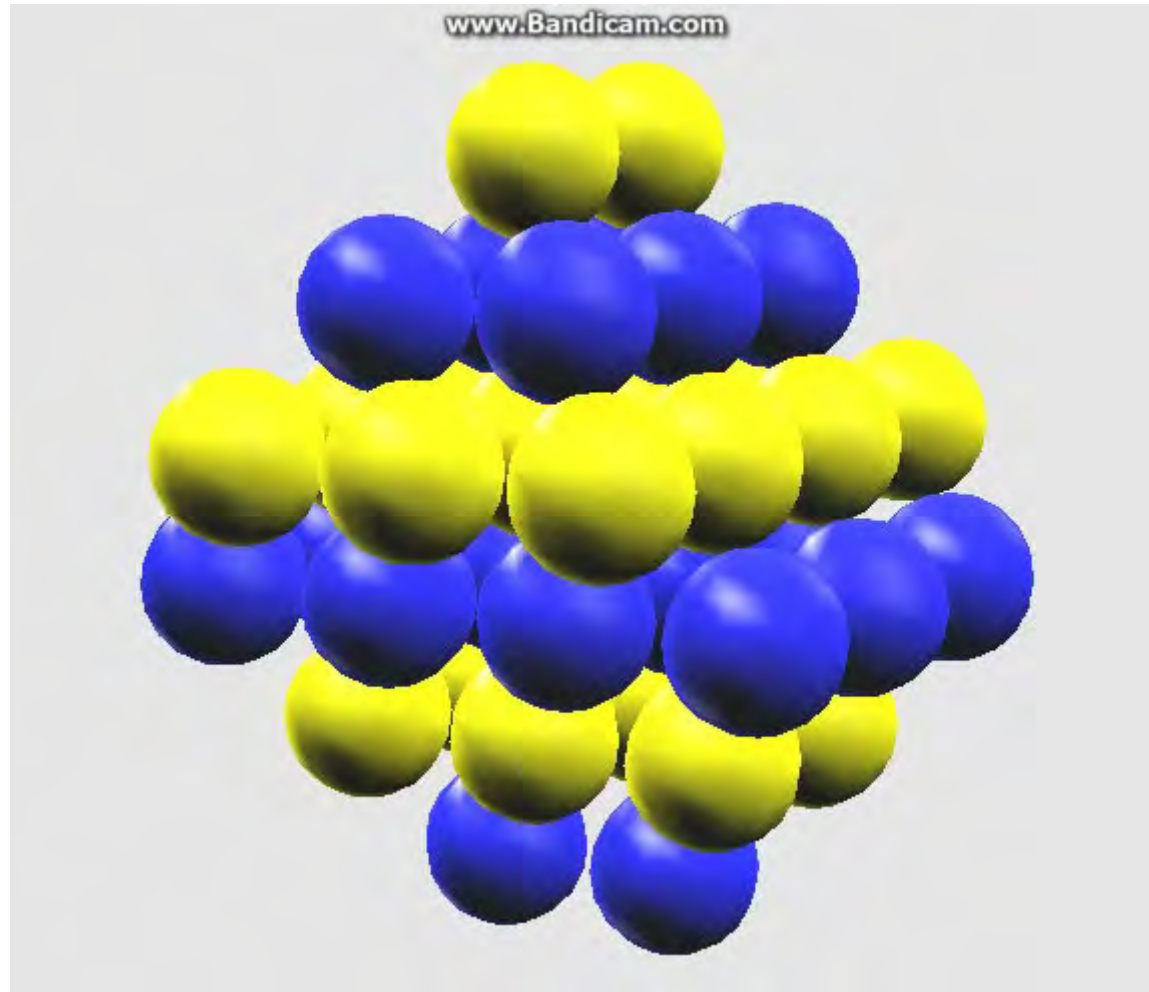


$^{40}\text{Ca}$

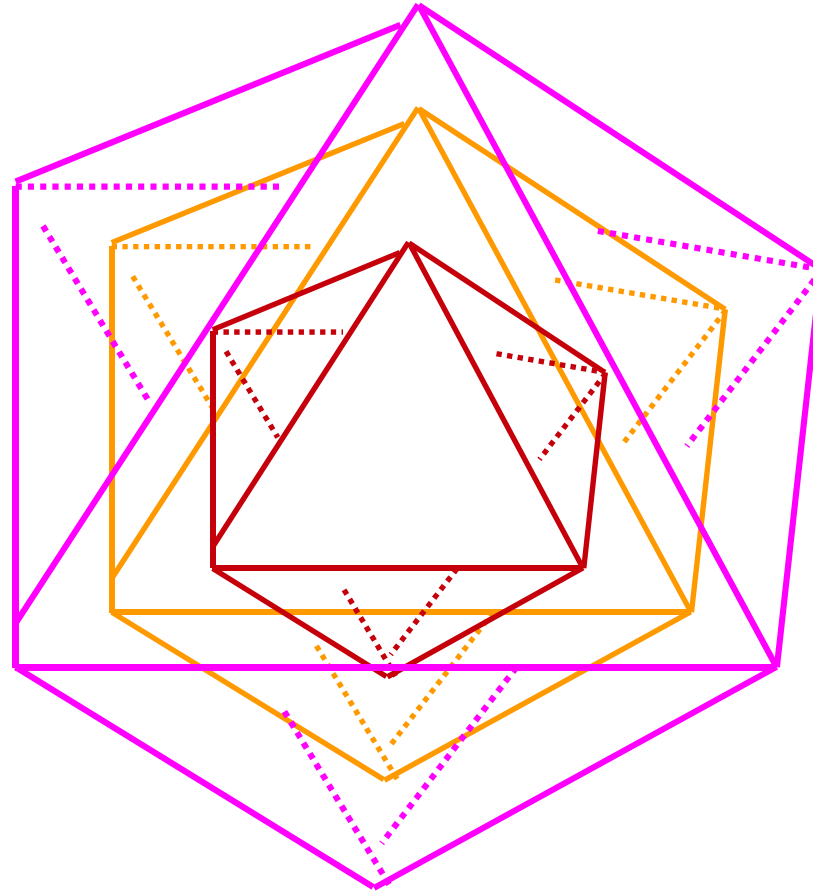


**s**-shell – red  
**p**-shell – yellow

$^{40}\text{Ca}$



$^{40}\text{Ca}$



**3 Nested Octahedra – s, p, d -shells**

# What's Further?

**Nested Octahedra –  $N = 0, 1, 2, \dots, \infty$**

**No!**

**Deviations from octahedral form:**

- Peculiarities of Nuclear synthesis
- Coulomb repulsion of protons

**Restricting factor from infinity:**

Coulomb repulsion of protons.

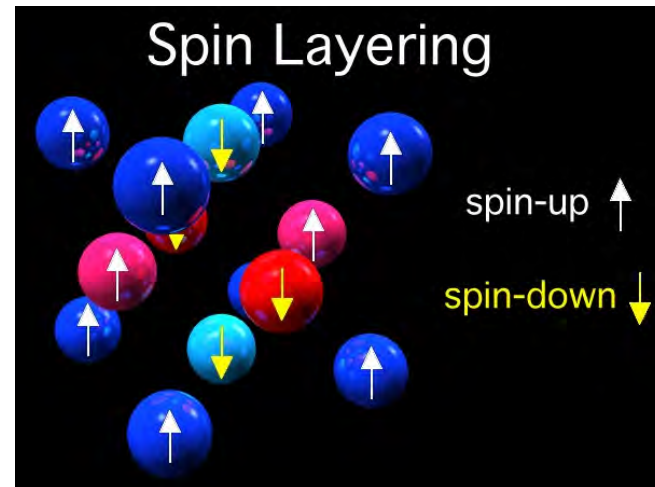
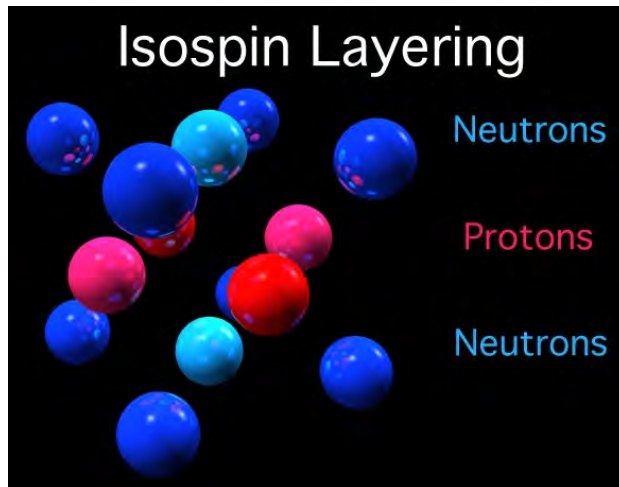
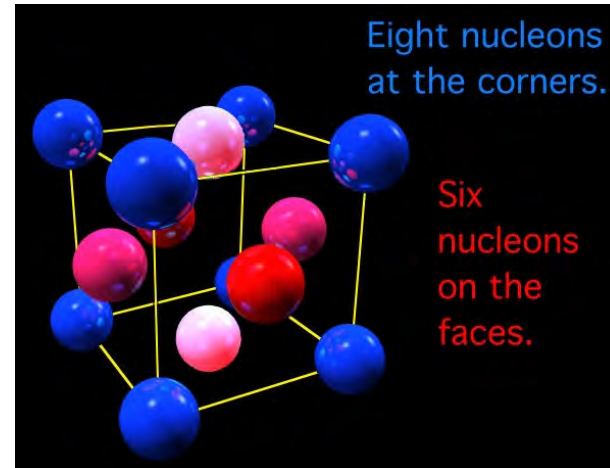
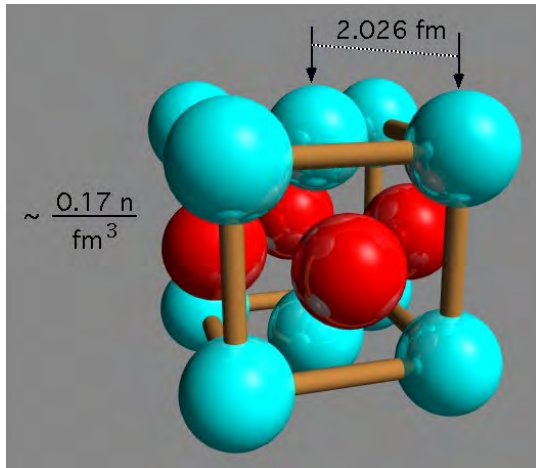


# SCQM to FCC symmetry of Nuclear Structure

- Nuclear shells correspond to faces of nested octahedra
- Nucleons are arranged in alternating isospin and spin layers
- Protons and neutrons are **strongly correlated**
- **It turned out that** nucleons occupy the nodes of **Face Centered Cubic Lattice (FCC)**

# SCQM → Face-Centered Cubic Lattice

Nucleons are arranged in face-centered cubic lattice



# Lattice Models of Nuclear structure

In terms of nucleons

- Simple Cubic Lattice
- Body Centered Lattice
- Hexagonal Close Packing
- **Face Centered Cubic Lattice (FCC)**

*E. Wigner, Phys. Rev. 51(1937)106*

*Cook N. and V. Dallacasa, Phys. Rev. C35(1987)1883*

# FCC Lattice Model

(N. Cook, 1987)

## Particle in 3D box

$$-(\hbar/2m) d^2\Psi/dr^2 + V(r) \Psi(r) = E\Psi(r)$$

For harmonic oscillator

$$E = \hbar\omega_0(n_x + n_y + n_z + 3/2) = \hbar\omega_0(N + 3/2)$$

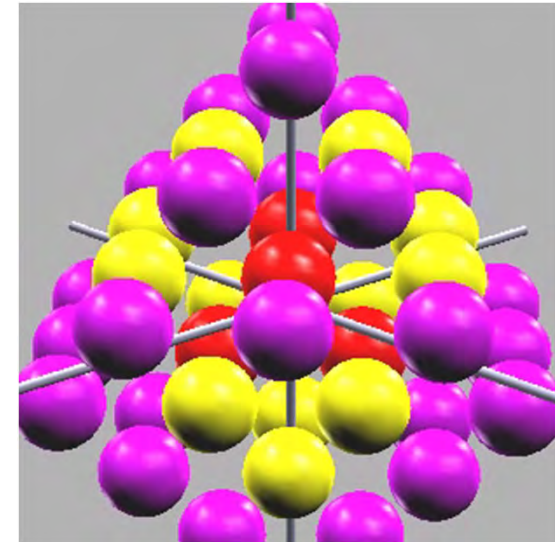
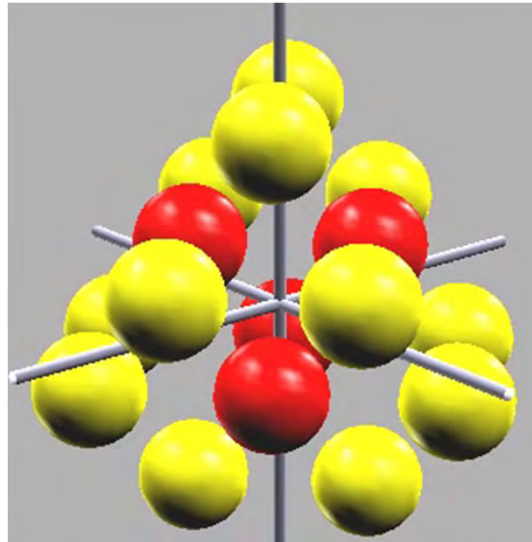
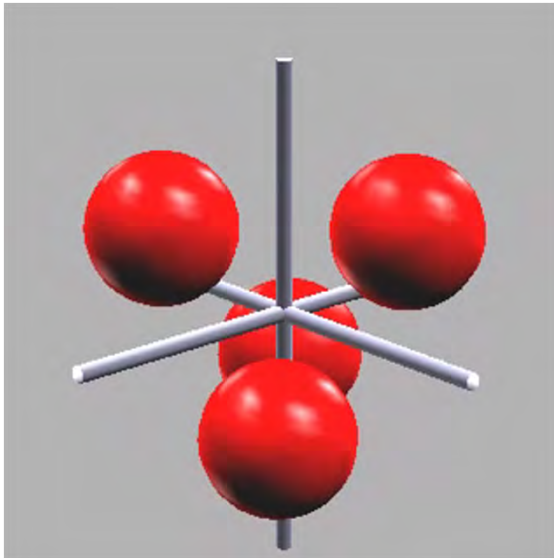
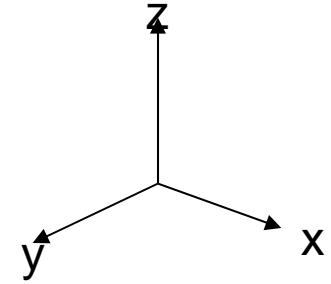
$$N = 0, 1, 2, 3, \dots$$

- Different combinations  $\mathbf{n}_x, \mathbf{n}_y, \mathbf{n}_z$ , giving the same total  $\mathbf{N}$ , denote the **number** of “degenerate” states with the same energy

# FCC Lattice Model

(N. Cook, 1987)

s, p, d - shells



- Origin of the coordinate system - at the center of the central tetrahedron
- The closure of each consecutive, symmetrical ( $x=y=z$ ) shell in the lattice composes **precisely** the numbers of nucleons in the shells derived from the three-dimensional Schrodinger equation

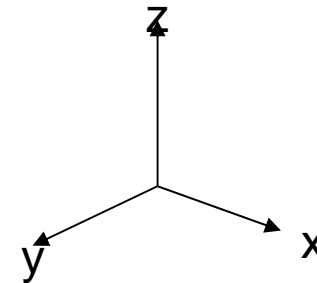
# FCC Lattice Model

(*N. Cook, 1987*)

- Principal quantum number, **N**

Assuming **x**, **y** and **z** coordinates of nucleons are **odd** – integers,

$$\mathbf{N} = (|\mathbf{x}| + |\mathbf{y}| + |\mathbf{z}| - 3)/2$$



The first shell (**s**-shell, **N** = 0) contains 4 nucleons with coordinates 111, -1-11, 1-1-1, -11-1.

