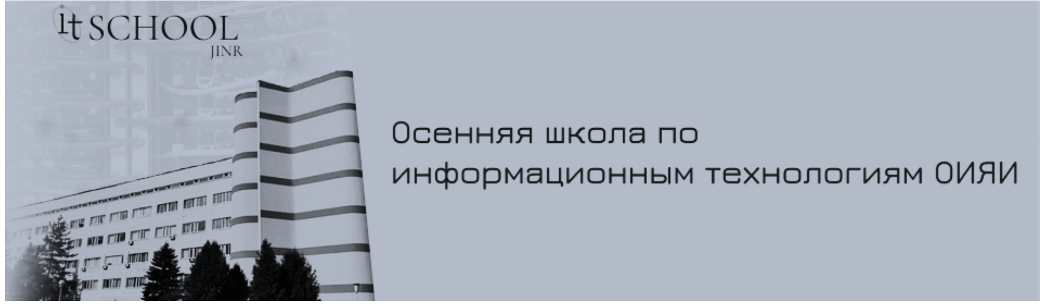


Machine Learning for
Neutron stars

Hovik Grigorian

JINR , AANL, *YSU*

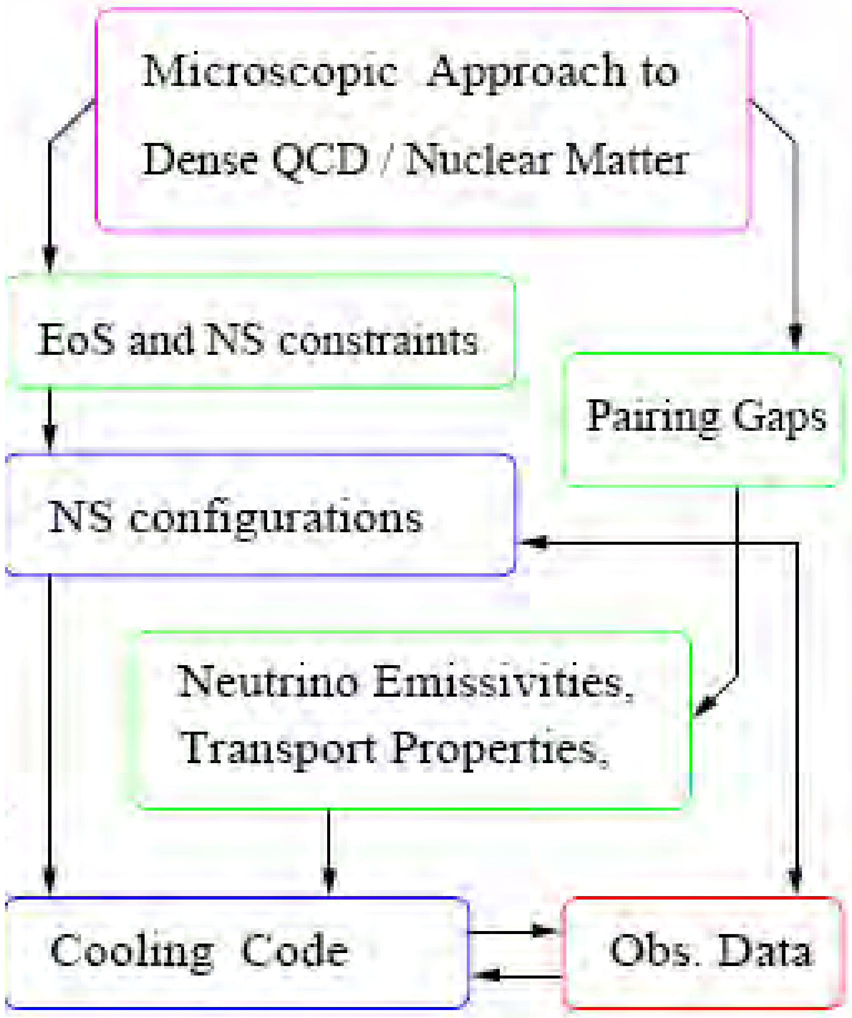
in cooperation with A. Ayriyan

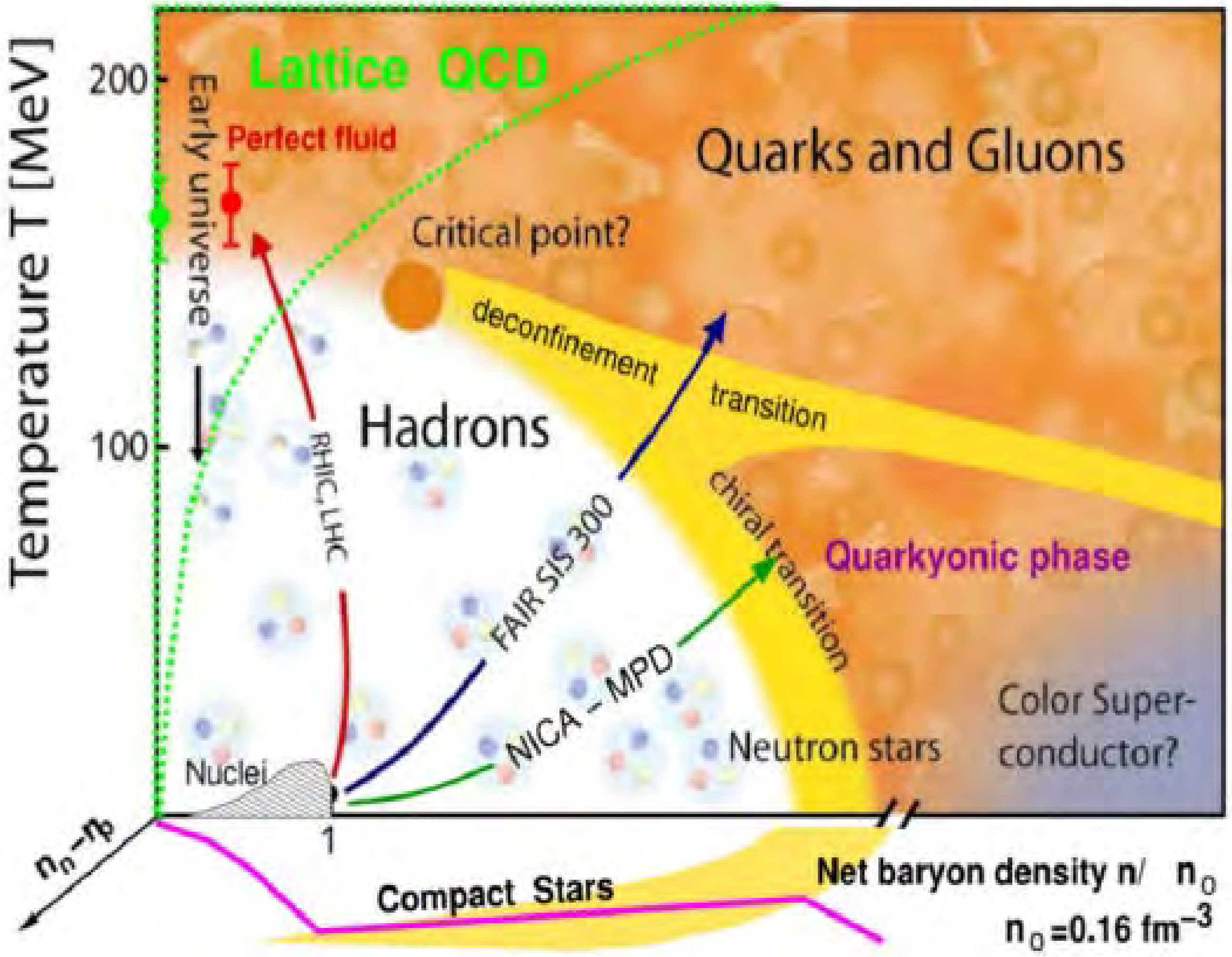
project

Content

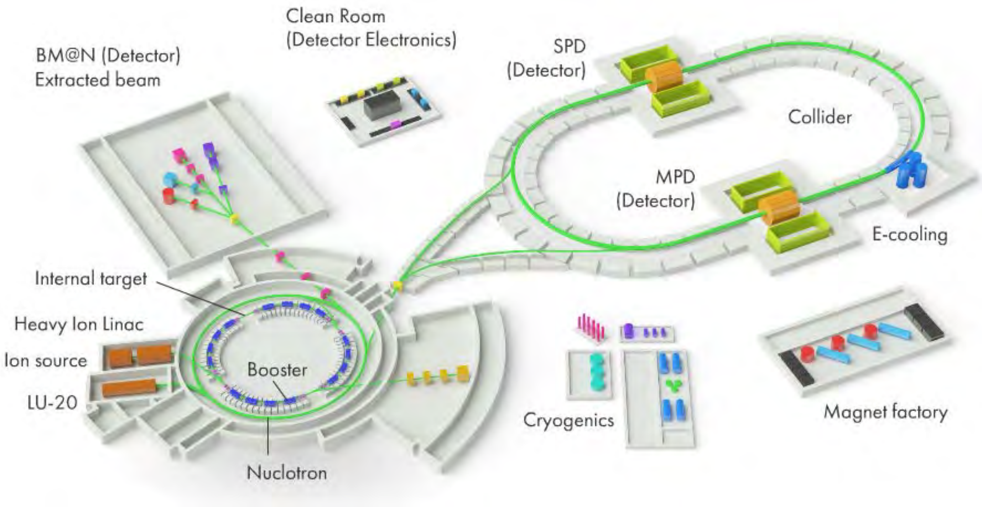
- Motivation from Neutron Star Physics
- Neutron Star Structures
- EoS & NS Configurations
- Thermal Evolution of NS
- Gravitational Wave Signal
- Comparison with observational data
- Machine Learning methods
- Conclusions