The second shell (**p**-shell, **N** = 1): 12 nucleons  
31-1, 3-11, -311, -3-1-1, 1-31, -131, 13-1, -1-3-1, -113,  
11-3, 1-13, -1-1-3

The **d**-shell ...

and so on ...

- Total angular momentum, **j**

$$\mathbf{j} = (|\mathbf{x}| + |\mathbf{y}| - 1)/2$$

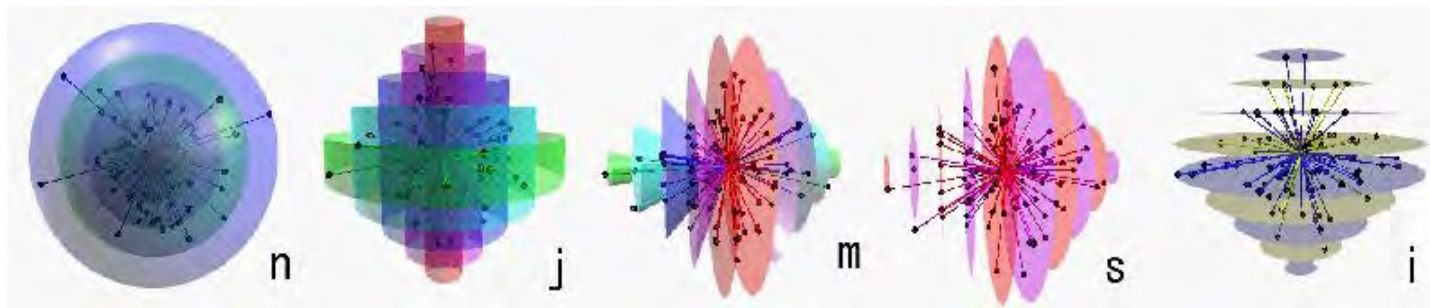
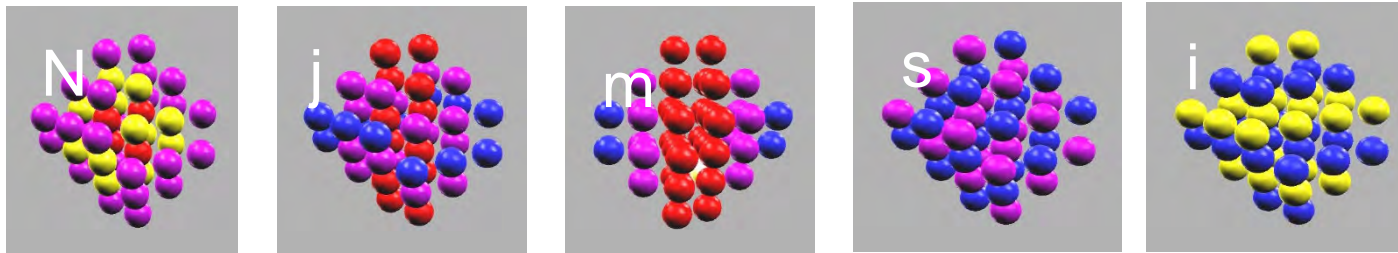
- Magnetic quantum number, **m**

$$\mathbf{m} = |x|/2$$

# FCC Lattice Model

(N. Cook, 1987)

Different colors correspond to  
different quantum numbers



# FCC-SCQM vs Shell Model

Close relation between

nucleon location in FCC-SCQM and quantum numbers of SM

$$n = (|x| + |y| + |z| - 3)/2$$

$$j = |l + s| = (|x| + |y| - 1)/2$$

$$m = (|x|/2)(-1)^{(x-1)}$$

$$s = (-1)^{(x-1)}/2$$

$$i = (-1)^{(z-1)}/2$$

and reversely

$$x = |2m|(-1)^{(m + 1/2)}$$

$$y = (2j + 1 - |x|)(-1)^{(i+j+m+1/2)}$$

$$z = (2n + 3 - |x| - |y|)(-1)^{(i+n-j-1)}$$

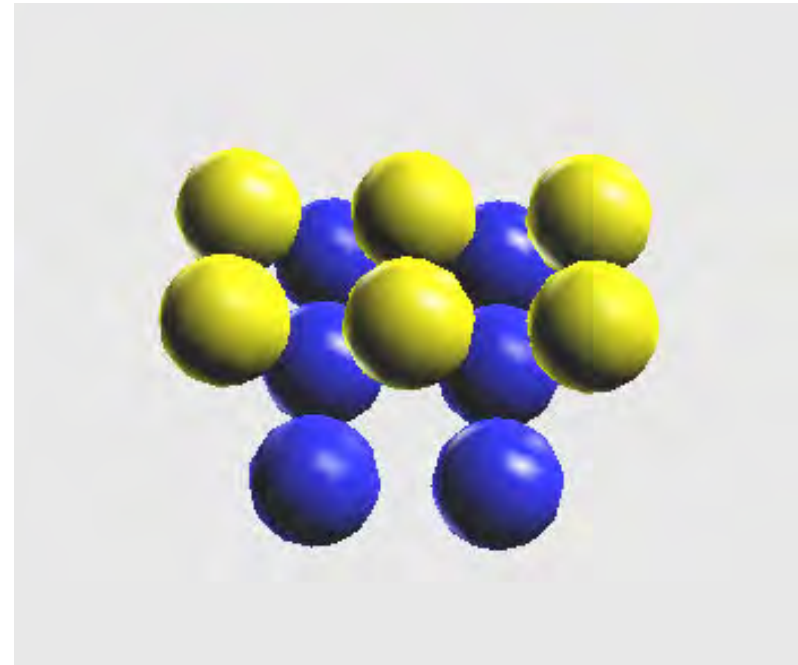
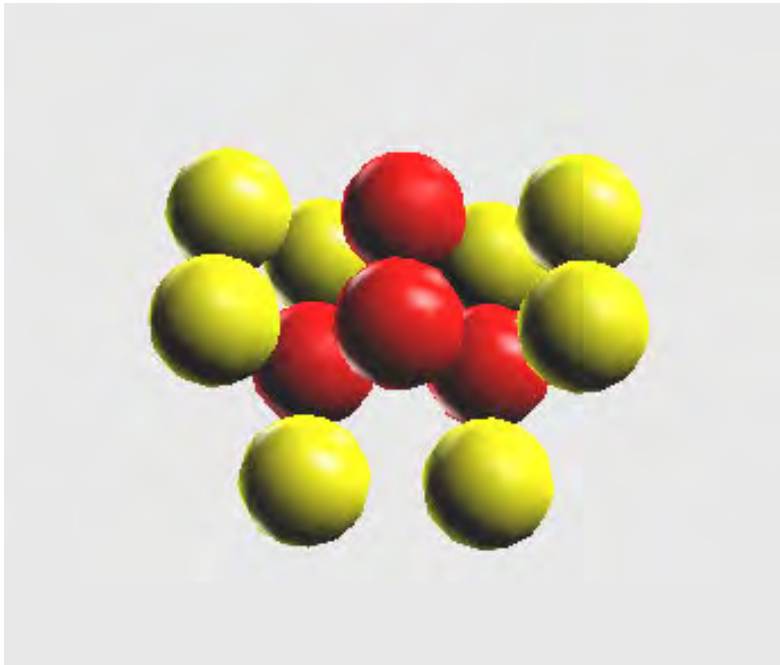


$^{12}\text{C}$

6 protons, 6 neutrons

$n$ , principal number  
 $n=0$ , red;  $n=1$ , yellow

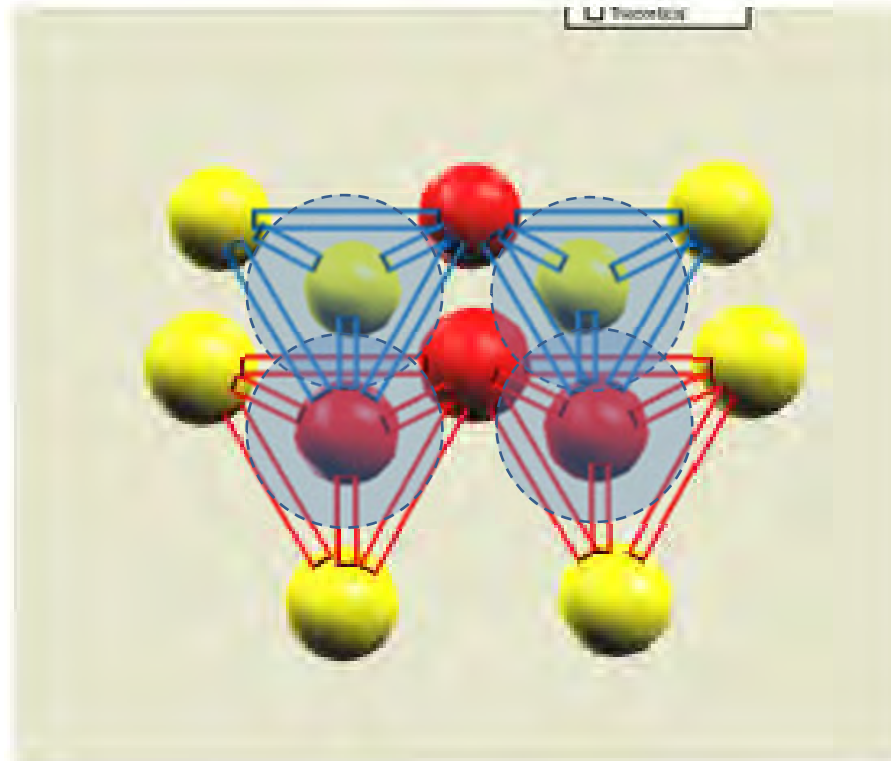
$i$ , isospin  
yellow – protons  
blue - neutrons



Problem for SM:  
**Why  $^{12}\text{C}$  is so stable?**

# Virtual $\alpha$ -clusters

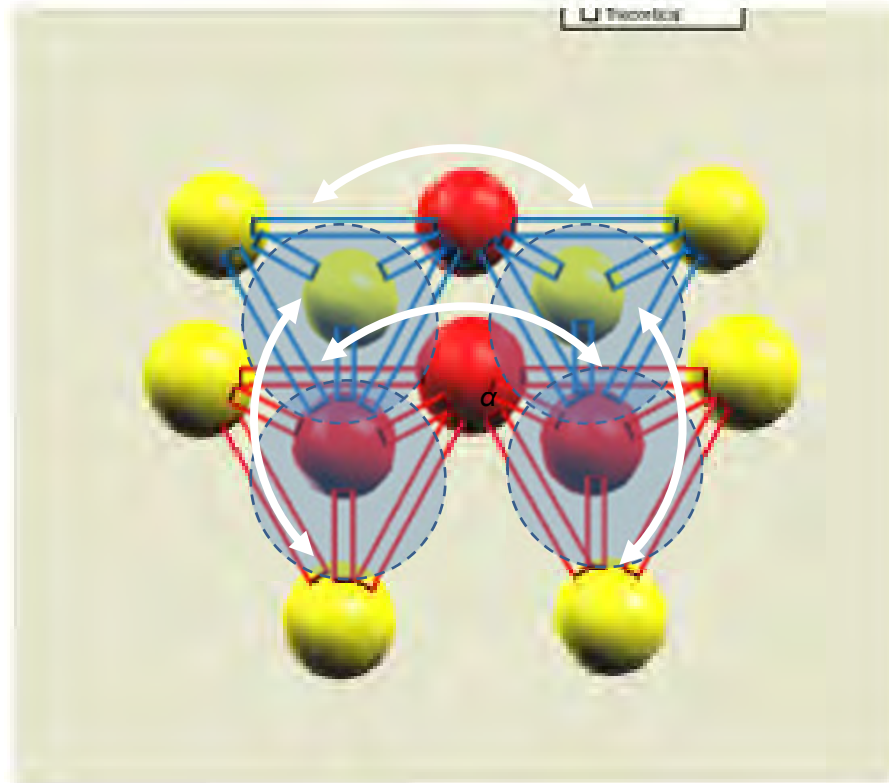
$^{12}\text{C}$  - 4 virtual  $\alpha$ -clusters



- 4 nucleons of  $s$ -shell (red) form with 6 nucleons of  $p$ -shell (yellow) 4 virtual  $\alpha$ -clusters.
- $s$ -shell nucleons are exchange particles

# $^{12}\text{C}$

Crosswise bindings of 4 virtual  $\alpha$ -clusters  
by exchange (red) nucleons of s-shell



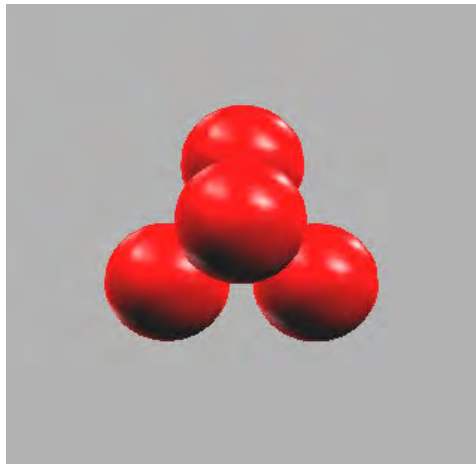
- exchange nucleons acquire larger binding energy as belonging simultaneously to 2 alpha clusters
- s-shell core is rearranged and disappears

# FCC-SCQM vs SM

## Spin-orbital coupling

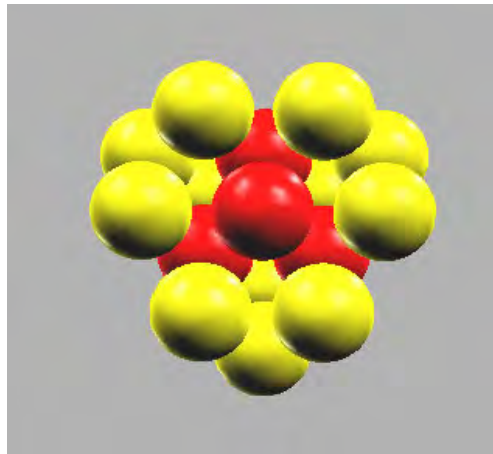
In SCQM Increasing number of exchange nucleons leads to **Lowering** of levels with higher **J**