Phase Diagramm & Neutron Stars

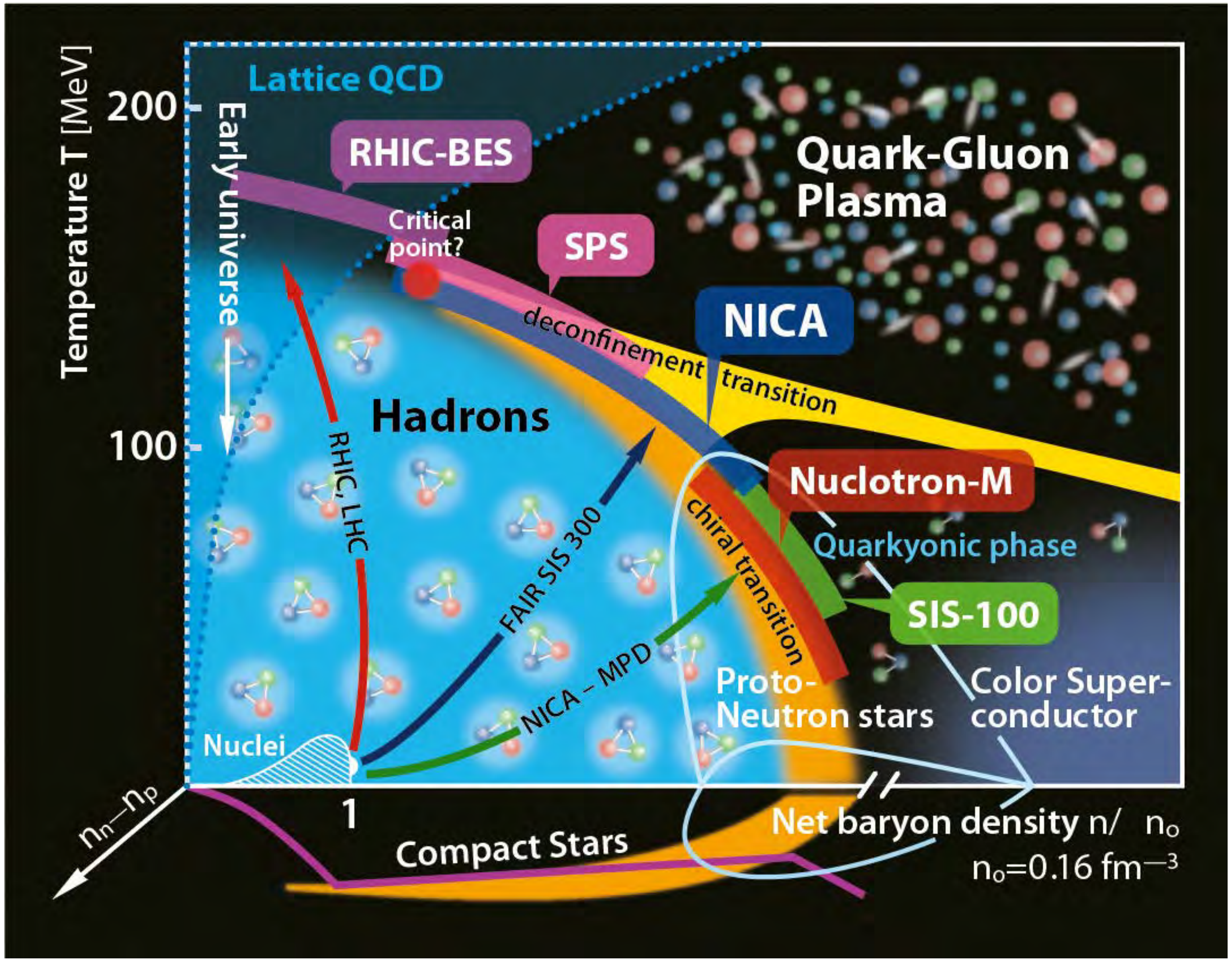




MPD Experiment. Physics



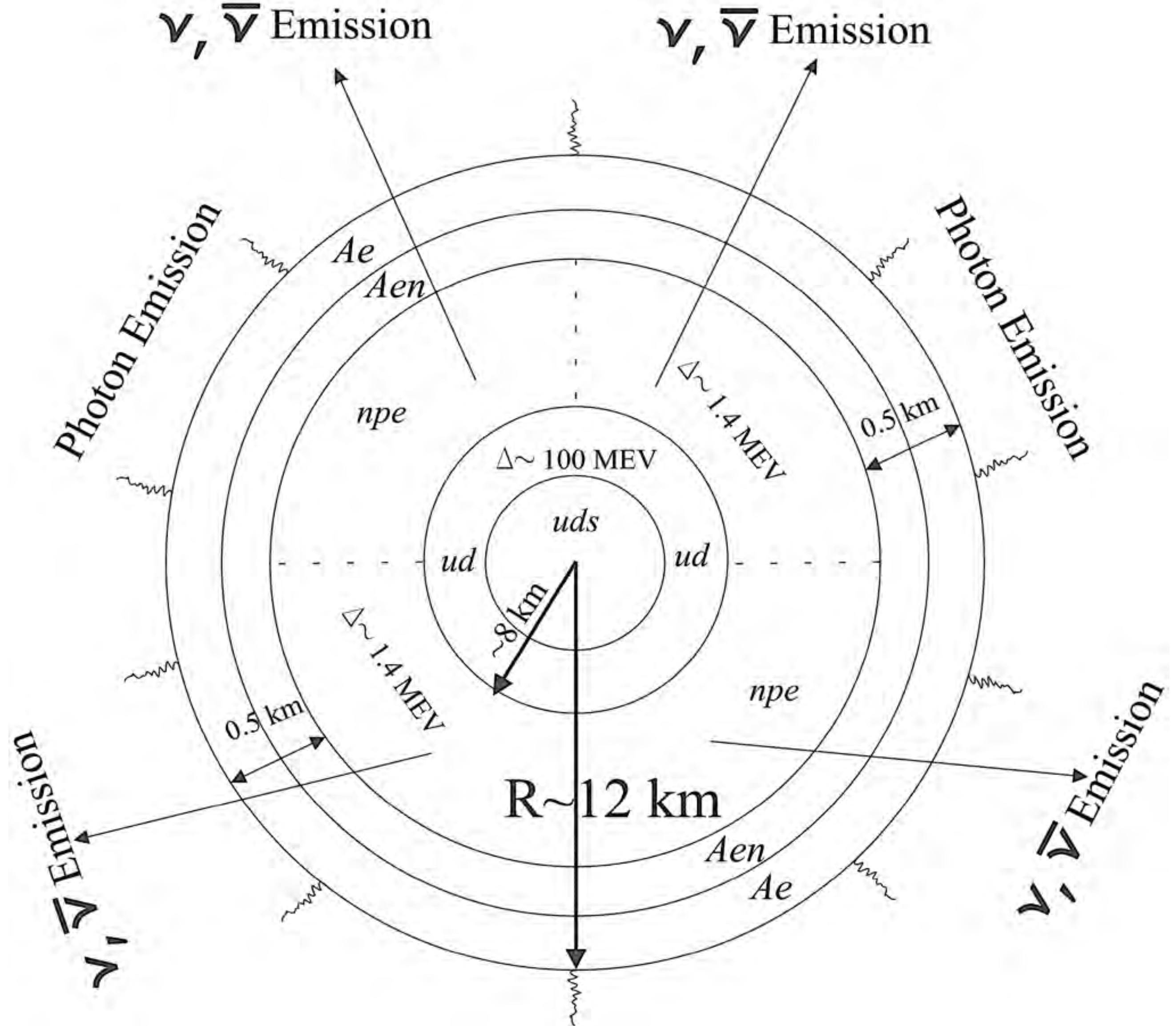
$\sqrt{s_{NN}}$ = 3-11 GeV



Facilities (chronologically):

- **AGS** in Brookhaven (heavy ions 1991-1999) - now injector for RHIC **3 ≤ $\sqrt{s_{NN}}$ ≤ 5 GeV**
- **SPS** at CERN - now mostly injector for LHC **6.4 ≤ $\sqrt{s_{NN}}$ ≤ 17.3 GeV**
- **RHIC** in Brookhaven (since 2000) - beam-energy-scan program **7.7 ≤ $\sqrt{s_{NN}}$ ≤ 200 GeV**
- **LHC** at CERN (since 2009) **$\sqrt{s_{NN}}$ = 2.76 - 5.6 TeV**
- **NICA** in Dubna - under construction **3 ≤ $\sqrt{s_{NN}}$ ≤ 9 GeV**
- **FAIR** in Darmstadt - under construction **3 ≤ $\sqrt{s_{NN}}$ ≤ 5 GeV**

Structure Of Hybrid Star



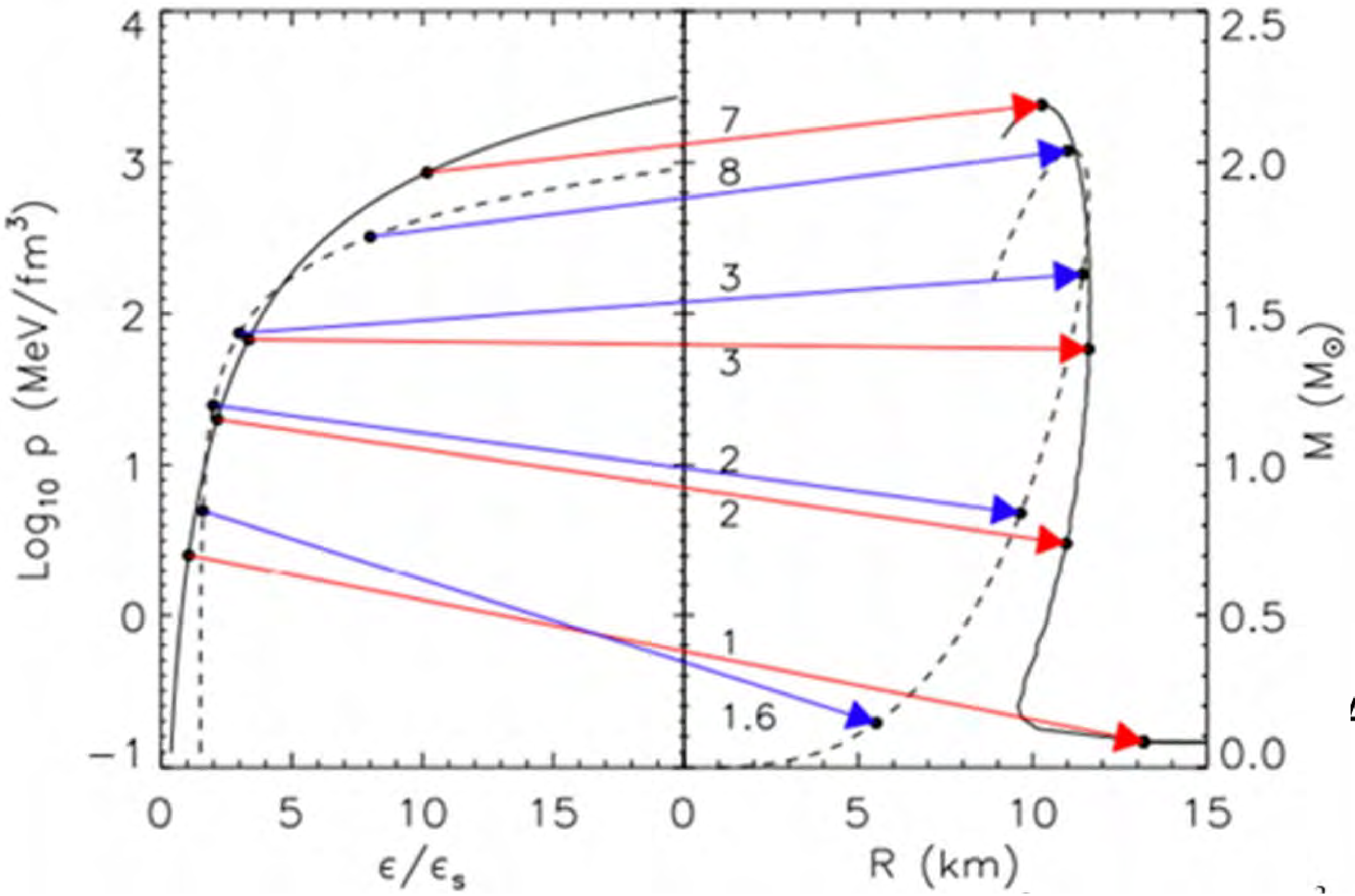
Static Neutron Star Mass and Radius

The structure and global properties of compact stars are obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations[1,2]:

$$
\begin{cases}
\dfrac{dP(r)}{dr} = -\dfrac{GM(r)\varepsilon(r)}{r^2}\dfrac{\left(1+\dfrac{P(r)}{\varepsilon(r)}\right)\left(1+\dfrac{4\pi r^3 P(r)}{M(r)}\right)}{\left(1-\dfrac{2GM(r)}{r}\right)}; \\
\dfrac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r); \\
\dfrac{dN_B(r)}{dr} = 4\pi r^2 \left(1-\dfrac{2GM(r)}{r}\right)^{-1/2} n(r).
\end{cases}
$$

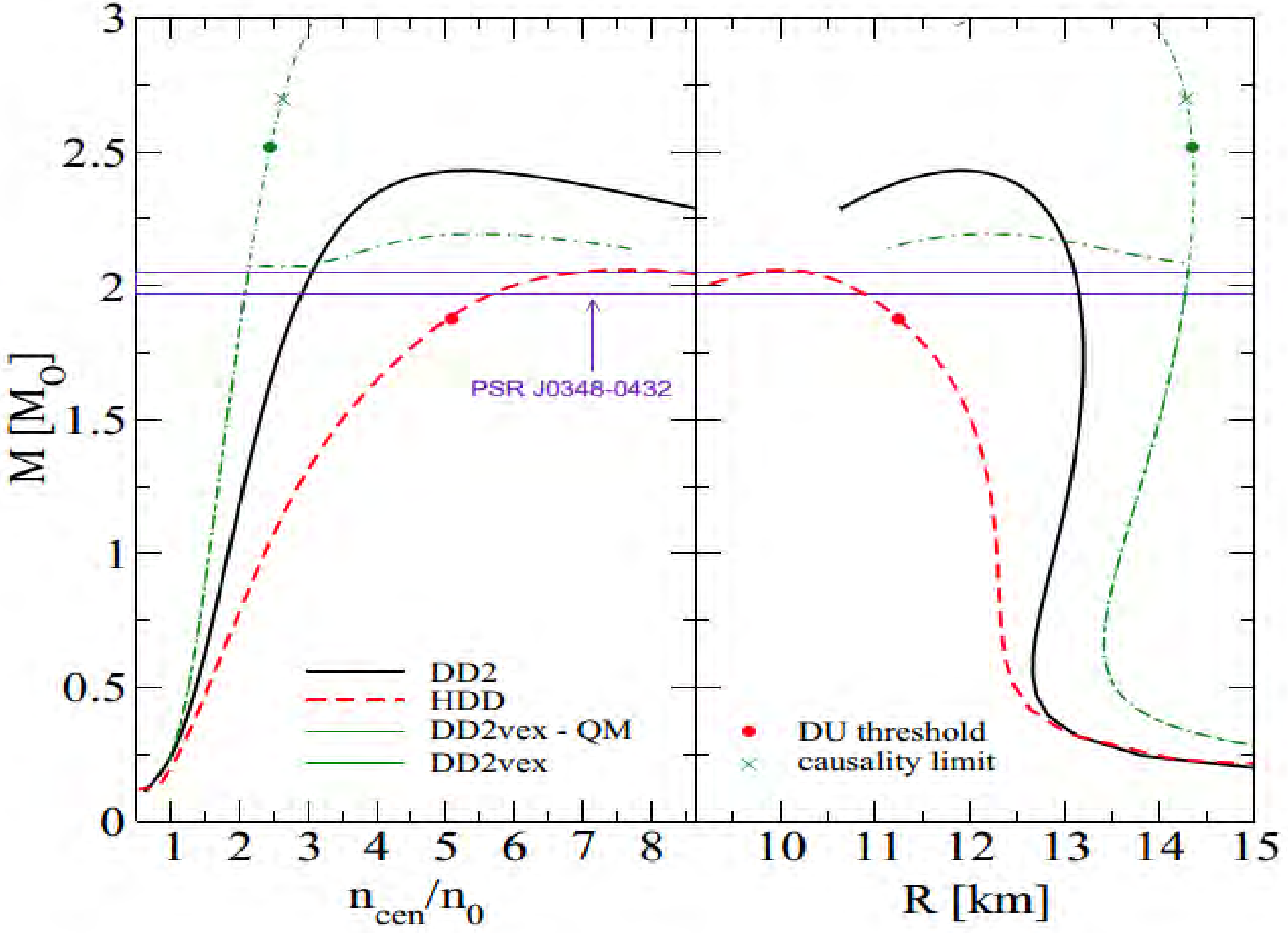
[1] R. C. Tolman, Phys. Rev. **55**, 364 (1939). [2] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. **55**, 374 (1939).

EoS vs. Mass Radious of NS

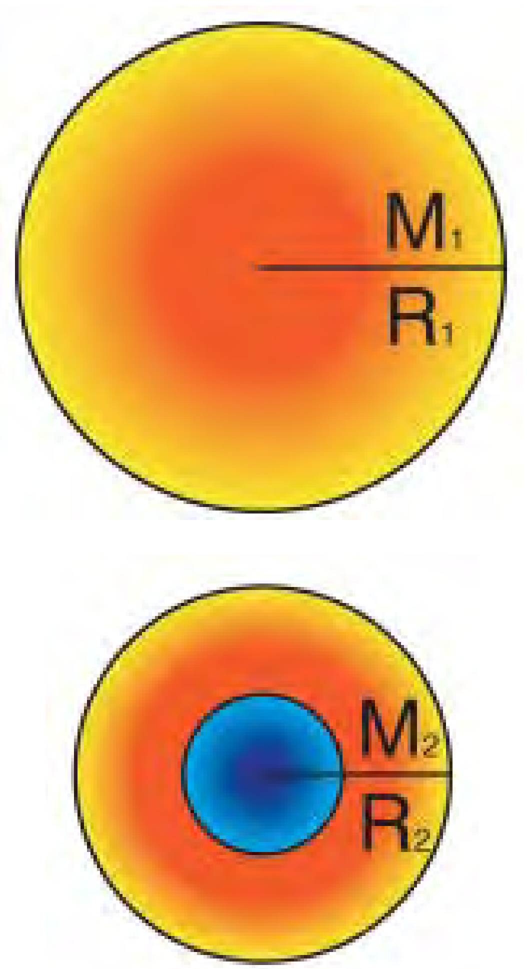


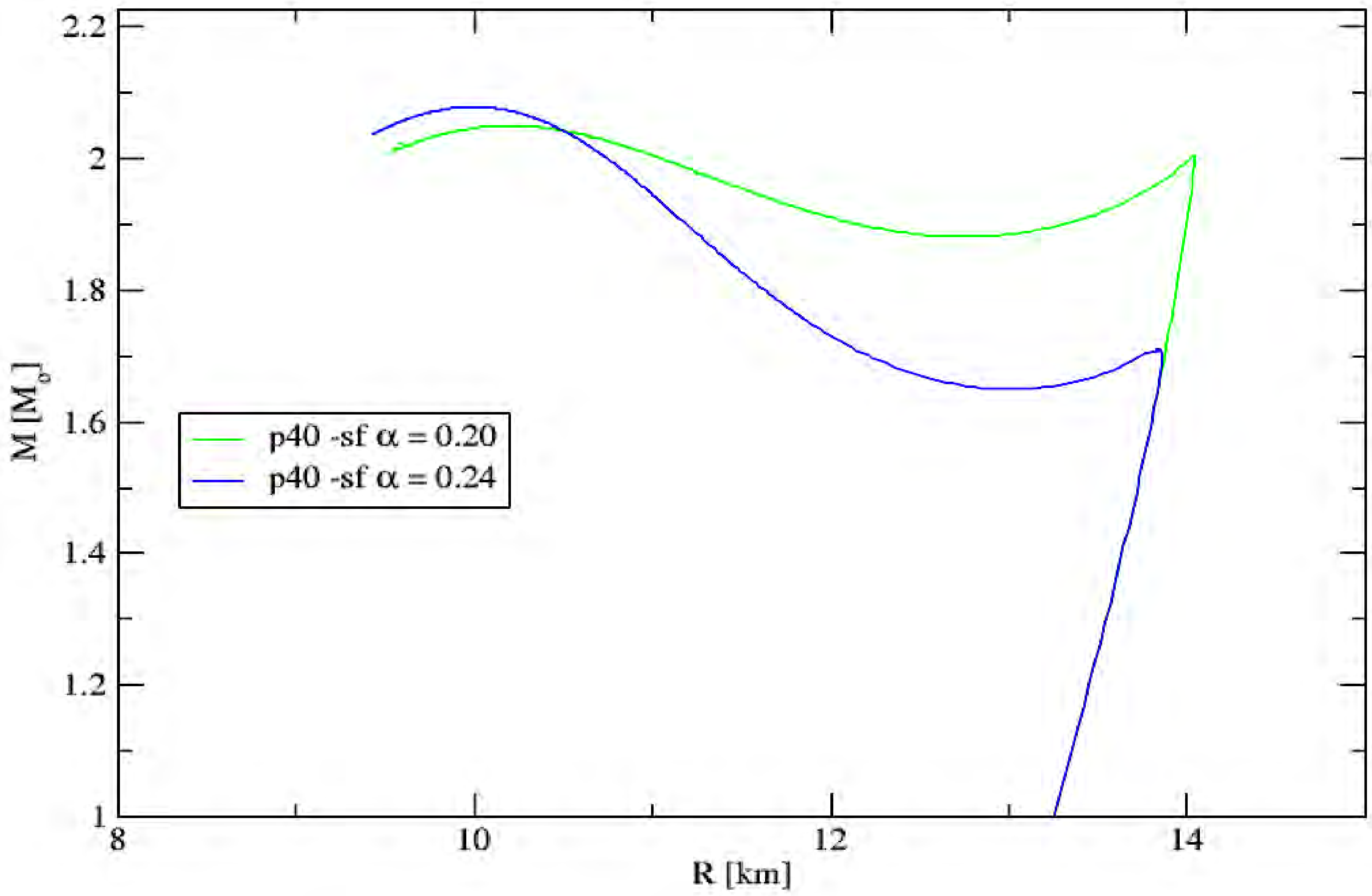
Lattimer,
Annu. Rev. Nucl. Part. Sci. 62, 485 (2012)
arXiv: 1305.3510