$J = 1/2$



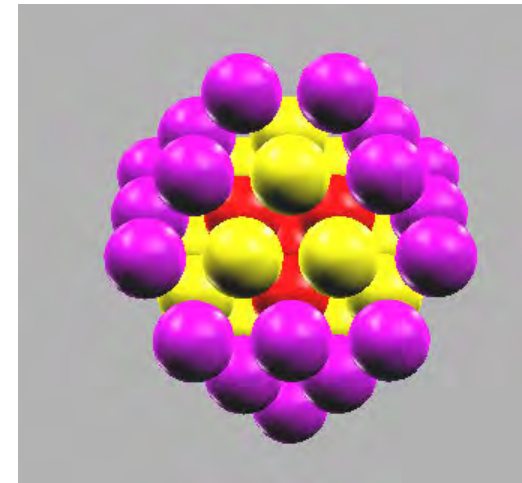
s:  $n=0, l=0$   
1 alpha

$J = 1/2, 3/2$



s:  $n=1, l=1$   
6 virtual alpha

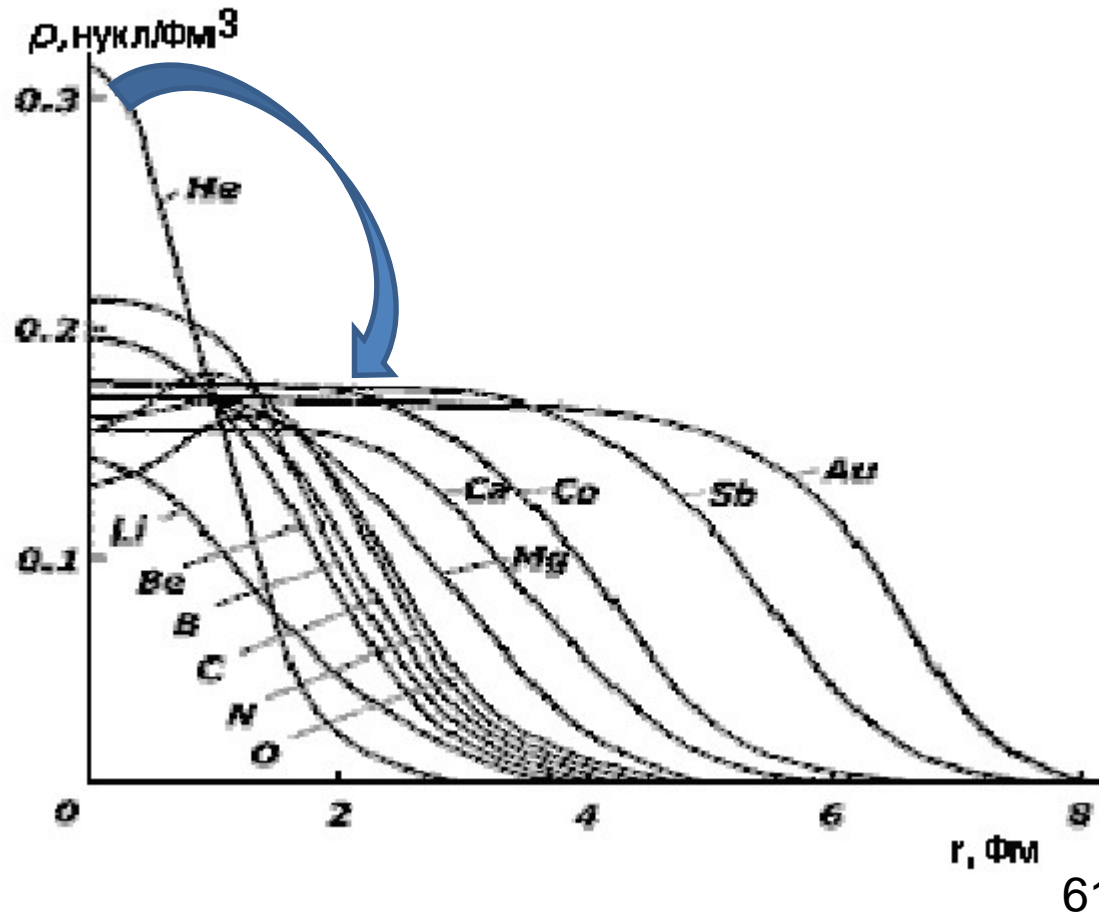
$J = 1/2, 3/2, 5/2$



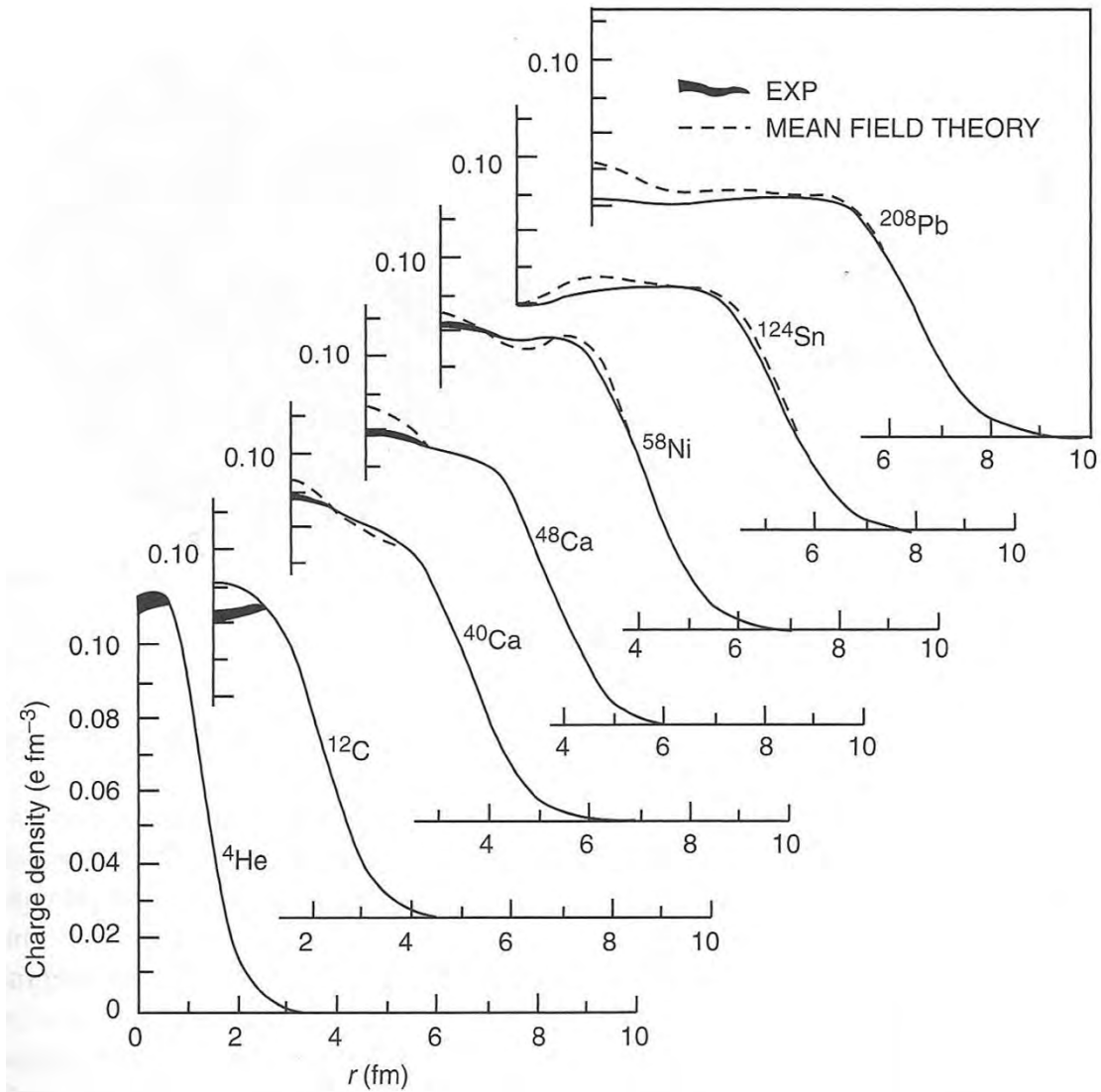
s:  $n=2, l=0, 2$   
22 virtual alpha

# Nuclear density

Result of rearrangement – of  $s$ -shell  
No  $s$ -core structure for  $A \geq 12$



# Fluctuation of central nuclear density



# Fluctuation of central nuclear density

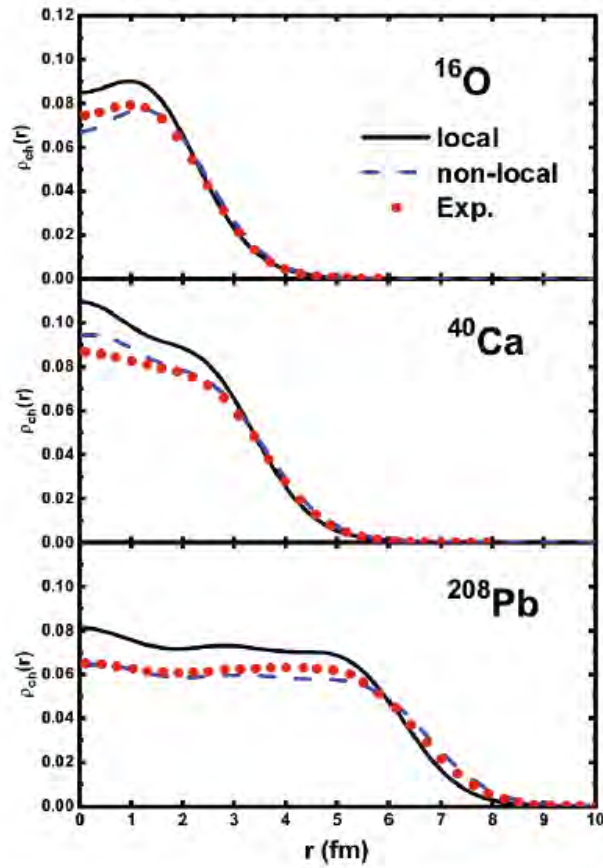
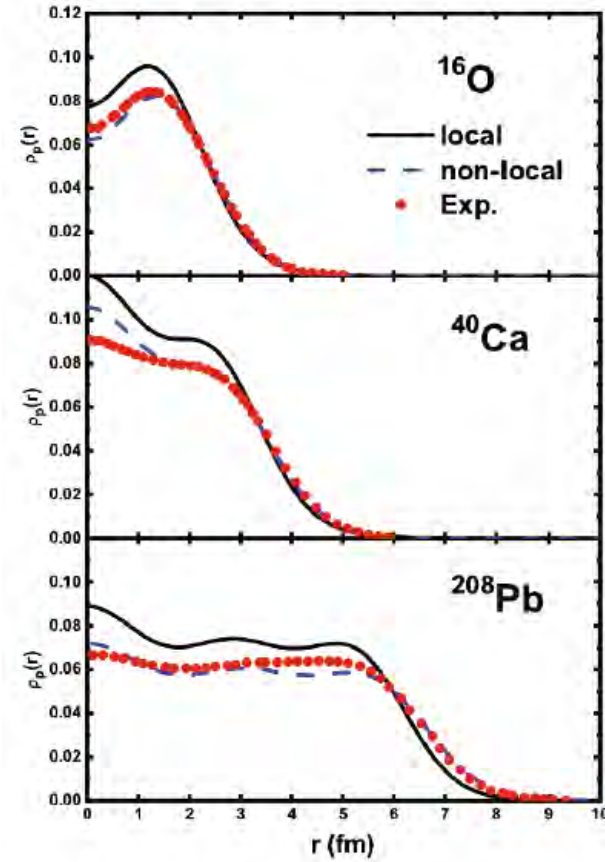


Fig. 3. (Color online) Same as Fig. 1 but for the charge

Charged density distribution



Point proton density distribution

# Nuclear Size and Shape

## Experimental Observations

- Compactness of and a hole inside  ${}^4\text{He}$

Point-nucleon charge distributions of  ${}^3\text{He}$  and  ${}^4\text{He}$   
Hole inside  ${}^3\text{He}$  and  ${}^4\text{He}$

*I. Sick, PRC, vol. 15, No.4; LNP, vol. 87, p.236*

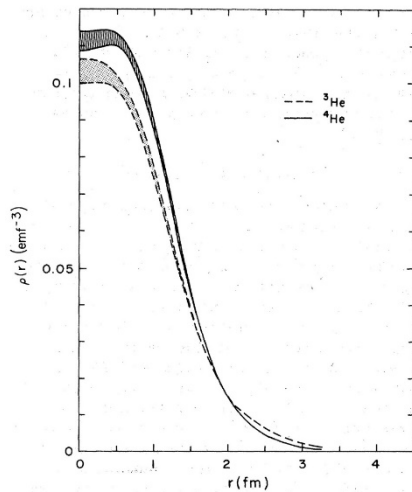
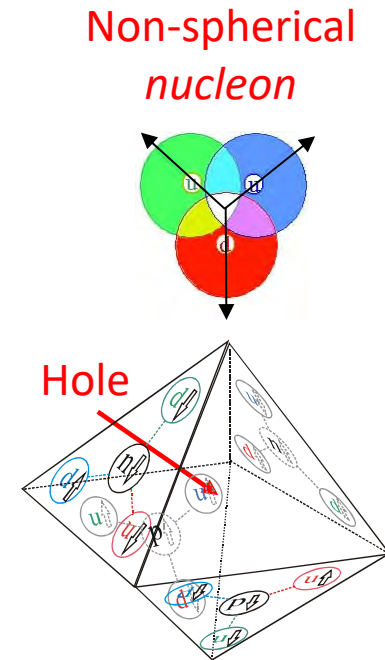


FIG. 11.  ${}^3,4\text{He}$  model-independent charge densities.

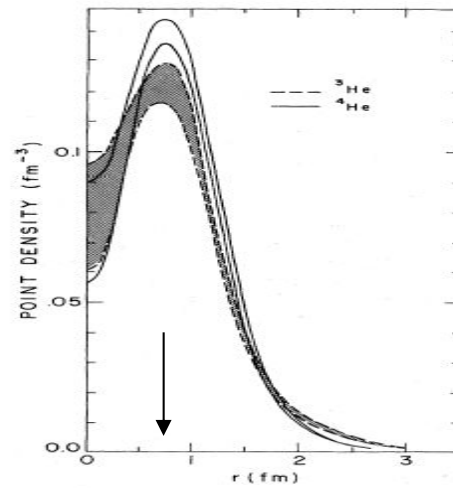
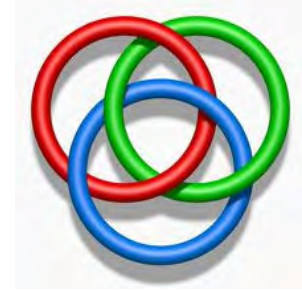


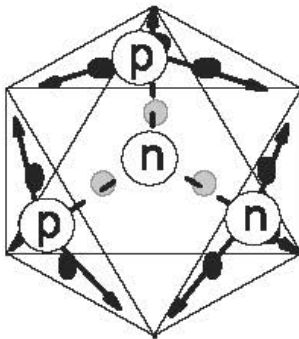
FIG. 15. Model-independent densities of pointlike ones in  ${}^3,4\text{He}$ .



# Helium Isotopes Borromean Nuclei

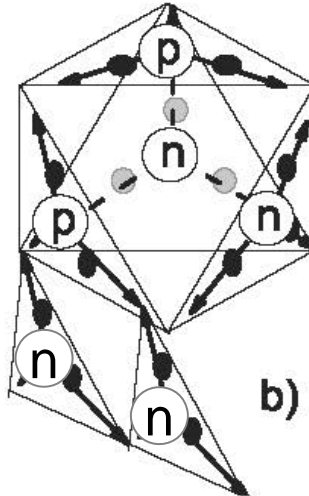


$^4\text{He}$   
Core



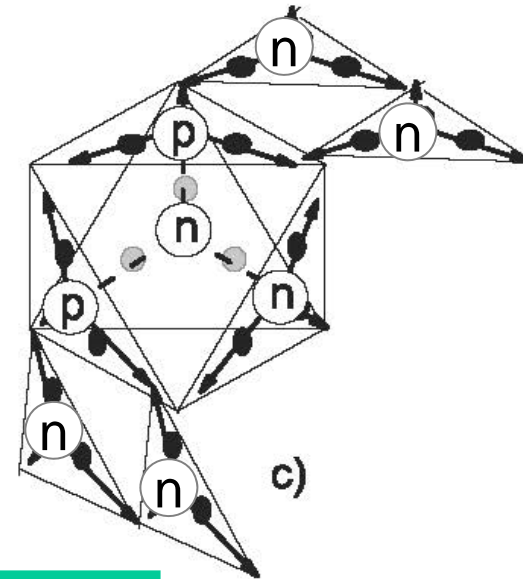
a)

$^6\text{He}$



b)

$^8\text{He}$



c)

Nucleus	$^4\text{He}$	$^6\text{He}$	$^8\text{He}$
Model	1.6	2.2	2.4
Exp.	1.57	2.4	2.5

# Fluctuation of central nuclear density

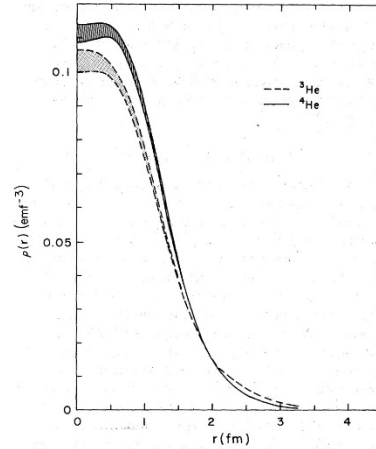
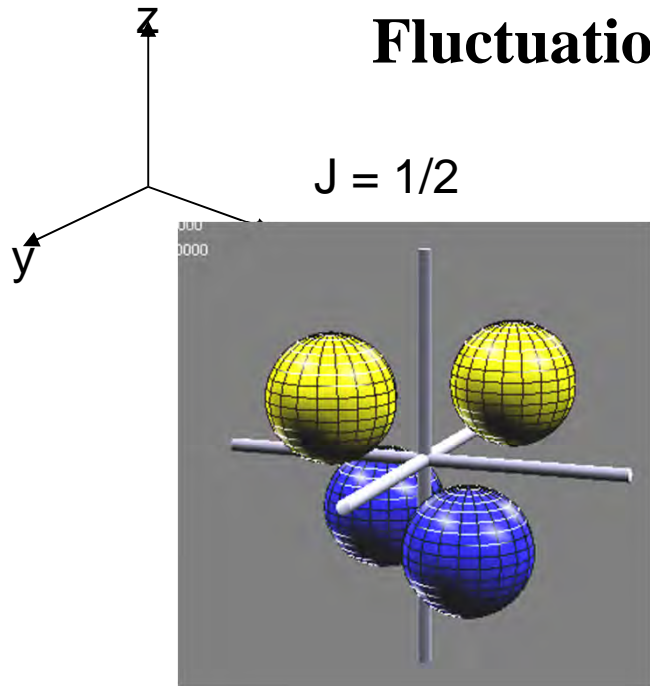


FIG. 11.  $^3\text{He}$  model-independent charge densities.

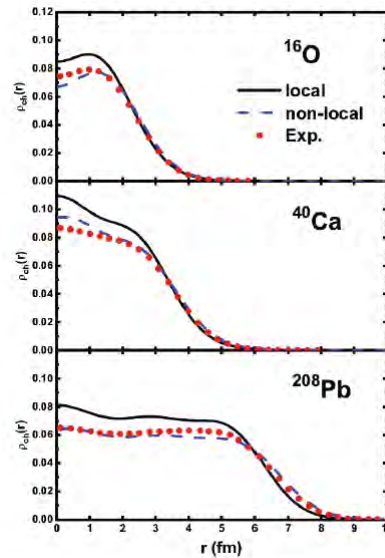
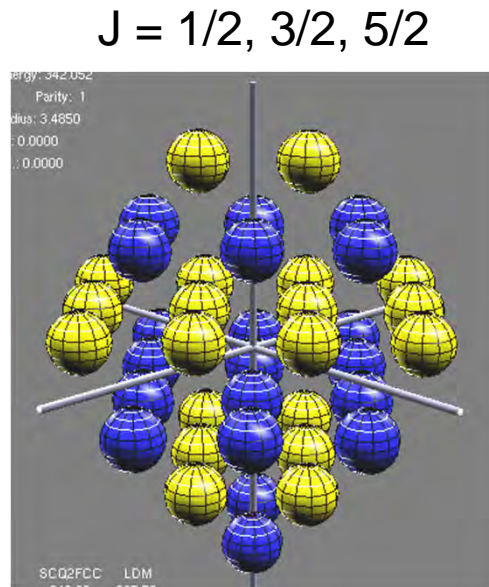
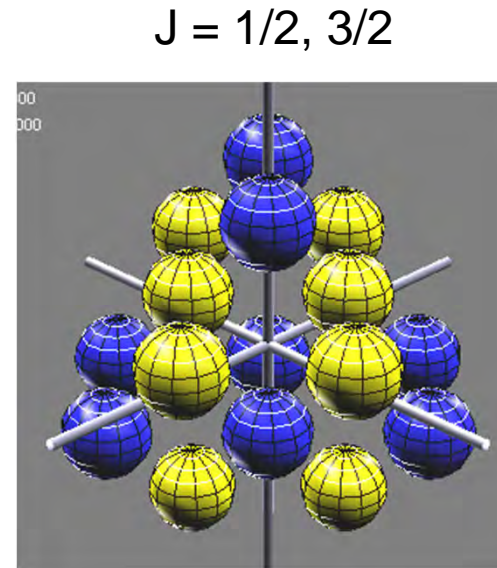
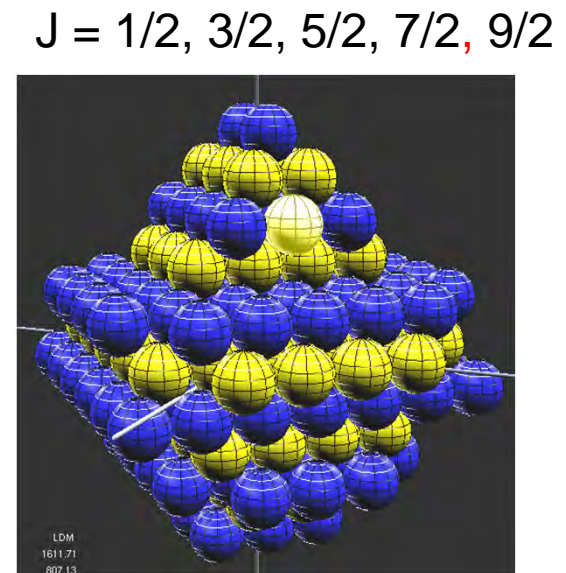
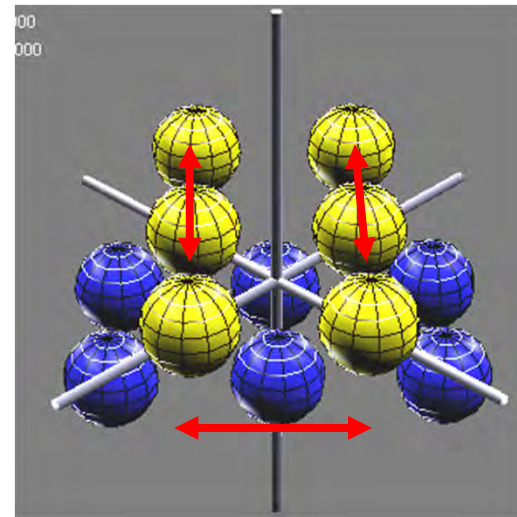
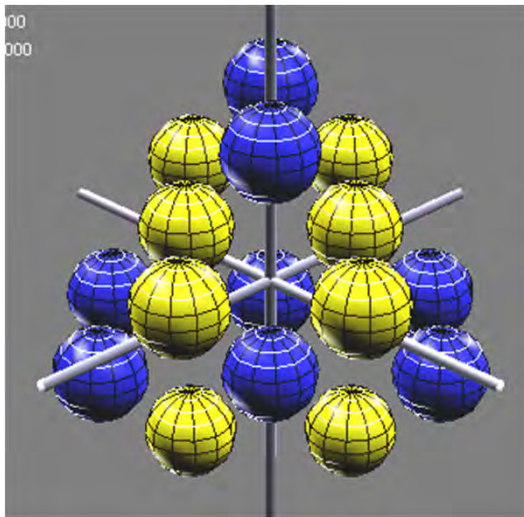
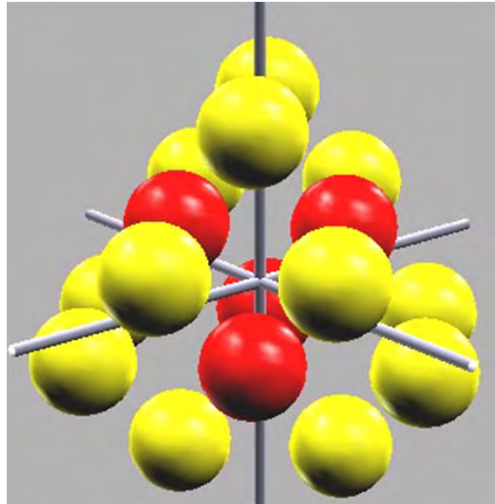


Fig. 3. (Color online) Same as Fig. 1 but for the charge.



# Fluctuation of central nuclear density

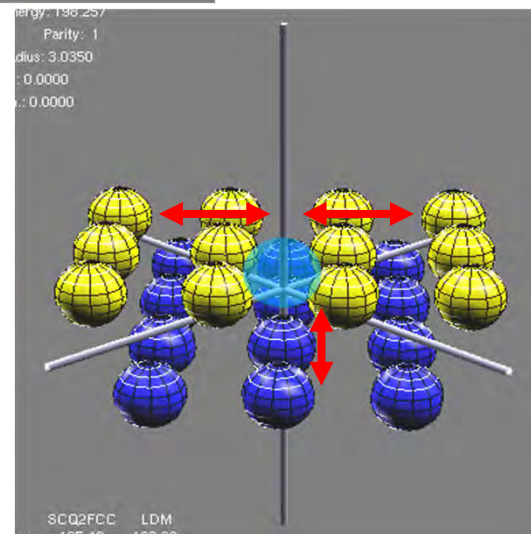
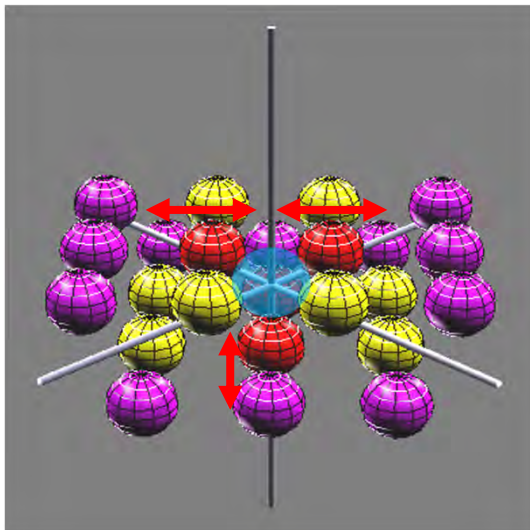
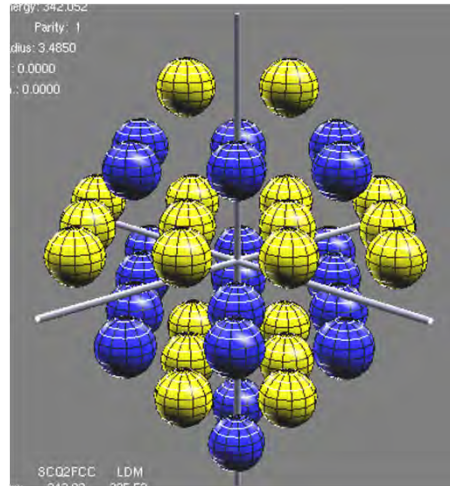
$^{16}\text{O}$   $J = 1/2, 3/2$



**Central Density Depression** 67

# Fluctuation of central nuclear density

$^{40}\text{Ca}$   
 $J = 1/2, 3/2, 5/2$



**Central Density Rize**

# Fluctuation of central nuclear density

## FCC-SCQM

Proton Closed Shells – Octahedra with filled faces  
2, 8, 20, 40, 70, 112, ... magic numbers,  
as given by HO potential

Central Density **Rise**:  $N_p = 2, 20, 70$

There is a central **virtual  $\alpha$ -cluster**

For all nuclei nearby these  $N_p$

Central Density **Depression**:  $N_p = 8, 40, 112$

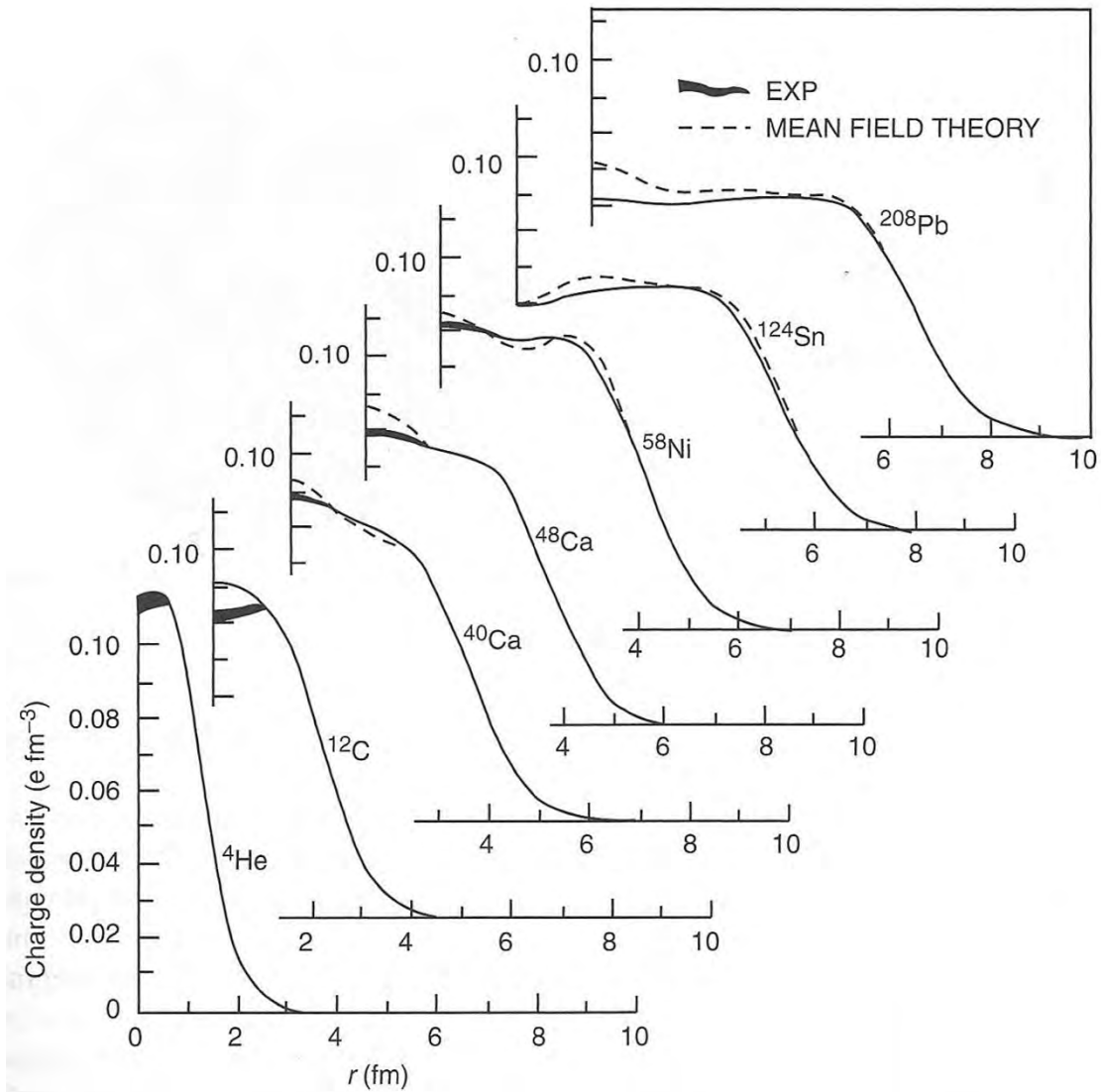
There is **No** a central **virtual  $\alpha$ -cluster**