Stability of stars
HDD, DD2 & DDvex-NJL EoS models

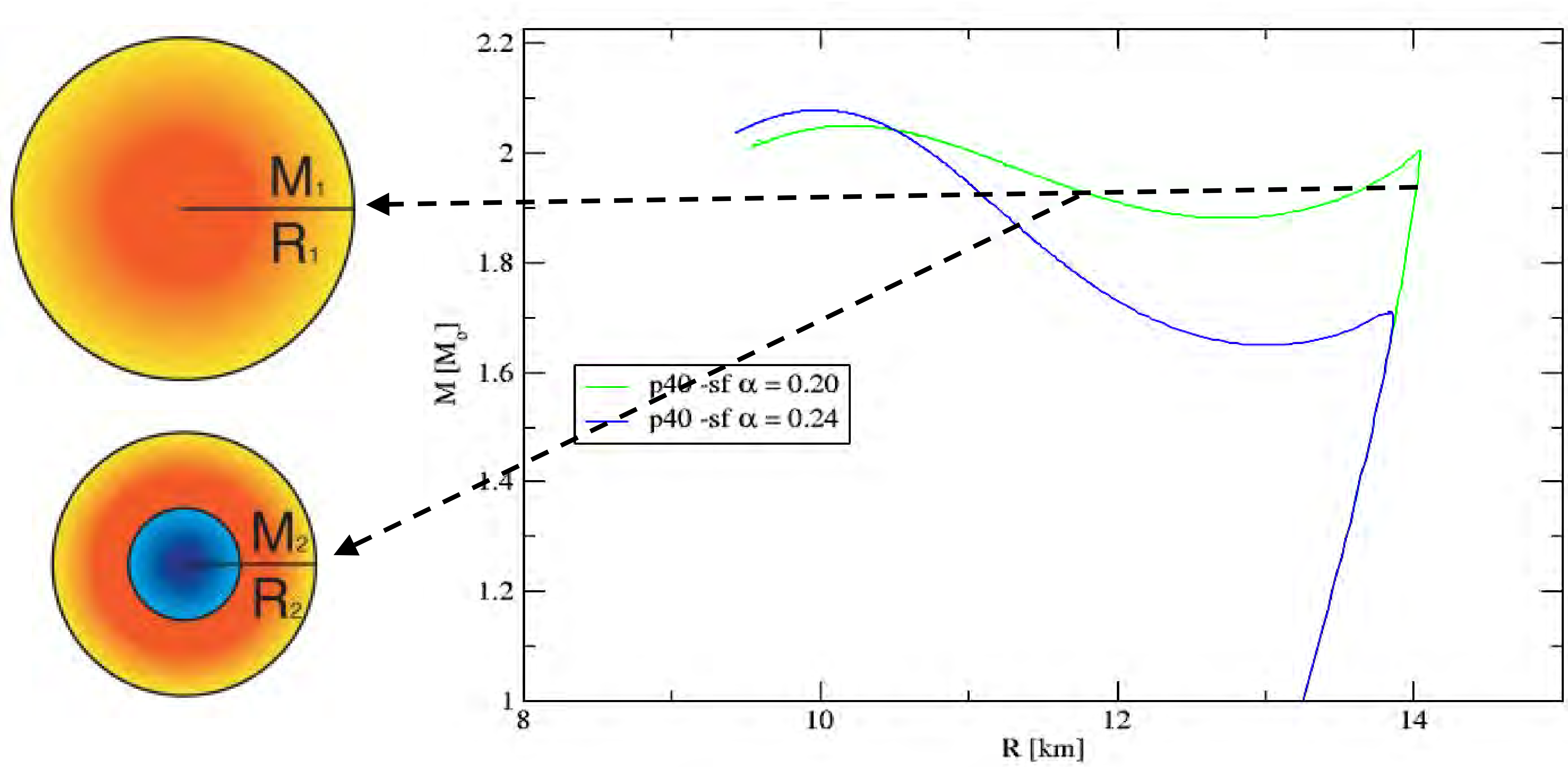


High Mass Twin CS

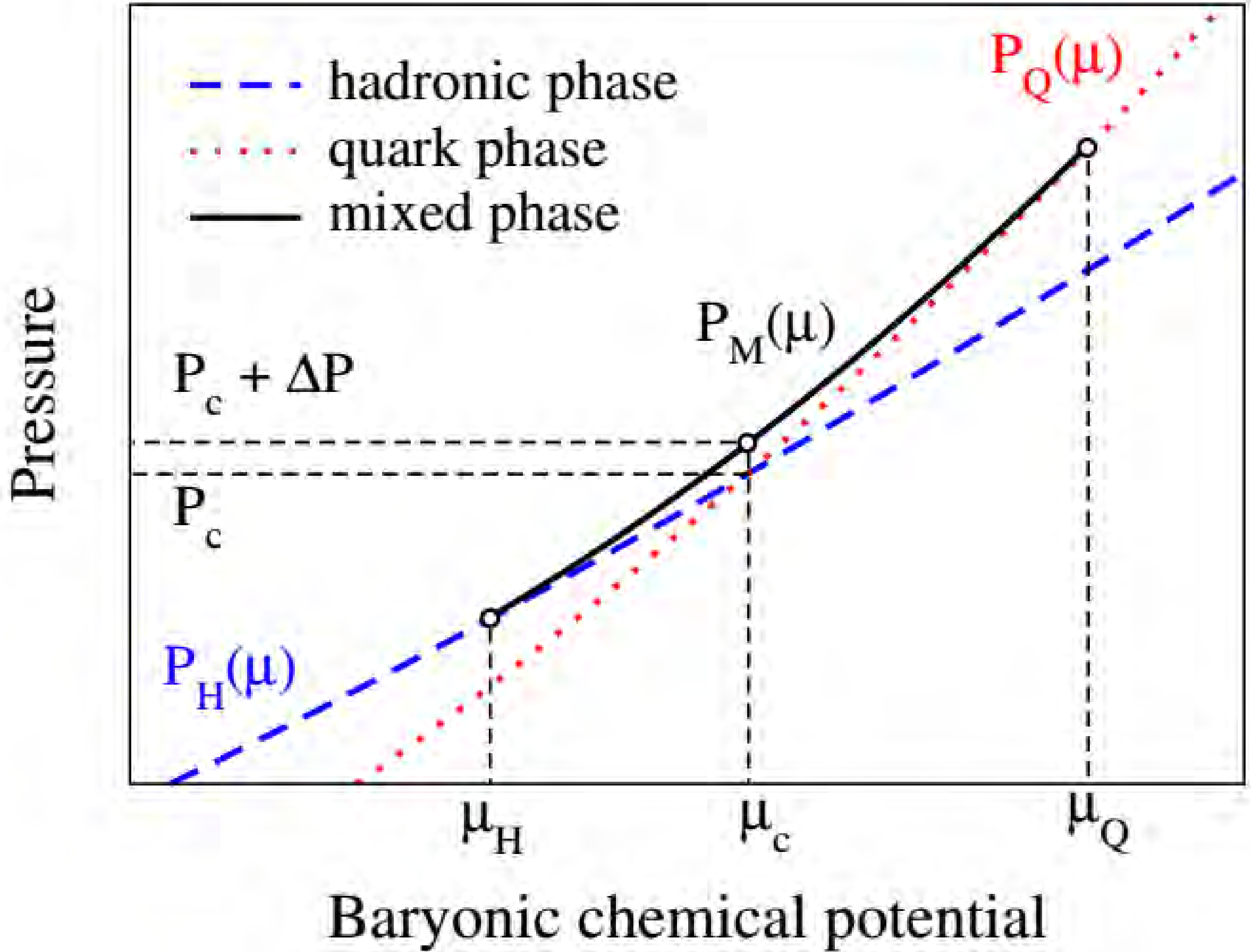




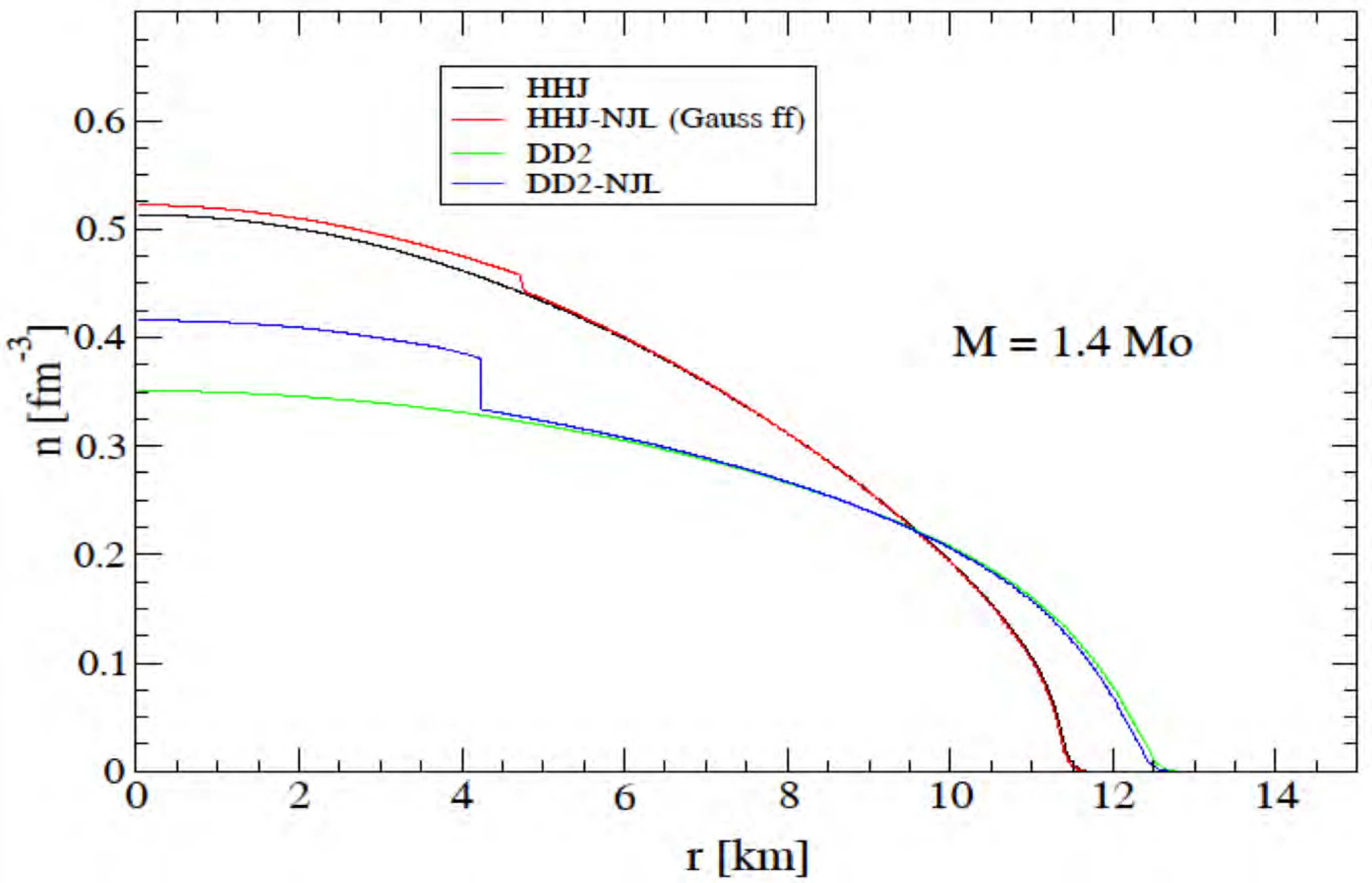
High Mass Twin CS



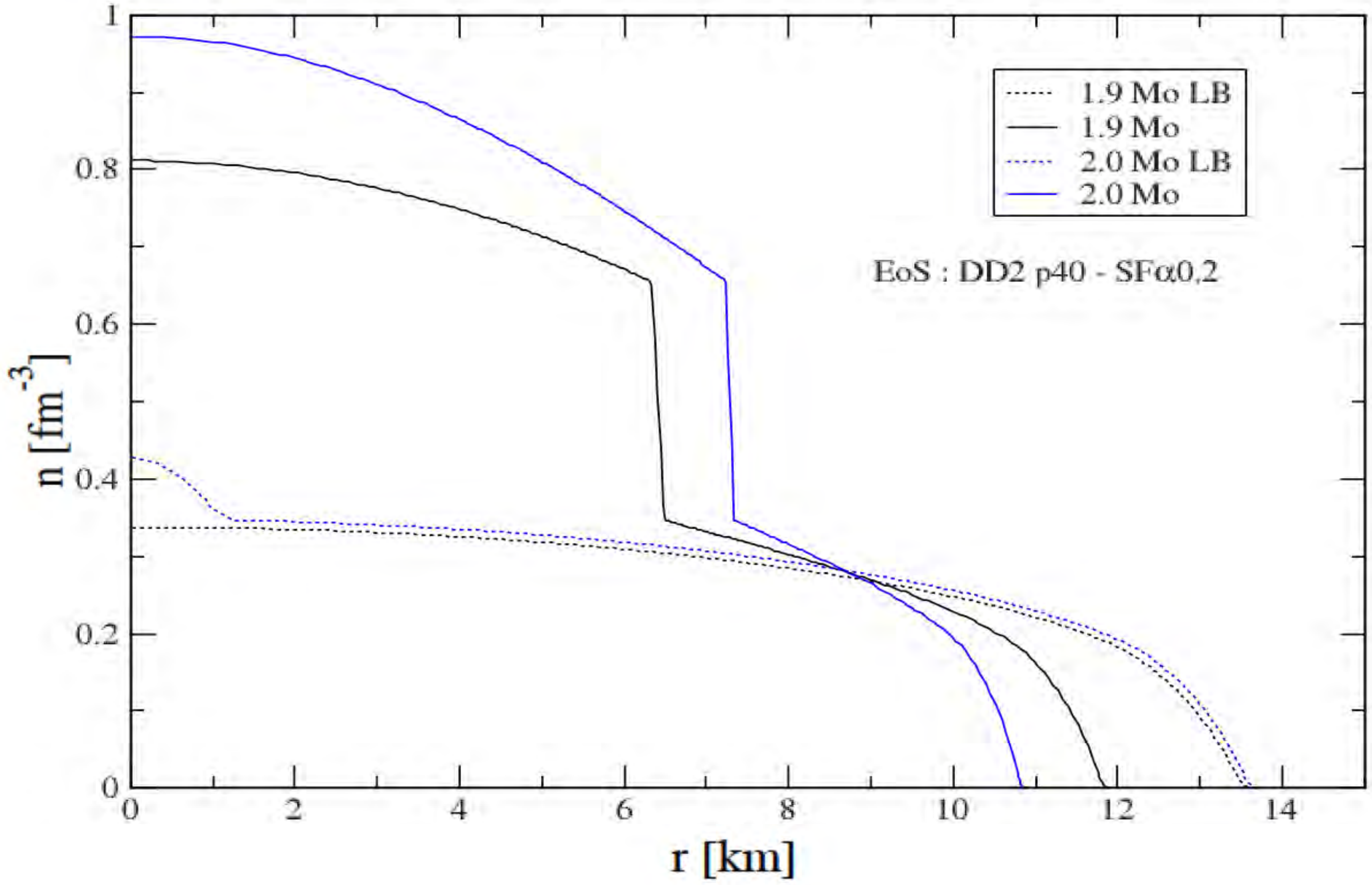
Mixed Phase in Quark-Hadron Phase Transition



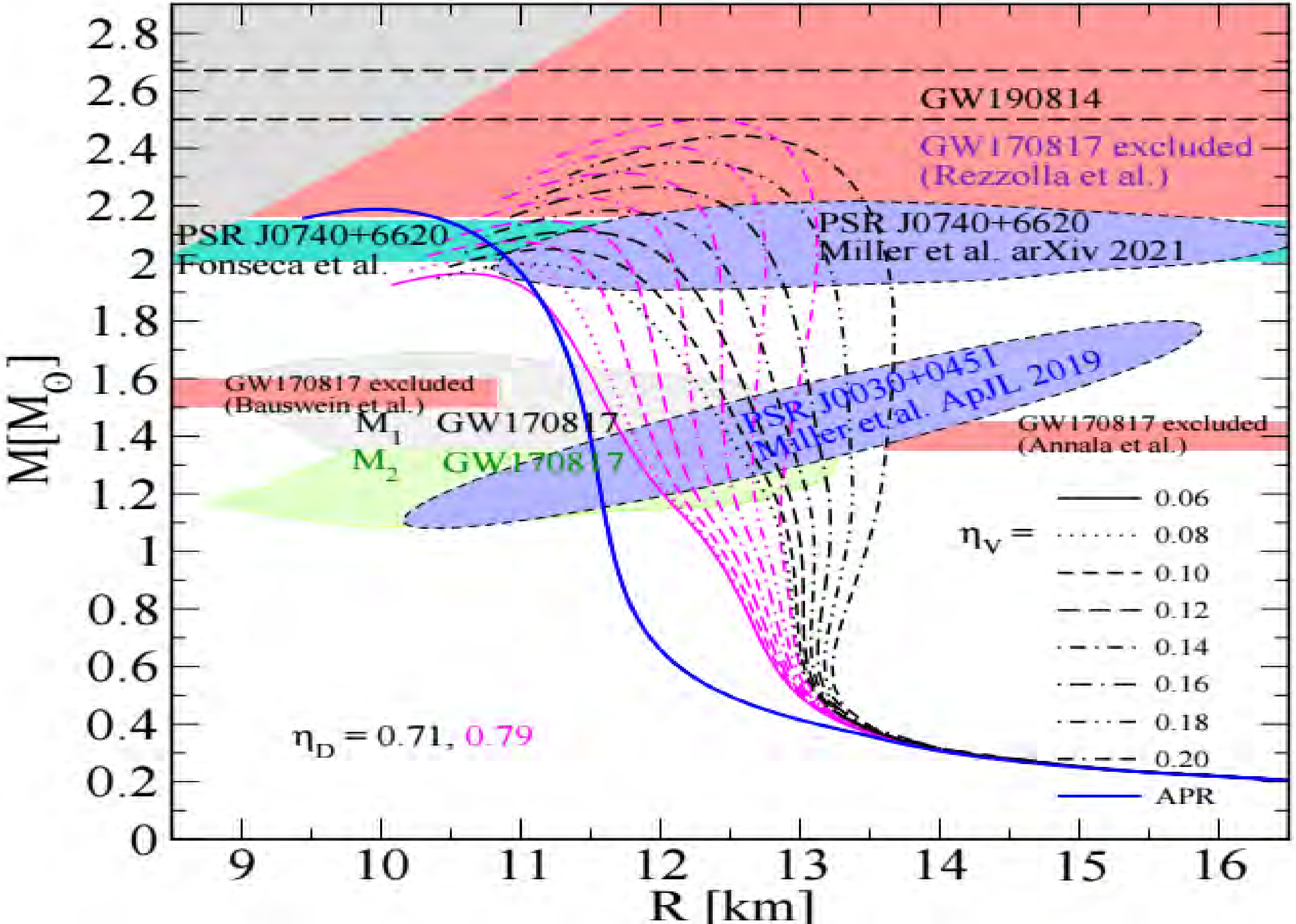
Different Configurations with the same NS mass