For all nuclei nearby these  $N_p$

**In Agreement with Experiment**

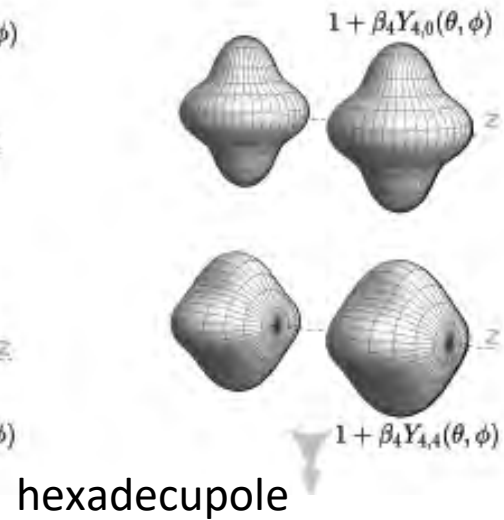
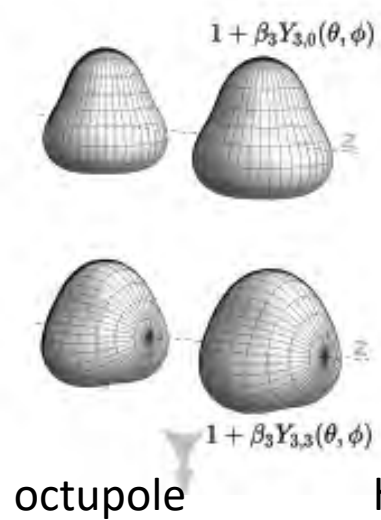
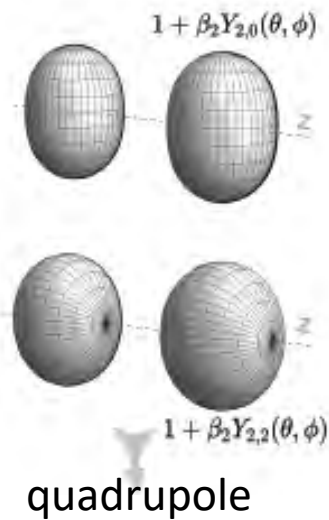
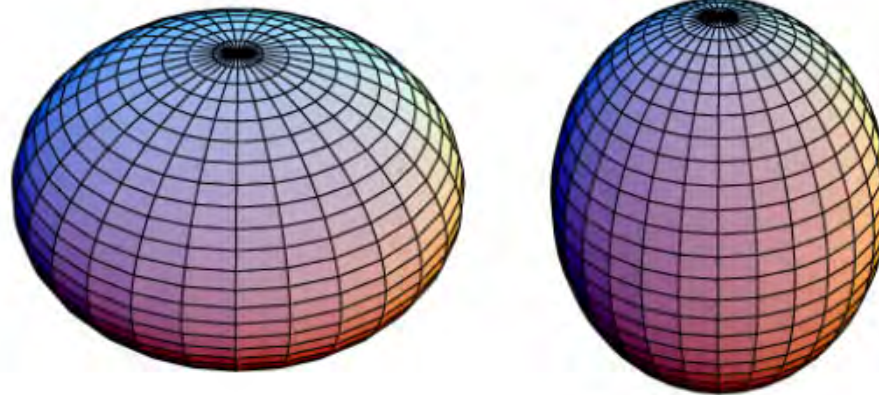
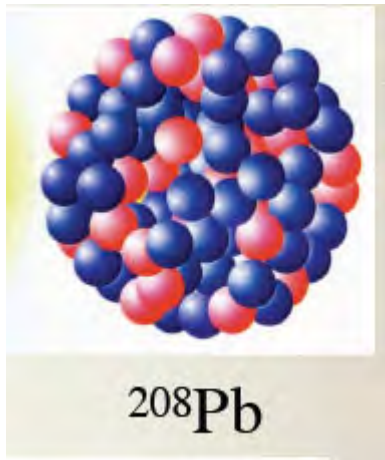


# Fluctuation of central nuclear density



# Nuclear Deformation

Nuclei are not spherically symmetric

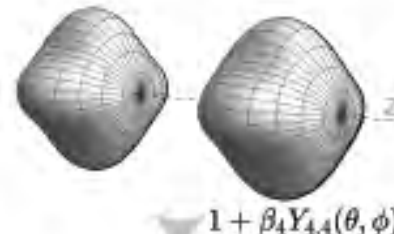
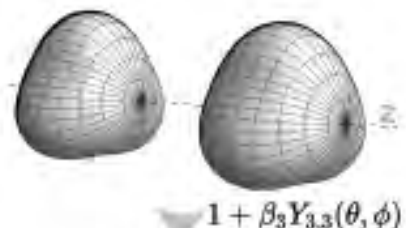
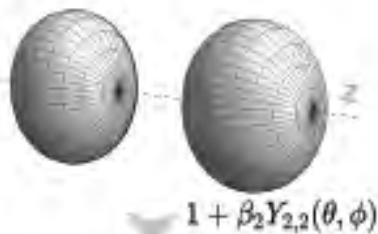
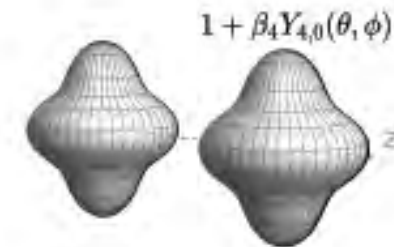
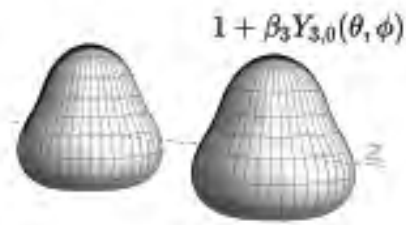
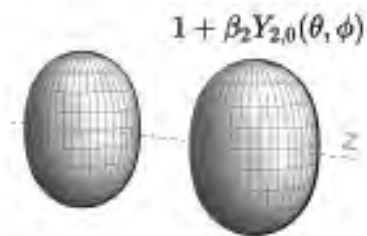


# Nuclear Deformation Theory

$$\rho(r, \theta, \phi) = \frac{\rho_0}{1 + e^{[r - R(\theta, \phi)]/a_0}} \quad \text{- Nuclear density}$$

$$R(\theta, \phi) = C(\alpha_{\lambda\mu})R_0 \left[ 1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} Y_{\lambda}^{\mu}(\theta, \phi) \right] \quad \text{- Nuclear radius}$$

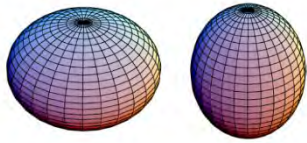
$$R(\theta, \phi) = R_0 \left( 1 + \beta_2 [\cos \gamma Y_{2,0} + \sin \gamma Y_{2,2}] + \beta_3 \sum_{m=-3}^3 \alpha_{3,m} Y_{3,m} + \beta_4 \sum_{m=-4}^4 \alpha_{4,m} Y_{4,m} \right),$$



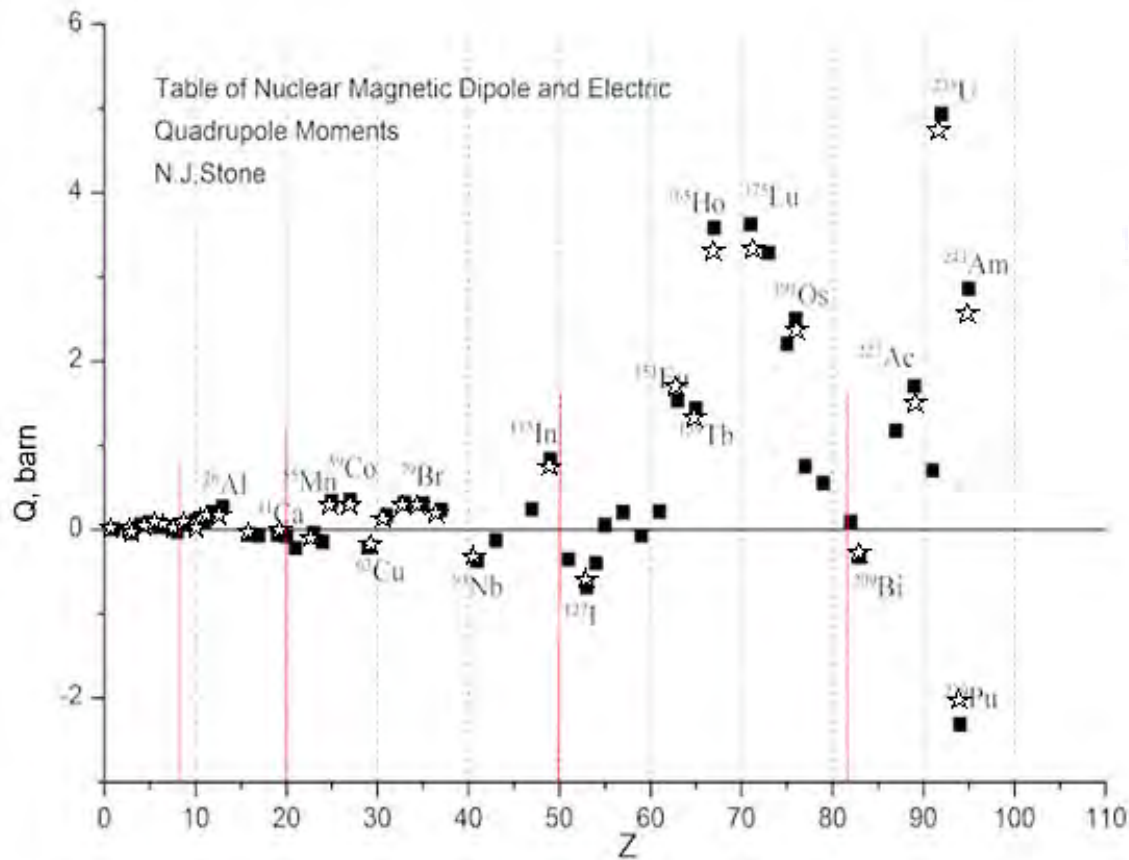


# SCQM+FCC vs Experiment

## Electric Quadrupole Moment



■ - Exp,    ☆ Model

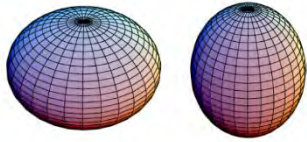


Model

$$Q = \frac{J(2J-1)}{(J+1)(2J+3)} Q_0$$

$Q_0$  – Intrinsic Quadrupole Moment

$$Q_0 = \sum_{k=1}^Z (2z_k^2 - x_k^2 - y_k^2)$$



# Nuclear Deformation

## Model vs Experiment

Charged(proton) Quadrupole Moments

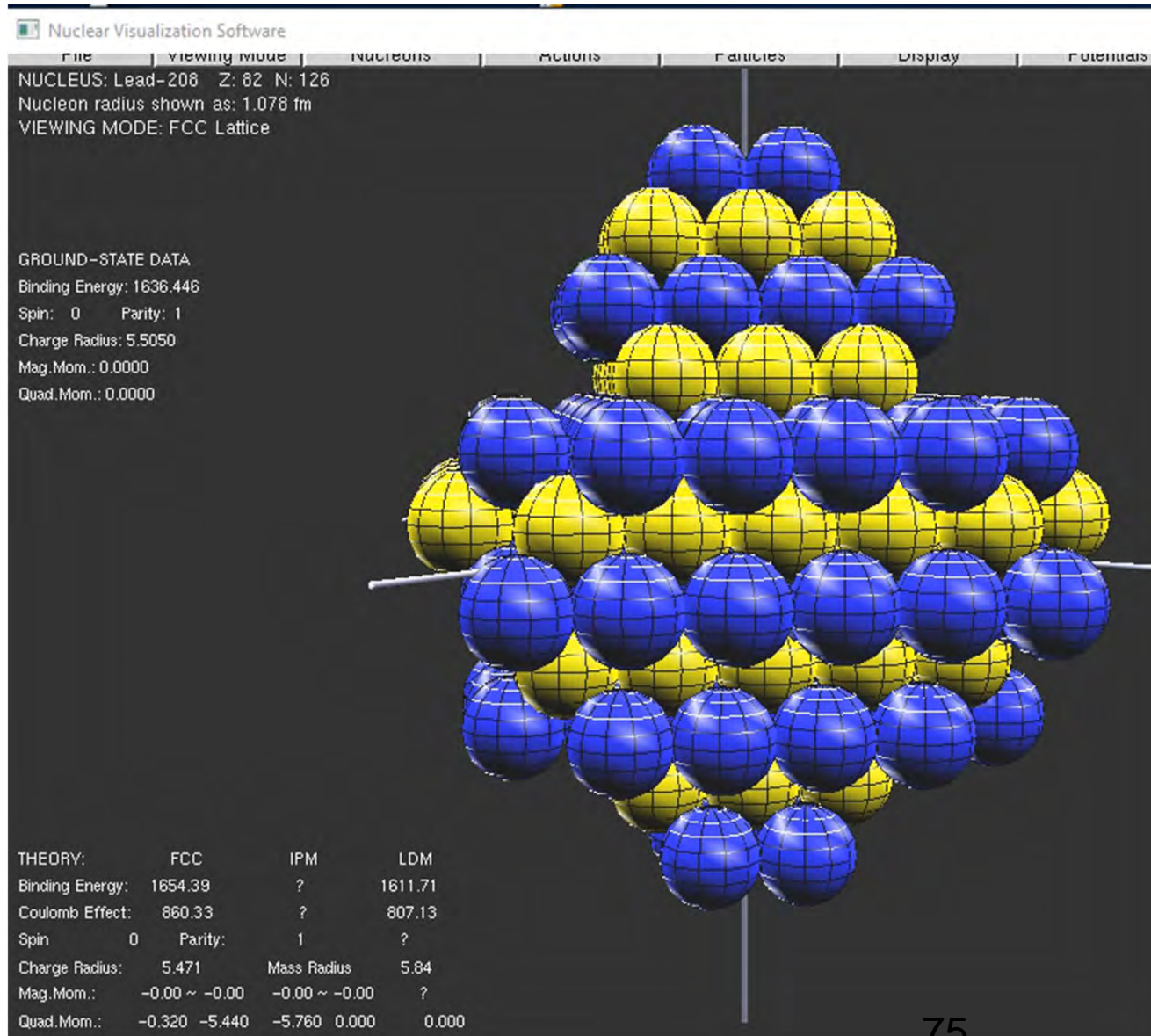
Neutron Quadrupole Moments

Nuclear Matter Quadrupole Moments

$$Q_0 = \sum_{k=1}^Z (2z_k^2 - x_k^2 - y_k^2) \quad \text{Intrinsic Quadrupole Moment}$$

Nucleus		C	Al	Ar	Cu	<sup>115</sup> In	<sup>118</sup> Sn	<sup>131</sup> Xe	<sup>197</sup> Au	<sup>208</sup> Pb	<sup>209</sup> Bi	<sup>235</sup> U
Charged Q	Exp.	0	0.15	0	-0.21	0.8	0	-0.12	0.54	0	-0.37	4.9
	Model	0	0.18	0	-0.02	0.7	0	-0.6	0.58	0	-0.26	4.7
Model												
Charged Q <sub>0</sub> ,		-0.08	0.49	0.16	-0.1	1.28	0.32	-1.92	2.96	-0.34	-0.49	10.1
Neutron Q <sub>0</sub>		-0.08	0.	0.64	0	-2.56	-0.32	0.72	-1.28	-5.42	-3.96	2.3
Nuclear Matter Q <sub>0</sub>		-0.16	0.49	0.80	-0.1	-1.28	0	-1.2	1.68	-5.76	-4.45	12.4

# $^{207}\text{Pb}$

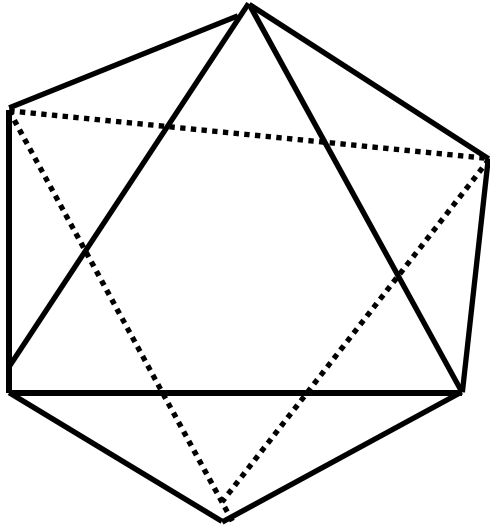


# Summary

## Nuclei possess **crystal-like** structure:

- Quarks-quark interactions in nuclei lead to strong proton-neutron correlations.
- Nucleon centers are arranged according to FCC lattice
- All bound nuclei are composed of **virtual triton-like** and  **$^4\text{He}$ -like clusters**
- Closed Shells = Octahedral Faces
- All nuclei are deformed
- **Symmetry energy** is a consequence of strong quark correlations  $\rightarrow$  strong correlations of protons and neutrons.
- The **pairing effect** is a consequence lattice structure

Thank you for your  
attention!



# Back Slides

# SCQM

## Motivation

### **proton-proton interactions**

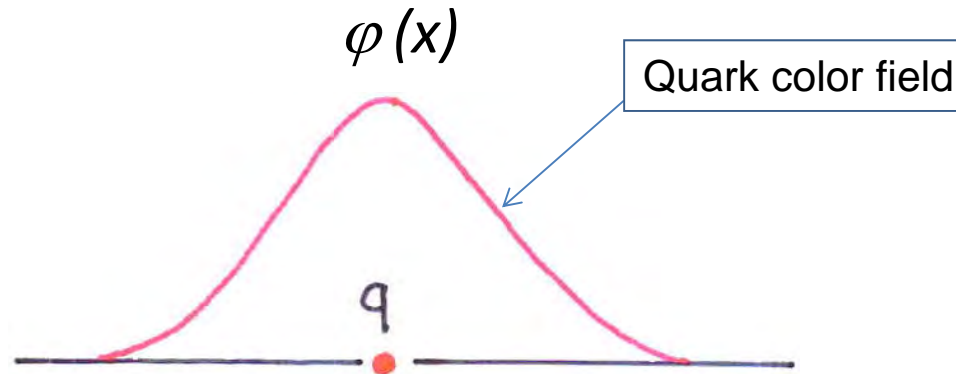
- soft elastic scattering
- hard elastic scattering
- single diffractive scattering
- double diffractive scattering
- inelastic non-diffractive scattering

“Elementary particles are  
no more than holes in  
vacuum.”

*Henry Poincare*

# SCQM

## Single Colored Quark inside Vacuum



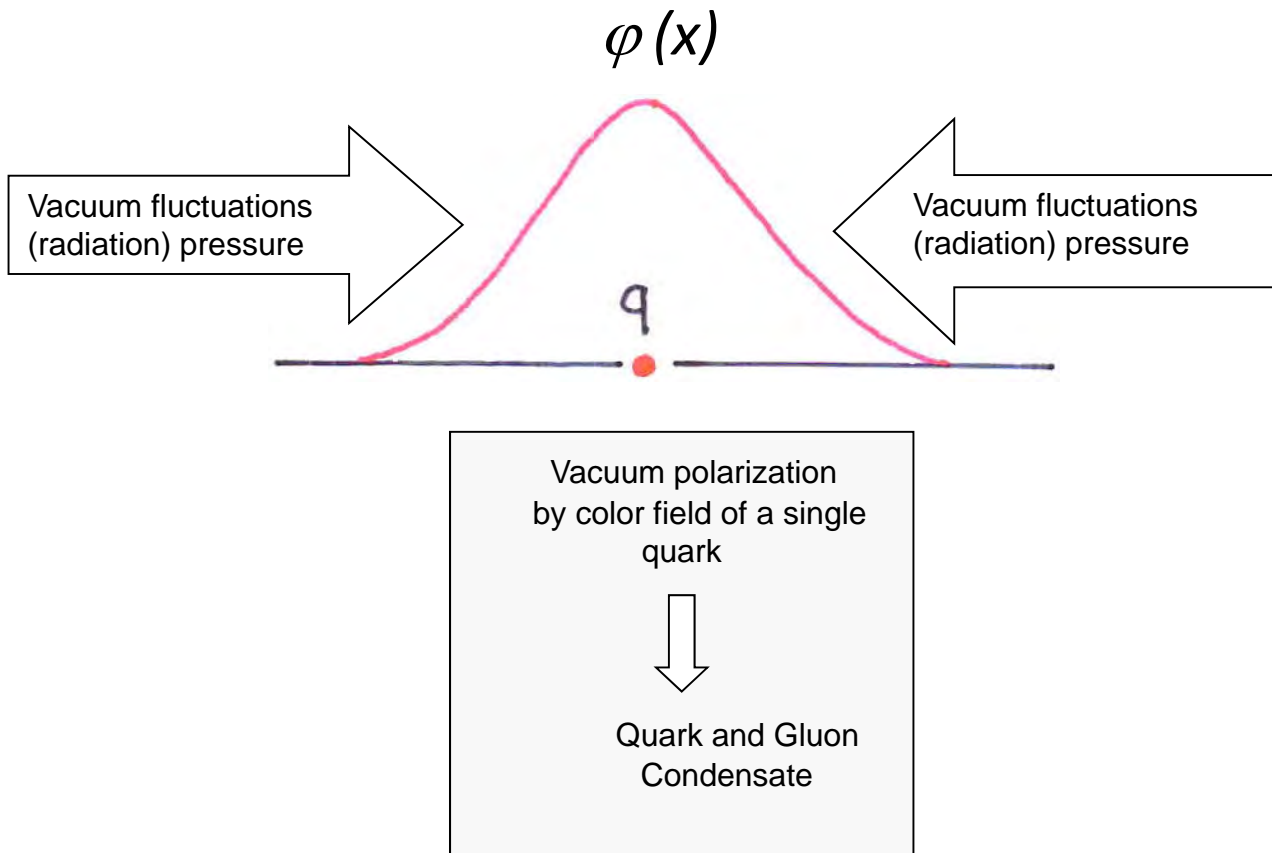
Vacuum polarization  
by color field of a  
single quark

↓

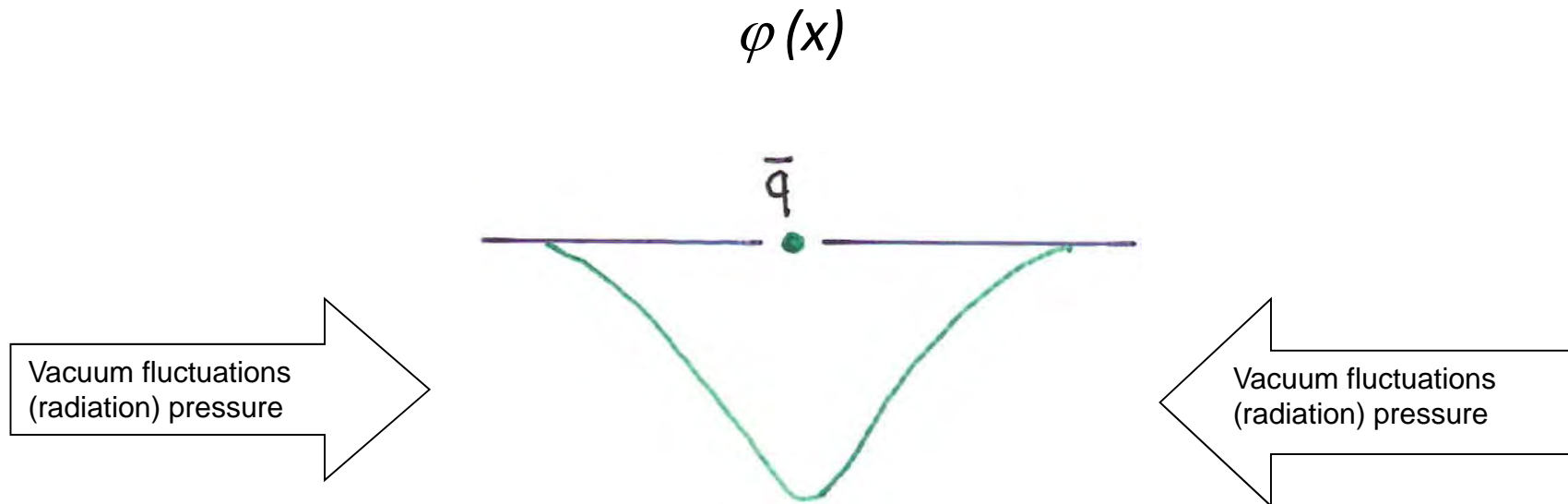
Quark and Gluon  
Condensate



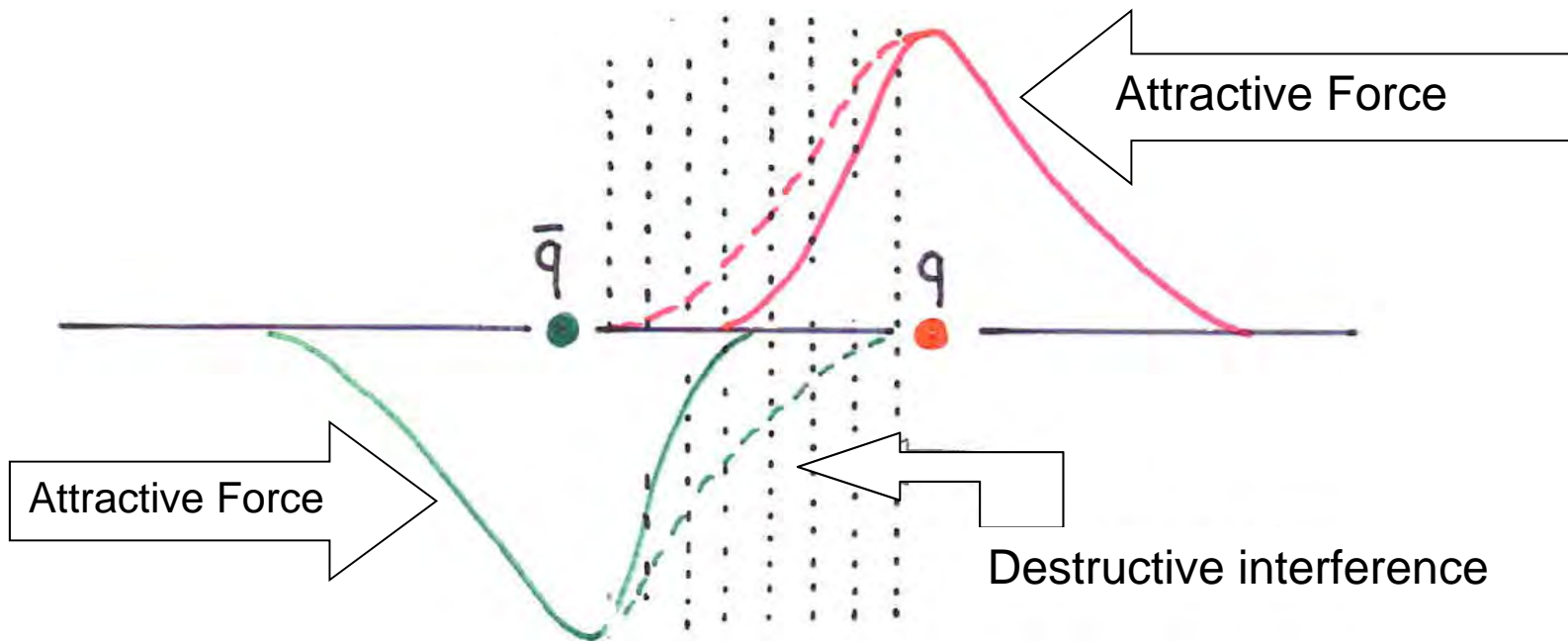
# Strongly Correlated Quark Model (SCQM)



# Strongly Correlated Quark Model (SCQM)

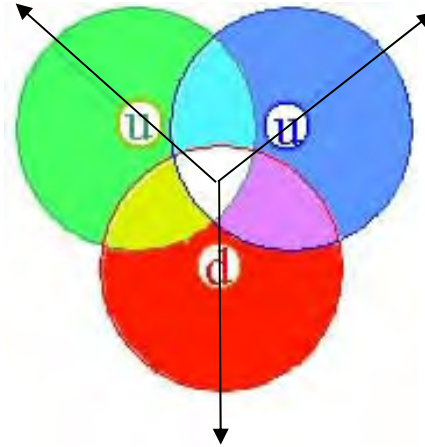


# Strongly Correlated Quark Model (SCQM)



Overlap of opposite color fields  $\rightarrow$  attraction force between quark and antiquark  
"Color Casimir" effect

# Nucleon



Nucleon wave function composed of color quarks

$$\psi(x) \rightarrow \frac{1}{\sqrt{6}} \sum_{ijk} e_{ijk} |c_i\rangle |c_j\rangle |c_k\rangle$$

Where  $|c_i\rangle$  are orthonormal states with  $i,j,k \rightarrow R,G,B$

g1

## SCQM $\implies$ The Local Gauge Invariance Principle

**Destructive Interference of color fields  $\equiv$  Phase rotation of the quark w.f. in color space:**

$$\psi(x)_{Color} \rightarrow e^{ig\theta(x)}\psi(x)$$

Phase rotation in color space  $\iff$  quark dressing (undressing)  $\equiv$  the gauge transformation

$$A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu\theta(x)$$

Therefore, during quark oscillation its

**color charge**

**momentum**

**mass**

**are continuously varying function of time.**

g1

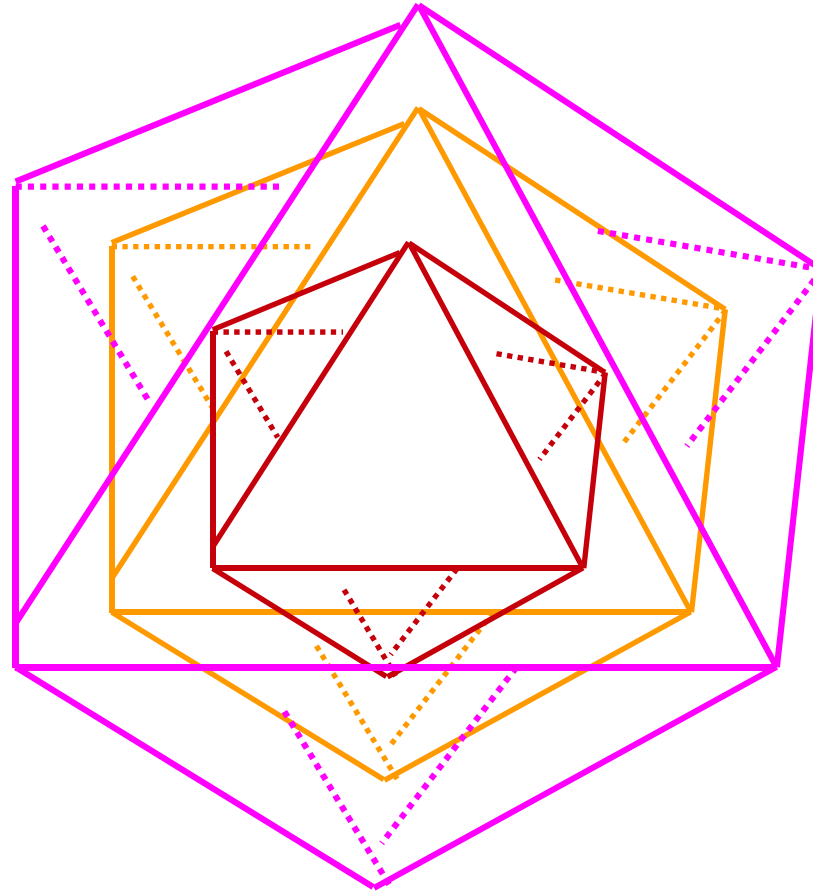
## Relation SCQM to QCD

We reduce interaction of color quarks via **non-Abelian** fields to its **E-M** analog:

$$A_a^\mu(x) \rightarrow A^\mu(x)$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - \lambda f^{abc} A_b^\mu A_c^\nu \rightarrow F_{ch}^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$^{40}\text{Ca}$



**3 Nested Octahedra – s, p, d -shells**

# FCC-SCQM vs SM

## Source of spin-orbital coupling in FCC-SCQM

Increasing number of exchange nucleons, belonging to adjacent virtual alpha clusters with increasing J-value of sub-shells.