Different Configurations with the same NS mass



Modern MR Data and Models



Thermal Evolution

The energy flux per unit time **l(r)** through a spherical slice at distance r from the center is:

$$l(r) = -4\pi r^2 k(r) \frac{\partial(Te^{\Phi})}{\partial r} e^{-\Phi} \sqrt{1 - \frac{2M}{r}}$$

The equations for energy balance and thermal energy transport are:

$$\frac{\partial}{\partial N_B}(le^{2\Phi}) = -\frac{1}{n}(\epsilon_\nu e^{2\Phi} + c_V \frac{\partial}{\partial t}(Te^{\Phi}))$$

$$\frac{\partial}{\partial N_B}(Te^{\Phi}) = -\frac{1}{k}\frac{le^{\Phi}}{16\pi^2 r^4 n}$$

where n = n(r) is the baryon number density, NB = NB(r) is the total baryon number in the sphere with radius r

$$\frac{\partial N_B}{\partial r} = 4\pi r^2 n(1 - \frac{2M}{r})^{-1/2}$$

F.Weber: Pulsars as Astro Labs. (1999);
D. Blaschke Grigorian, Voskresensky, A& A 368 (2001)561.

Equations for Cooling Evolution

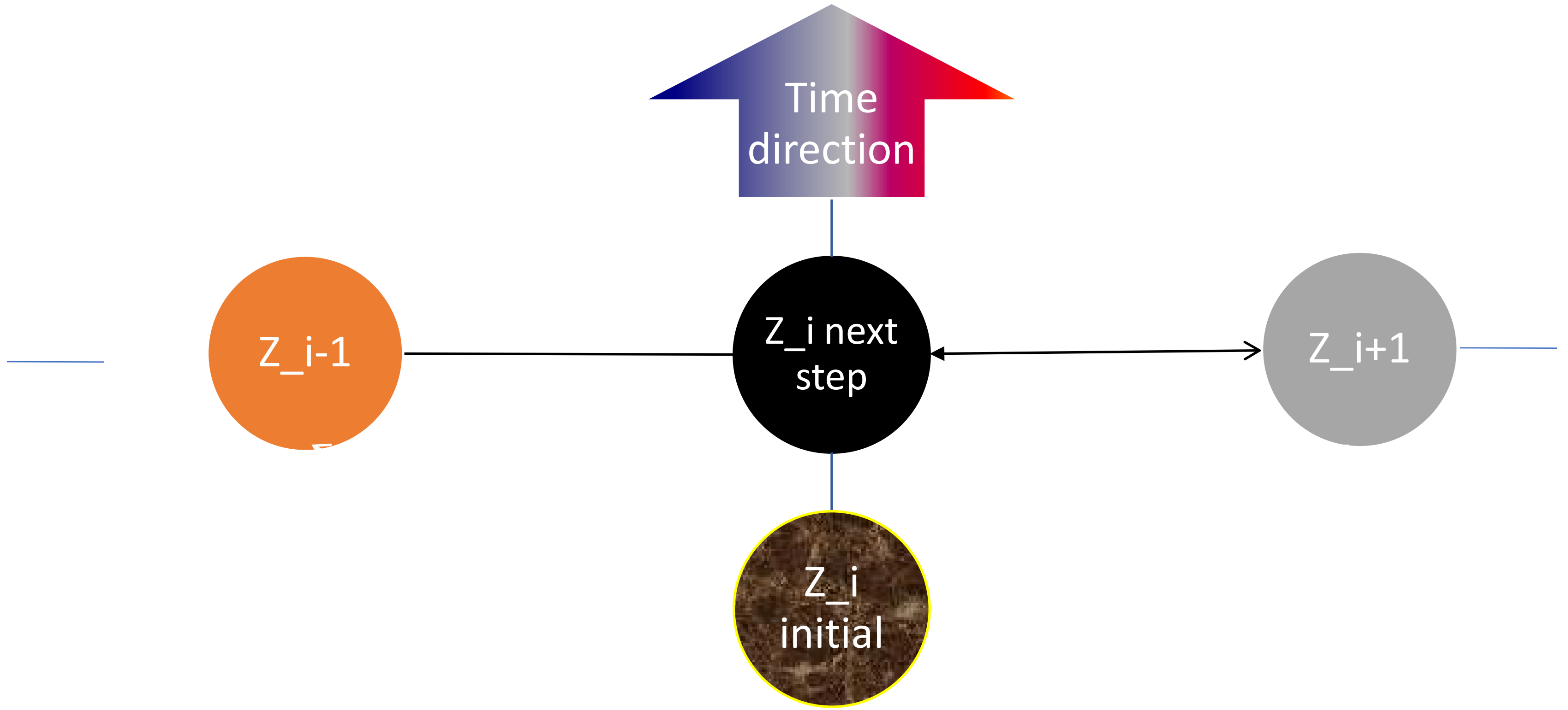
$$\begin{cases} \dfrac{\partial z(\tau,a)}{\partial \tau} = A(z,a)\dfrac{\partial L(\tau,a)}{\partial a} + B(z,a) \\ L(\tau,a) = C(z,a)\dfrac{\partial z(\tau,a)}{\partial a} \end{cases}$$

$$z(\tau,a) = \log T(\tau,a)$$

$$L_{i\pm1/2} = \pm\frac{C_i + C_{i\pm1}}{2}\frac{z_{i\pm1} - z_i}{\Delta a_{i-1/2(1\mp1)}}$$

$$\frac{\partial L_i}{\partial a} = 2\frac{L_{i+1/2} - L_{i-1/2}}{\Delta a_i + \Delta a_{i-1}}$$

Finite difference scheme



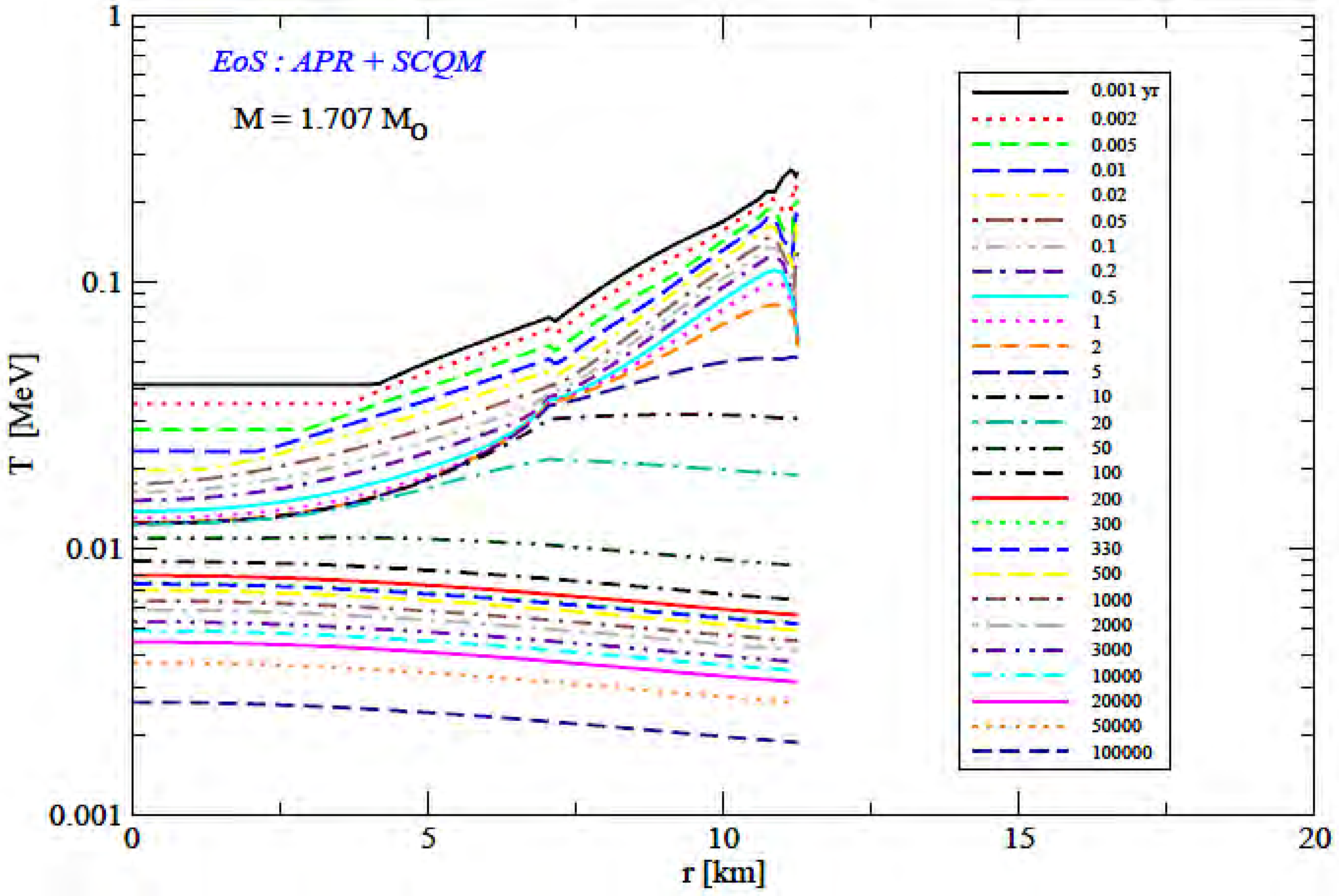
$$\alpha_{i,j-1} z_{i+1,j} + \beta_{i,j-1} z_{i,j} + \gamma_{i,j-1} z_{i-1,i} = \delta_{i,j-1}$$