**Lowering of levels with higher J**  
**Splitting of nuclear levels**



# FCC-SCQM vs SM

## What about magic numbers?

### **SM**

Describes observed magic numbers of protons and neutrons

**2, 8, 20, 28, 50, 82, 126**

### **FCC-SCQM**

Closed Shells – Octahedra with filled faces

**2, 8, 20, 40, 70, 112, ...** as given by HO potential

# FCC-SCQM vs SM

## What about magic numbers?

SM: 2, 8, 20, 28, 50, 82, 126

FCC-SCQM: 2, 8, 20, 40, 70, 112, ...

**But, in FCC-SCQM the more preferable to start filling the next shell by the subshell with highest J (from the base of octahedron).**

If these subsells are filled, we get the following magic numbers:

**2, 6, 8, 14, 20, 28, 40, 50, 70, 82, 112, 126, ...**

**Red numbers** arise from adding to filled faces (shell) of octahedra the subshells with highest value J.

**However, takes place only if both protons and neutrons fill this subshells forming virtual alpha clusters.**

# The role of Quarks in FCC

- Color fields of Quarks, responsible for strong interactions, arrange nuclear nucleons in FCC Lattice structure.
- Strong interactions are **tensorial**
- Quark loops form **virtual** 3- and 4-nucleon clusters inside bound nuclei
- Evidence of quark loops is **big separation energy** in even-even nuclei
- Halo nuclei are formed by core and virtual 3-nucleon clusters ( ${}^3\text{H}$ -type)
- Ground state nuclei are formed by virtual  ${}^3\text{H}$ - and  ${}^4\text{He}$ -type clusters.
- There are no real  ${}^4\text{He}$  cluster in ground state nuclei

# Summary (cont.)

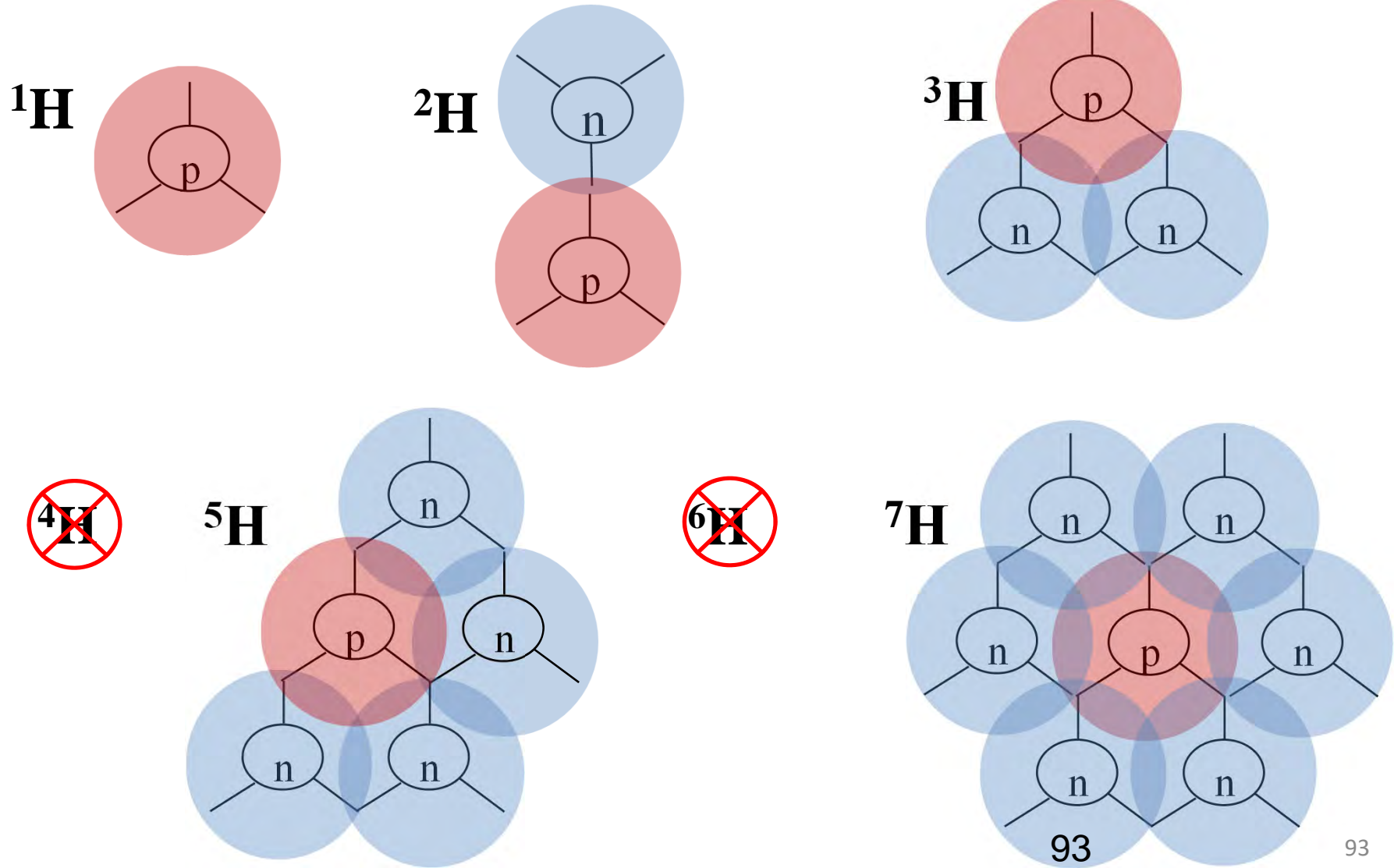
## Quantization

Rigid body quantization

**As a rigid body Nuclei can possess:**

- particle – hole excitations
- collective modes of excitations
  - Shape vibrations and fluctuations
  - Rotations
  - Isospin vibrations
  - Sissor fluctuations

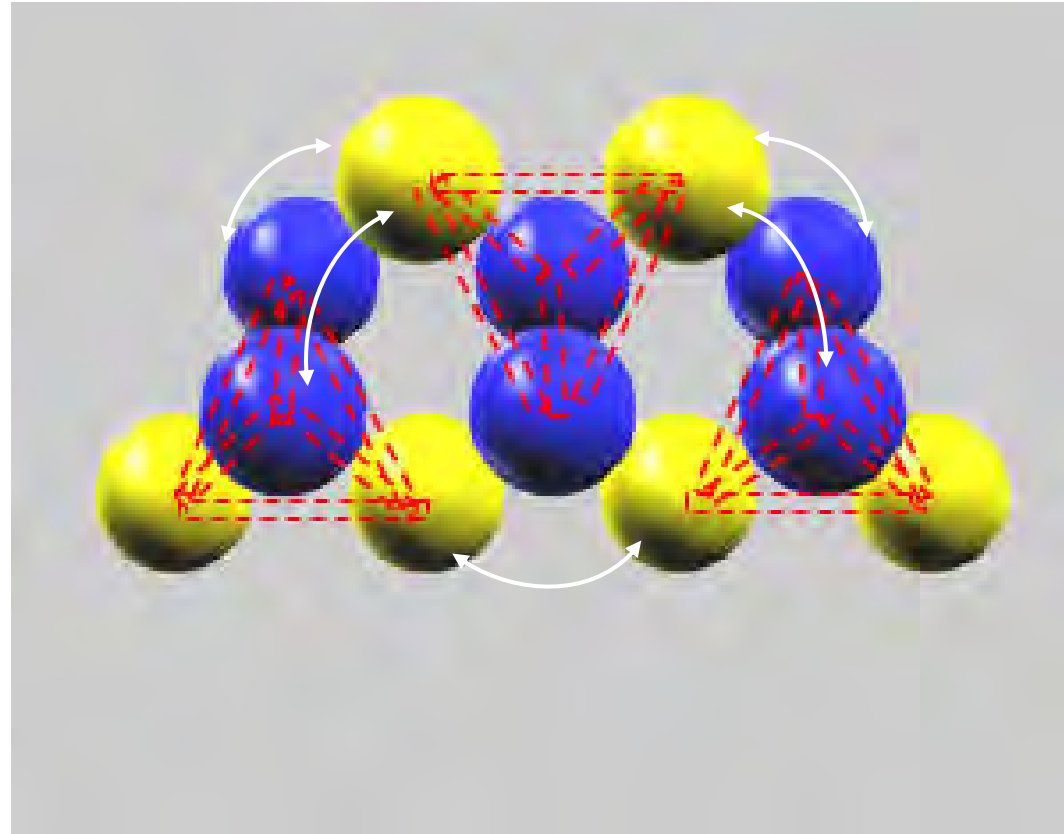
# Bound Hydrogen Isotopes



# $^{12}\text{C}$ Hoyle state

## Borromean nucleus

Loosely bound 3 **real**  $\alpha$ - cluster nucleus



Frames of  $\alpha$ -clusters are depicted as tetrahedrons. Neutrons of left and right  $\alpha$ -clusters are bound with protons of central  $\alpha$ -cluster (like in  $^8\text{He}$ ), and their 2 nearest protons are bound together.

# FCC-SCQM vs SM

What about spin-orbital coupling (SOC)?

## SOC

- Splitting of nuclear levels
- Lowering of levels with higher **J**
- Description of observed magic numbers of protons and neutrons

2, 8, 20, 28, 50, 82, 126

**Is it possible get the same numbers in FCC-SCQM?**

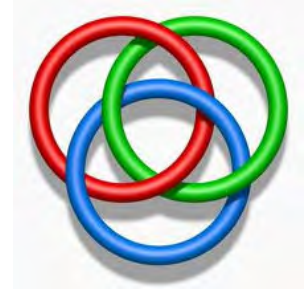
**YES !**

# Summary

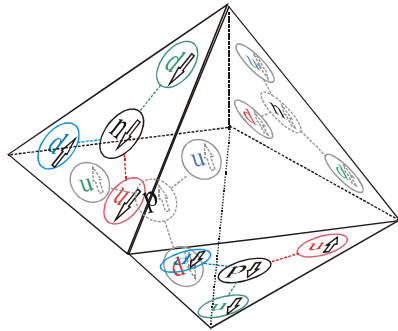
- Quarks play an explicit role in formation of the nuclear structure.
- Quark loops are building blocks of nuclear binding.
- Quarks and nucleons (protons and neutrons) inside nuclei are strongly correlated.
- ‘Halo’ nuclei – **fruits of quark-loop bindings**
- Effect of quark looping:  $E_{\text{sep}} < E_{\text{bound}}/A$



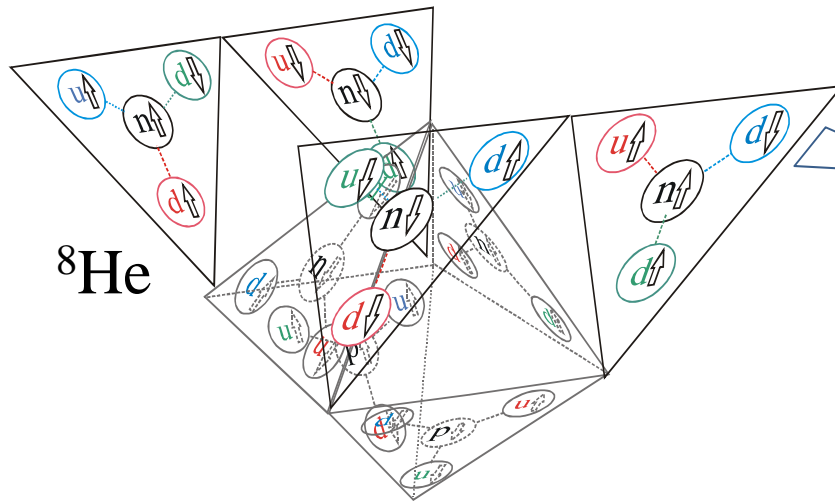
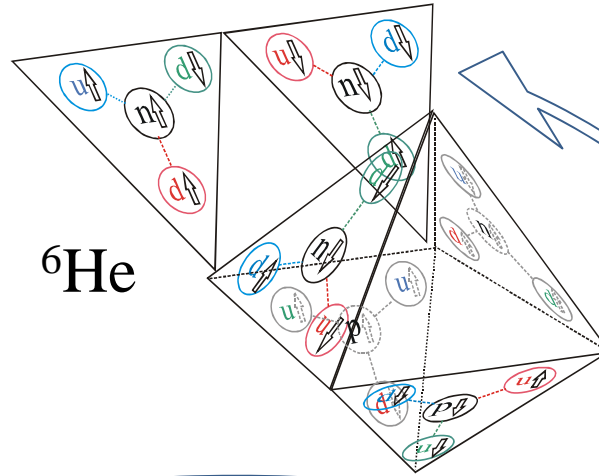
# Helium Isotopes Borromean Nuclei



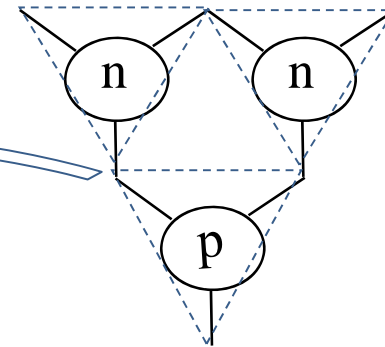
$^4\text{He}$   
Core



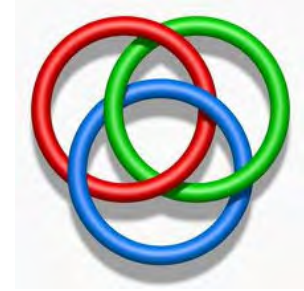
$^6\text{He}$



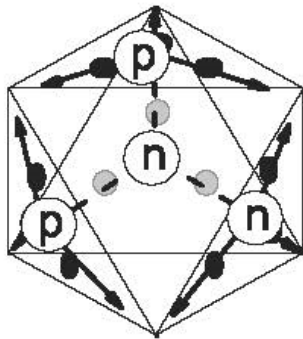
Quark loop



# Helium Isotopes Borromean Nuclei



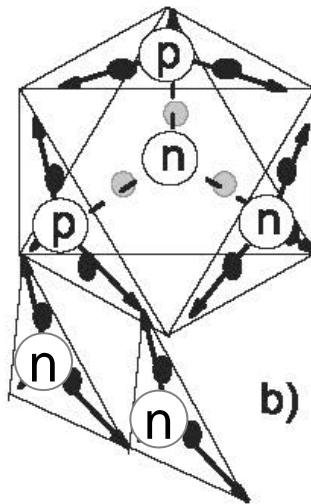
${}^4\text{He}$   
Core



a)

$R = 1.57 \text{ fm}$   
 $R_{\text{exp}} = 1.6 \text{ fm}$

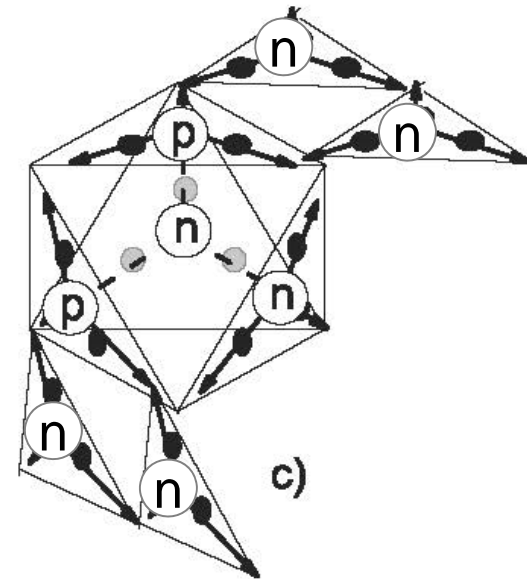
${}^6\text{He}$



b)

$R = 2.2 \text{ fm}$   
 $R_{\text{exp}} = 2.45 \text{ fm}$

${}^8\text{He}$

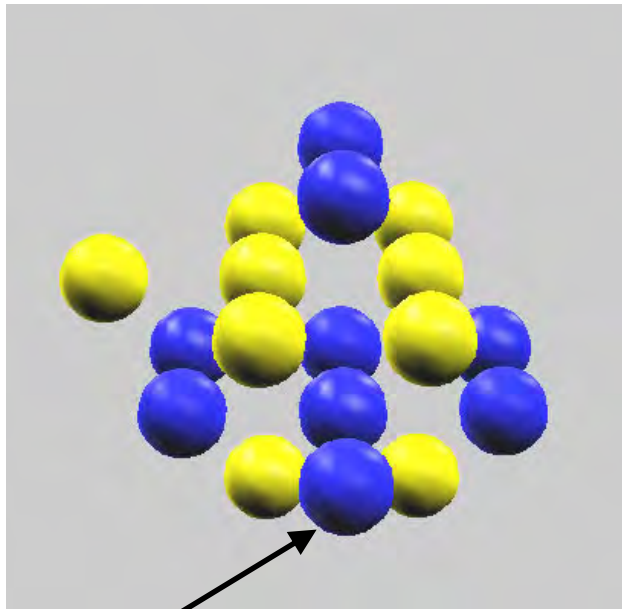


c)

$R = 2.4 \text{ fm}$   
 $R_{\text{exp}} = 2.53 \text{ fm}$

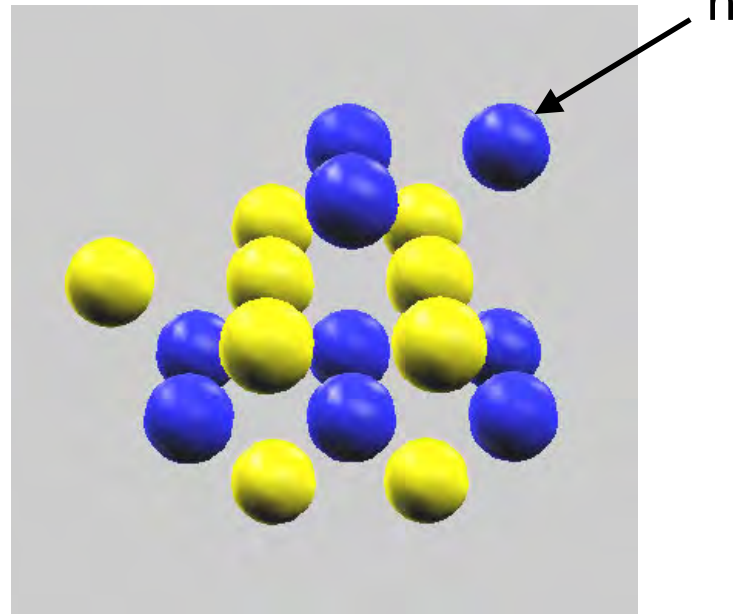
# Fluorine Isomers

$^{18}\text{F}_m$   
 $t_{1/2} \sim 200 \text{ ns}$



n  $5^+$

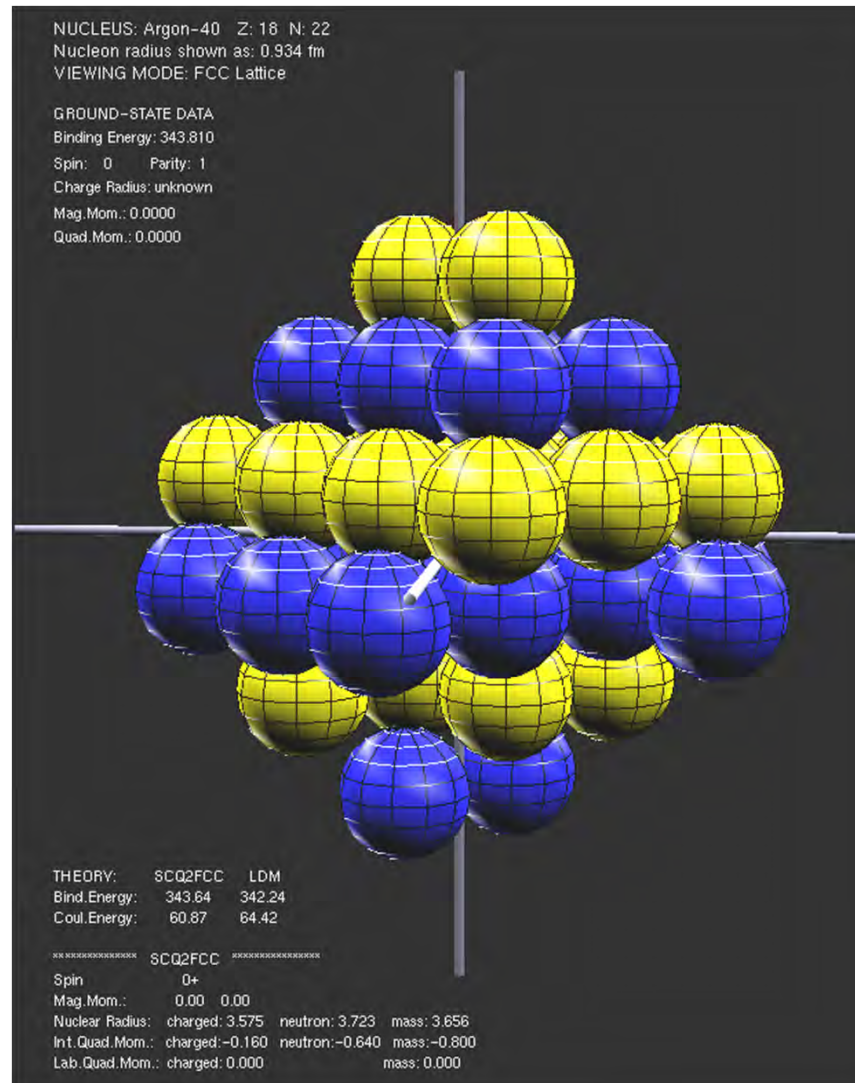
$^{18}\text{F}$   
 $t_{1/2} \sim 110 \text{ min}$



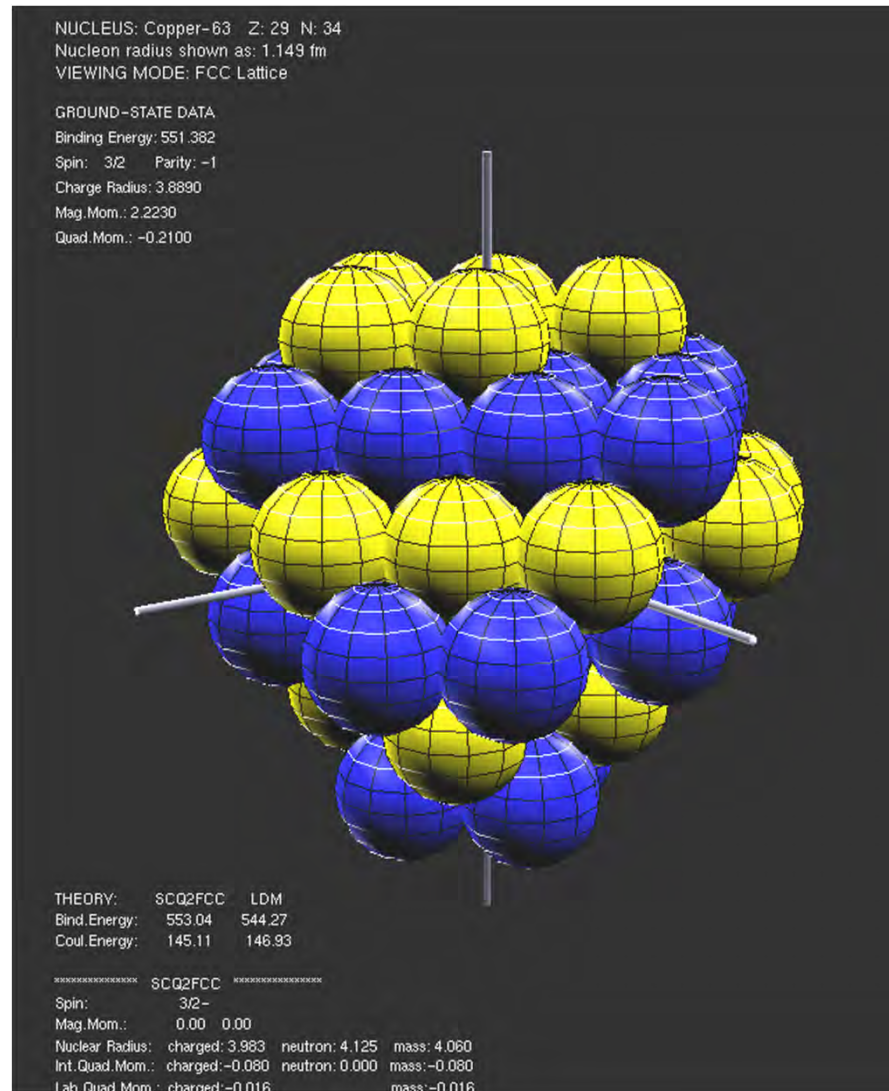
$1^+$

protons – yellow  
neutrons - blue

# $^{40}\text{Ar}$

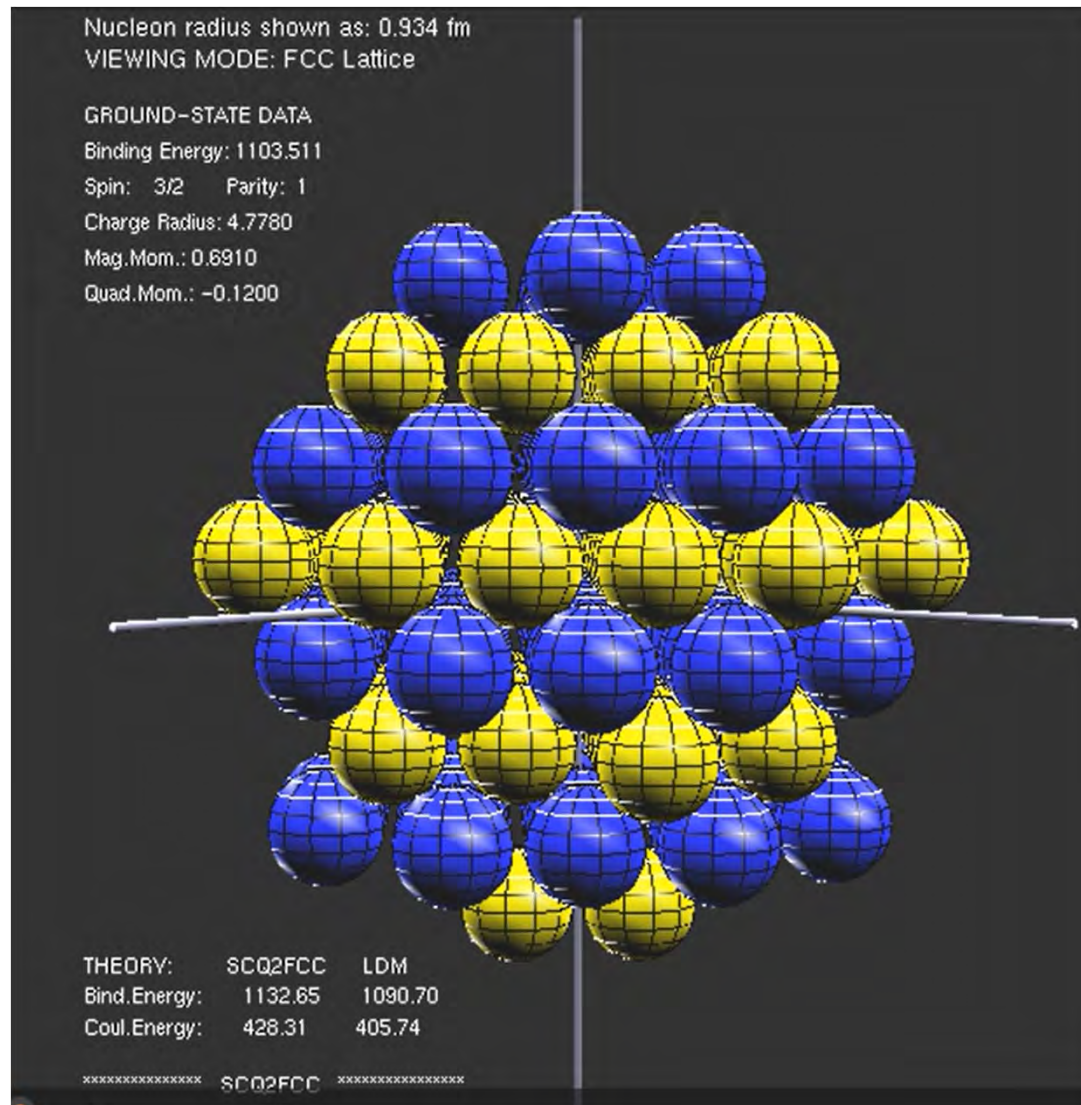


# $^{63}\text{Cu}$





# $^{131}\text{Xe}$



# $^{235}\text{U}$