Finite difference scheme

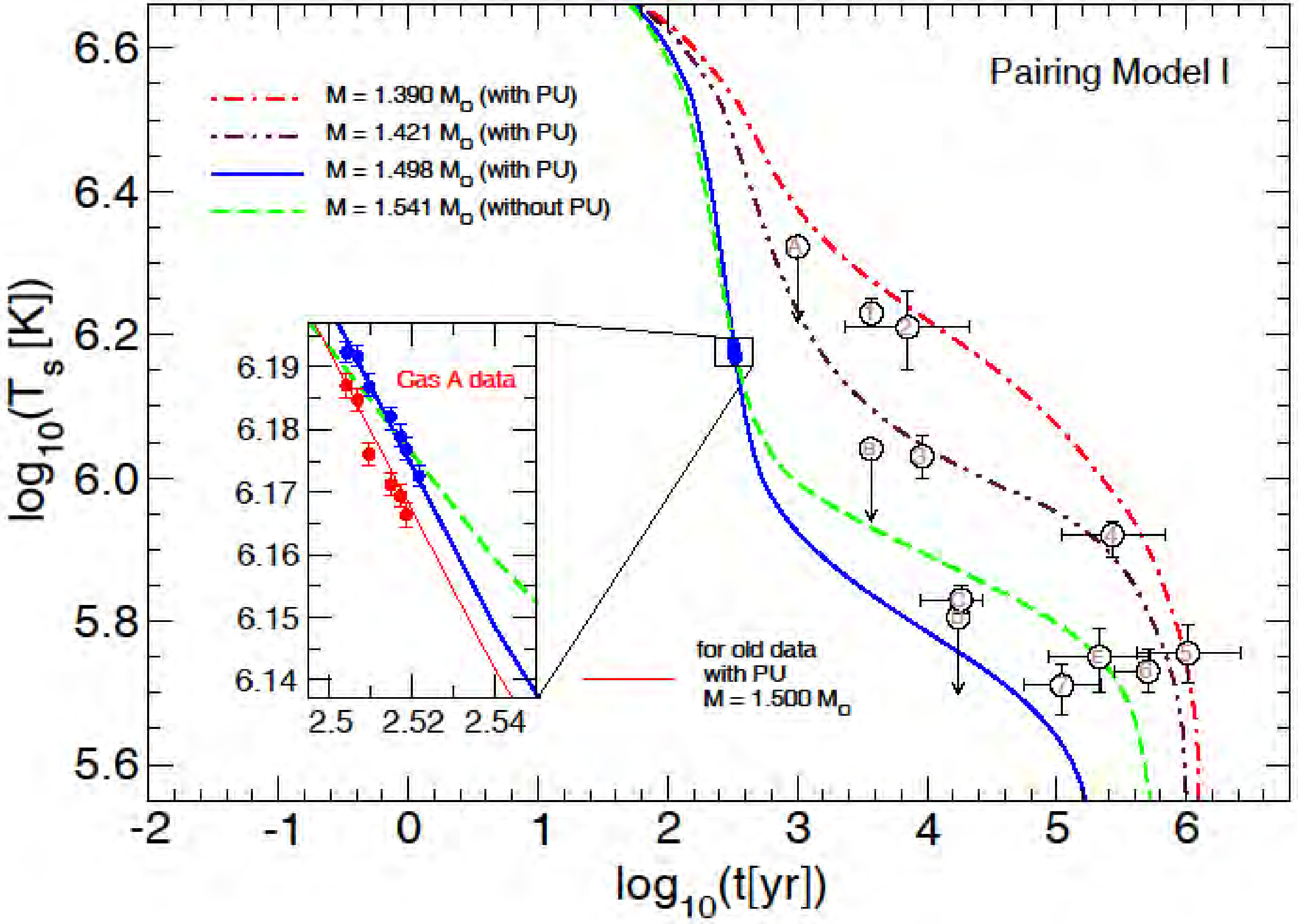
$$\begin{pmatrix} \beta_{0,j-1} & \alpha_{0,j-1} & & & 0 \\ \gamma_{1,j-1} & * & * & & \\ & * & * & * & \\ & & * & * & \alpha_{N-1,j-1} \\ 0 & & & \gamma_{N,j-1} & \beta_{N,j-1} \end{pmatrix} \begin{pmatrix} z_{0,j} \\ z_{1,j} \\ * \\ * \\ z_{N,j} \end{pmatrix} = \begin{pmatrix} \delta_{0,j-1} \\ \delta_{1,j-1} \\ * \\ * \\ \delta_{N,j-1} \end{pmatrix}$$

$$\alpha_{i,j-1} z_{i+1,j} + \beta_{i,j-1} z_{i,j} + \gamma_{i,j-1} z_{i-1,i} = \delta_{i,j-1}$$

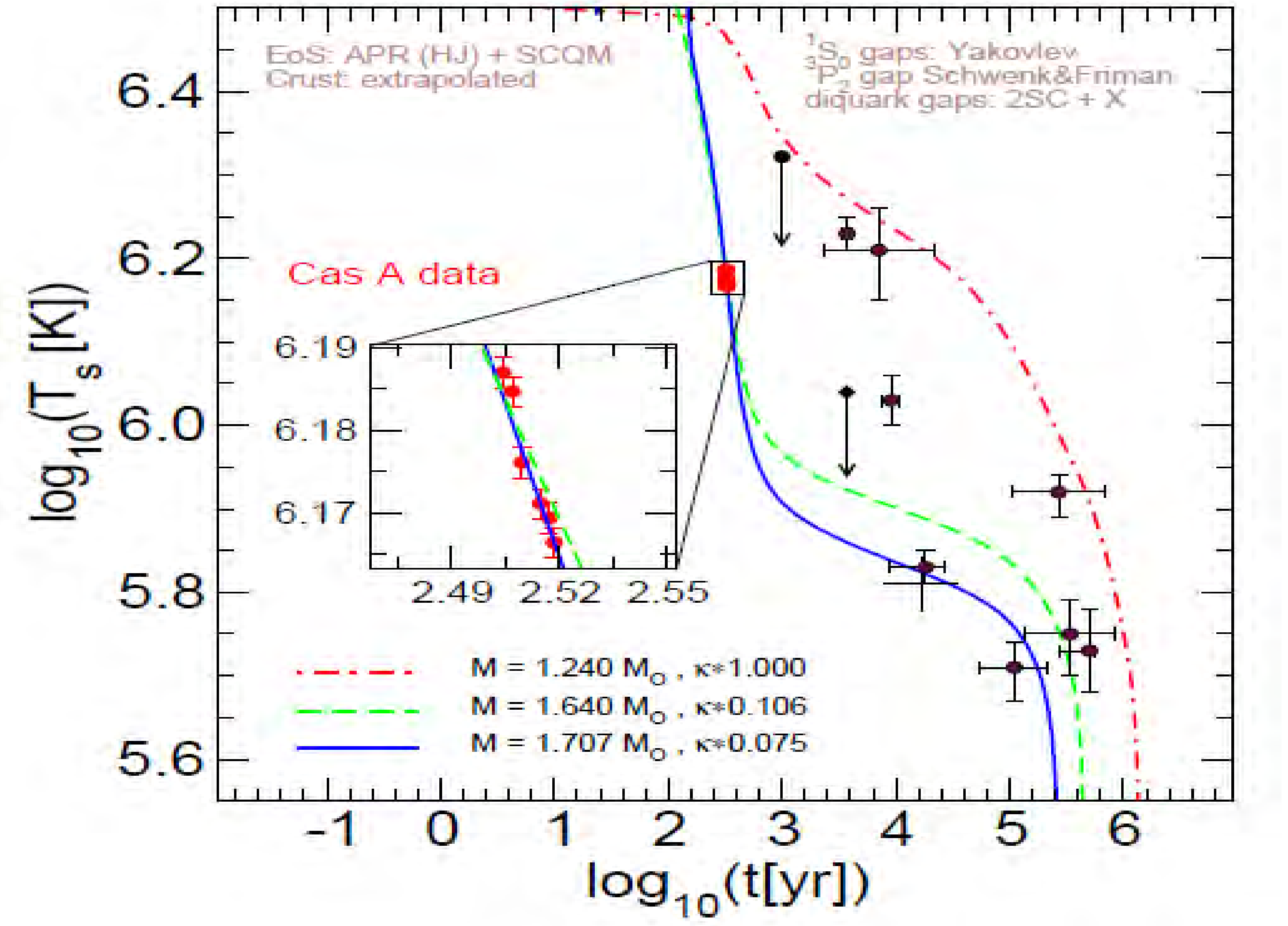
Temperature in the Hybrid Star Interior



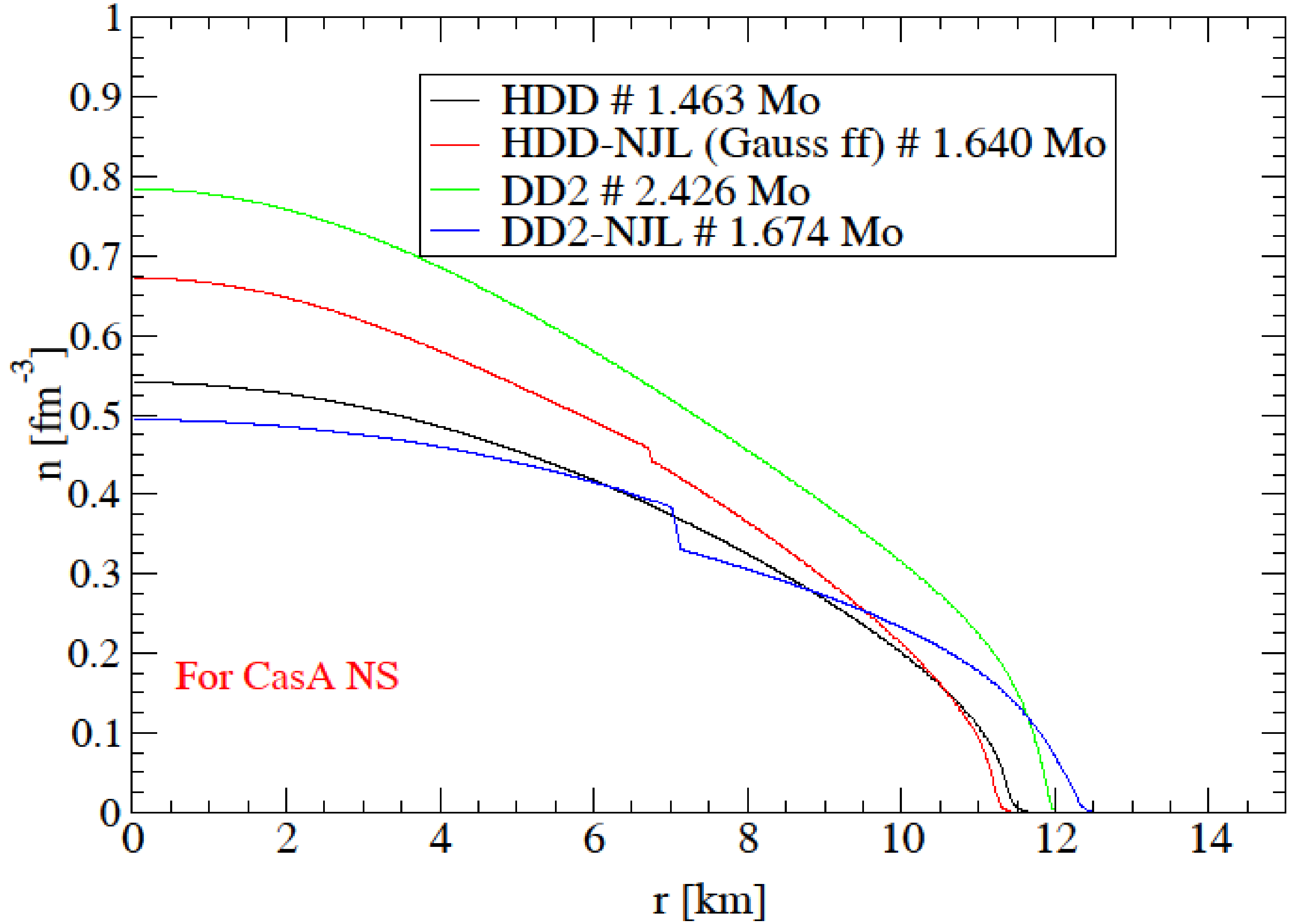
Cas A as an Hadronic Star



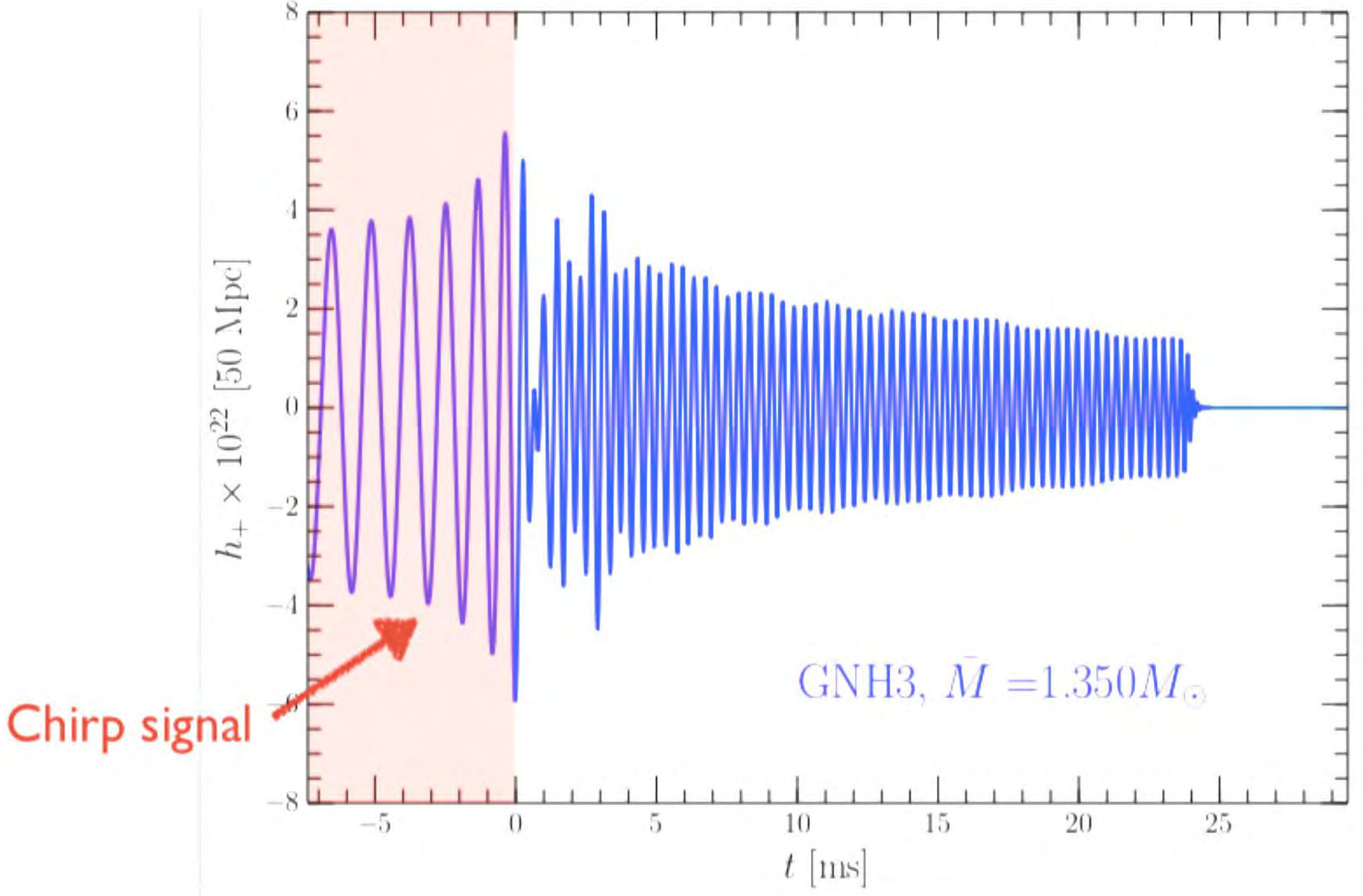
Cas A as an Hybrid Star



Possible internal structure of CasA



Anatomy of the GW signal



Computing the love number and tidal deformability

Ansatz for the metric including a l=2 perturbation

$$\begin{aligned} ds^2 &= -e^{2\Phi(r)}\left[1+H(r)Y_{20}(\theta,\varphi)\right]dt^2 \\ &\quad +e^{2\Lambda(r)}\left[1-H(r)Y_{20}(\theta,\varphi)\right]dr^2 \\ &\quad +r^2\left[1-K(r)Y_{20}(\theta,\varphi)\right]\left(d\theta^2+\sin^2\theta d\varphi^2\right) \end{aligned}$$

Following Hinderer et al. 2010

Integrate standard TOV system:

$$\begin{aligned} e^{2\Lambda} &= \left(1-\frac{2m_r}{r}\right)^{-1}, \\ \frac{d\Phi}{dr} &= -\frac{1}{\epsilon+p}\frac{dp}{dr}, \\ \frac{dp}{dr} &= -(\epsilon+p)\frac{m_r+4\pi r^3 p}{r(r-2m_r)}, \\ \frac{dm_r}{dr} &= 4\pi r^2\epsilon. \end{aligned}$$

EoS to be provided ε(p)

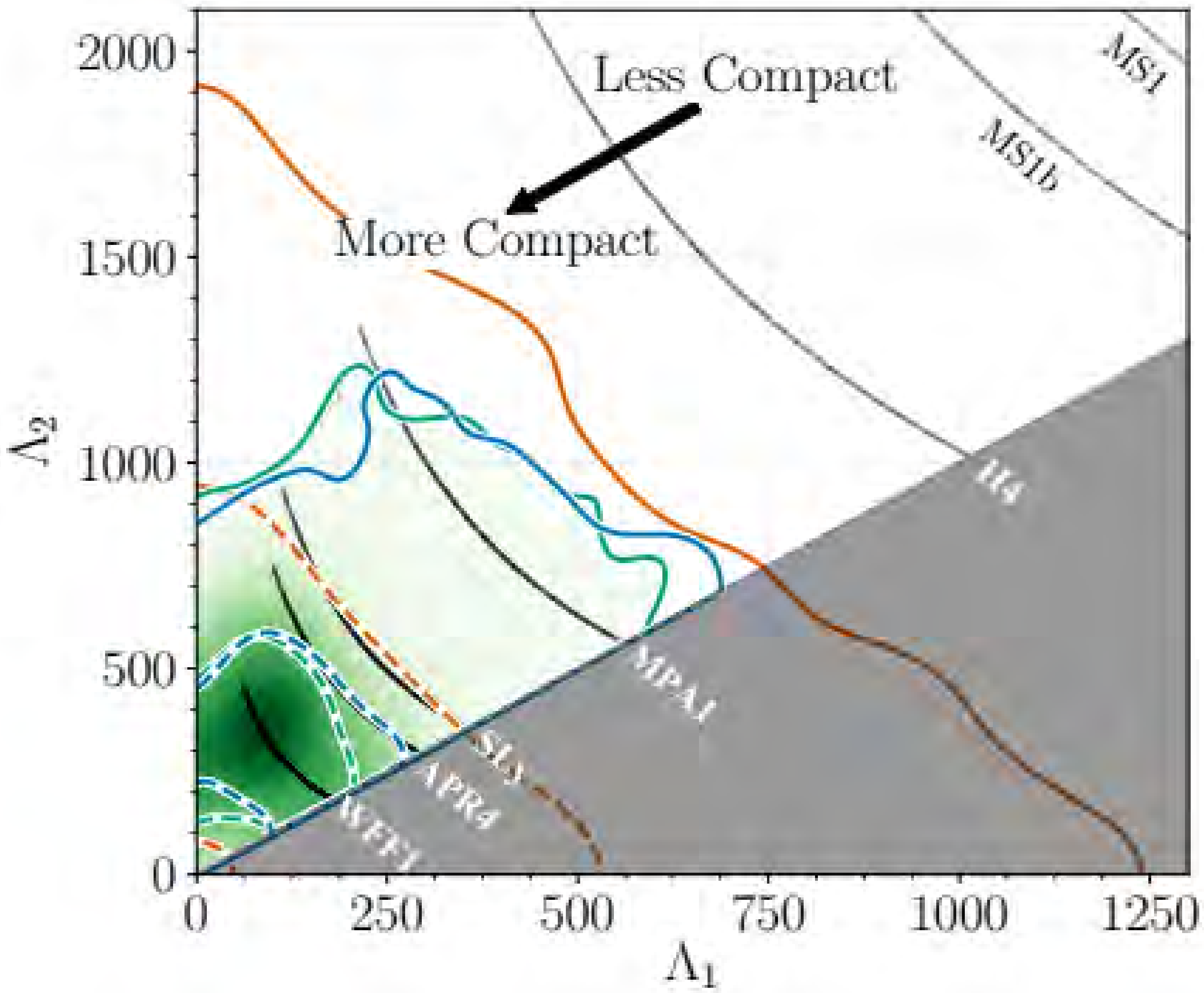
And additional eqs. for perturbations:

$$\begin{aligned} \frac{dH}{dr} &= \beta \qquad (11) \\ \frac{d\beta}{dr} &= 2\left(1-2\frac{m_r}{r}\right)^{-1} H\left\{-2\pi\left[5\epsilon+9p+f(\epsilon+p)\right]\right. \\ &\quad \left.+\frac{3}{r^2}+2\left(1-2\frac{m_r}{r}\right)^{-1}\left(\frac{m_r}{r^2}+4\pi rp\right)^2\right\} \\ &\quad +\frac{2\beta}{r}\left(1-2\frac{m_r}{r}\right)^{-1}\left\{-1+\frac{m_r}{r}+2\pi r^2(\epsilon-p)\right\}. \end{aligned}$$

(K(r) given by H(r))

Note: Although multidimensional problem – computation in 1D since absorbed in Y20

Tidal Deformability



$$y = \frac{R\,\beta(R)}{H(R)}$$

$$\begin{aligned} k_2 \;=\; & \frac{8C^5}{5}(1-2C)^2[2+2C(y-1)-y] \\ & \times\Big\{2C[6-3y+3C(5y-8)] \\ & \quad +4C^3[13-11y+C(3y-2)+2C^2(1+y)] \\ & \quad +3(1-2C)^2[2-y+2C(y-1)]\ln(1-2C)\Big\}^{-1} \end{aligned}$$

where $C = M/R$ is the compactness of the star.

$$\Lambda \equiv \frac{2}{3}k_2\left(\frac{R}{M}\right)^5$$

LIGO collab. arXiv:1805.11581 (2018)

Bayesian Inference

Bayesian analysis is a statistical paradigm that shows the most expected hypotheses using probability statements and current knowledge.

One of the most frequent case is analysis of probable values of model parameters.

Bayes' theorem:

Likelihood Prior

$$p(H_1 \mid D, I) = \frac{p(D \mid H_1, I)\, p(H_1 \mid I)}{p(D \mid I)}$$

Posterior Evidence

Prior: knowledge before experiment (logically)

Likelihood: Probability for data if the hypothesis was true

Posterior: Probability that the hypothesis is true given the data

Evidence: normalization; important for model comparison

Generally, maximum likelihood (parameters which maximize the probability for data) **does not** give the most likely parameters!!!

Bayesian Inference for NS

Formulation of set of models (set of hypothesis):
π_i here $i = 0..N-1$

↓

Finding the *a priori* probabilities of the models:
$$P(\pi_i) = 1/N \quad \text{for} \quad \forall i = 0..N-1$$

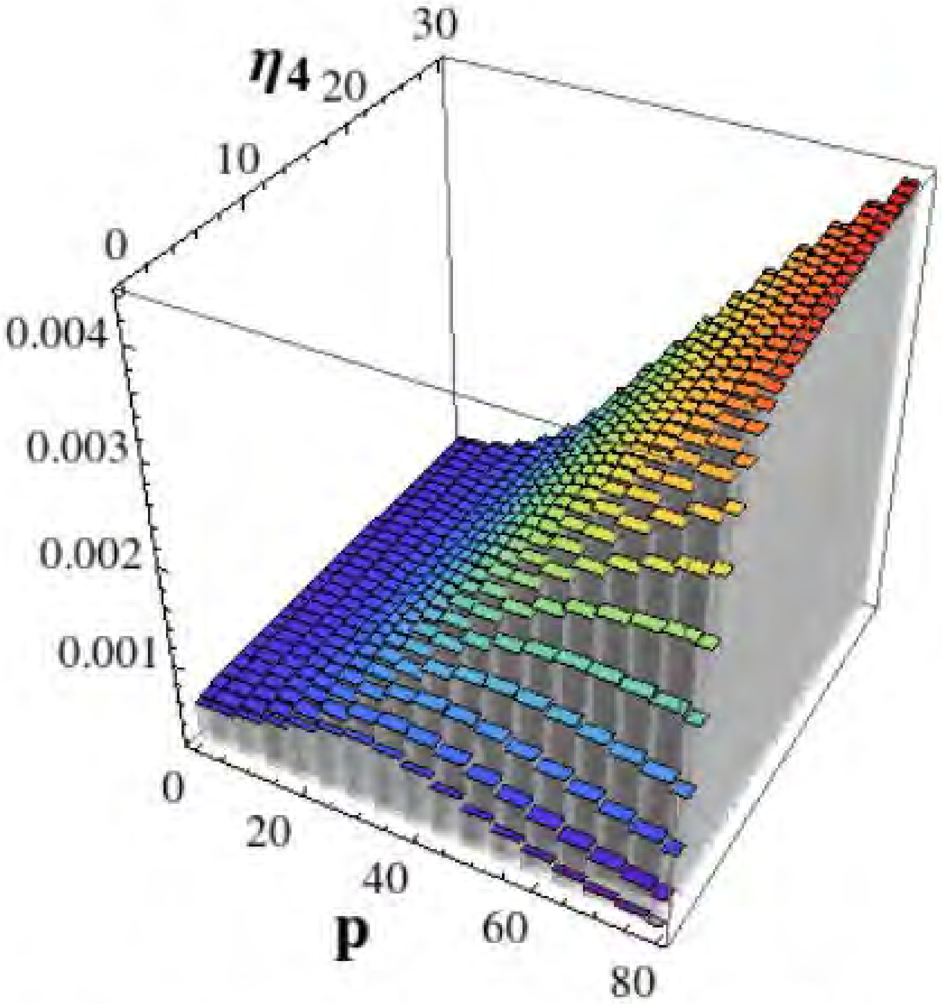
↓

Calculating the coditional probabilities of the events:
$$P(E\,|\overrightarrow{\pi}_i) = \prod_\alpha P(E_\alpha\,|\overrightarrow{\pi}_i),$$
where α is the index of the observational constraints.

↓

Calculating the *a posteriori* probabilities of the models:
$$P(\overrightarrow{\pi}_i\,|E) = \frac{P(E|\overrightarrow{\pi}_i)P(\overrightarrow{\pi}_i)}{\sum_{j=0}^{N-1} P(E|\overrightarrow{\pi}_j)P(\overrightarrow{\pi}_j)}$$

Example of Bayesian Inference Result



Alvarez, Ayriyan, Benic, Blaschke, Grigorian, Typel, EPJ A 52 (2016) 69

Possible Implementation of Machine Learning

- ✓Relation of Description of the stellar matter with Mechanical characteristics of NS
- ✓Integration of multidimensional integrals in modeling of EoS of super-dense matter
- ✓Comparison with observational data (Bayesian inference)

EoS – MR relation

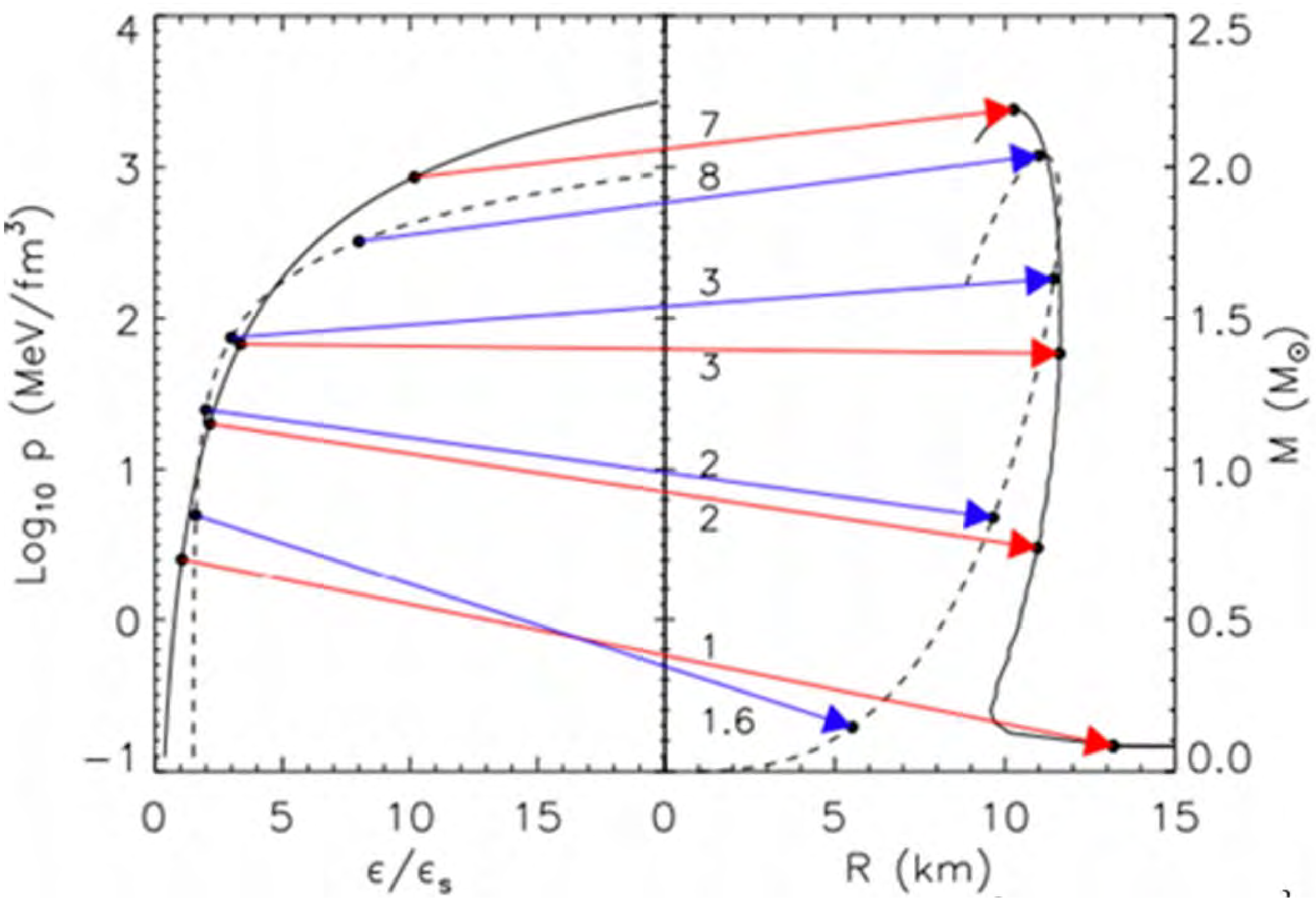


Image to Image neural Network:

GAN?

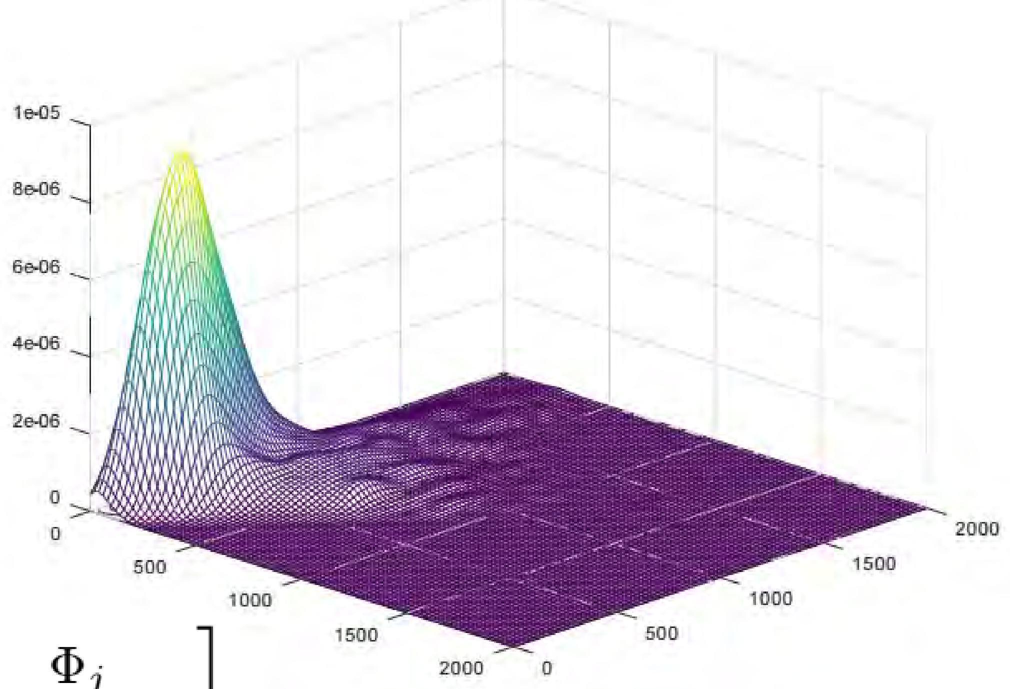
U-net?

Another one?

Multidimensional Integration

$$I[f] = \int_S f(\boldsymbol{x}) d\boldsymbol{x}$$

$$\widehat{f}(x) = b_2 + \boldsymbol{W}_2^T \sigma(\boldsymbol{b}_1 + \boldsymbol{W}_1 \boldsymbol{x}) = b^{(2)} + \sum_{j=1}^{k} w_j^{(2)} \sigma\left(b_j^{(1)} + \sum_{i=1}^{n} w_{ij}^{(1)} x_i\right)$$



$$\widehat{I}(f, \boldsymbol{\alpha}, \boldsymbol{\beta}) = I[\widehat{f}] = b_2 \prod_{i=1}^{n} (\beta_i - \alpha_i) + \sum_{j=1}^{k} w_j^{(2)} \left[\prod_{i=1}^{n} (\beta_i - \alpha_i) + \frac{\Phi_j}{\prod_{i=1}^{n} w_{ij}^{(1)}} \right]$$

$$\Phi_j = \sum_{r=1}^{2^n} \xi_r \mathrm{Li}_n \left(-\exp\left[-b_j^{(1)} - \sum_{i=1}^{n} w_{ij}^{(1)} \ell_{i,r} \right] \right).$$

Comparison with Observational Data

Bayes' theorem:

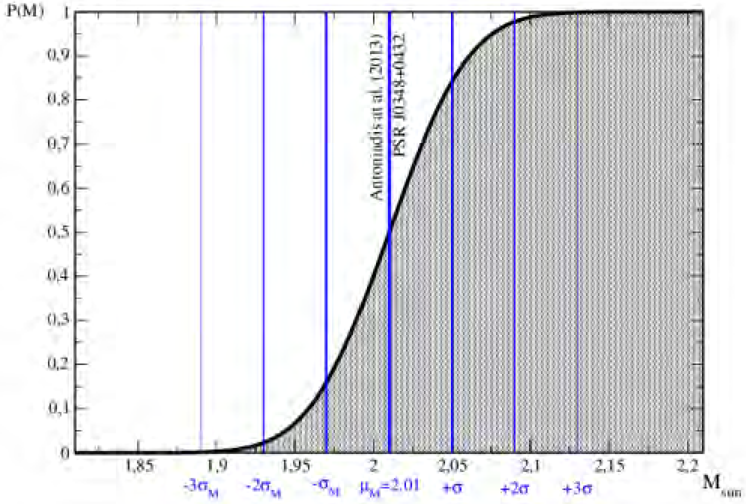
Likelihood   Prior

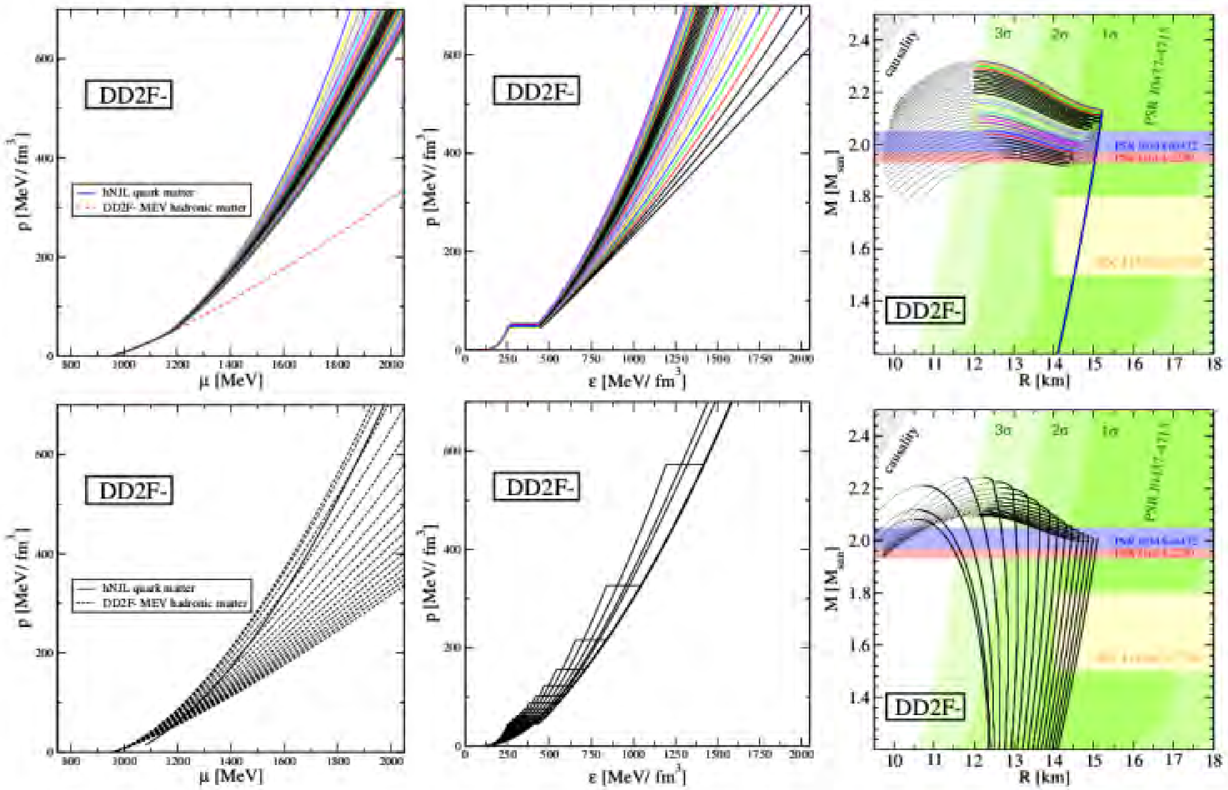
$$p(H_1 \mid D, I) = \frac{p(D \mid H_1, I)\, p(H_1 \mid I)}{p(D \mid I)}$$

Posterior   Evidence

Likelihood for Mass Constraint for given π_i

$P(E_A|\pi_i) = \Phi(M_i, \mu_A, \sigma_A)$, here M_i is max mass given by π_i.
$\mu_A = 2.01\ \mathrm{M}_\odot$ and $\sigma_A = 0.04\ \mathrm{M}_\odot$ [Antoniadis *et al.*, Science **340**, 6131 (2013)].





Thank you for your attention!

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